A coarse Fresnel zone plate (FZP) may be used as a large-area coded aperture for imaging incoherent sources. The longitudinal and lateral spatial resolutions are discussed on the basis of experiment using an FZP having 18 or 19 zones as an aperture. In this paper, it is then shown that the lateral spatial resolution is about 5 mm and the longitudinal is about 1 cm, respectively and it is appropriate to use the longitudinal resolution factor 3.7 as a guide to know the tomographic effect.

These results may be extended for imaging incoherent gamma-ray sources. According to our experiments, radioactivity of several tens of a millicurie is necessary for imaging with $^{99m}$Tc, when using X-ray film as a detector. Therefore, it is important to improve the detector in order to apply to the human organ imaging generally.

1. Introduction

Mertz and Young developed an X-ray star camera in 1961. In this camera, a Fresnel zone plate (FZP) was used in place of the camera lens. In 1965 to 1966, Einighammer experimentally verified the theoretical lateral resolution of Mertz-Young process for X-ray star imaging by Fresnel transform. Barrett, in 1972, suggested that the image of a gamma-ray source could be reconstructed from a shadow hologram of the said source recorded by Fresnel transform. Afterwards, gamma-ray imaging was rapidly advanced by Barrett, Rogers, Tipton, et al. However, no sufficient studies on the lateral spatial resolution, longitudinal spatial resolution (tomographic effect) and reconstructed images have been made yet. We, therefore, have studied the relationship between these resolutions using diffuse white light sources for simplicity and a coarse FZP having 18 or 19 zones and the reconstructed images.

2. Resolution Factors of Fresnel Zone Plate

The amplitude of an FZP derives from alternate transparent and opaque concentric annuli except that the first or central zone is a circle. The radius, $r_n$ of each annulus or circle is given by

$$r_n = r_1 \sqrt{n}, \quad n = 1, 2, 3, \ldots n. \quad (1)$$

The radius of the first zone is $r_1$. If the first zone is transparent, the FZP is positive. If it is opaque, the FZP is negative. When a plane wave of wavelength $\lambda$ is directed perpendicularly towards a positive or negative FZP as shown in Fig. 1, the intensity distribution, $I(z)$, on the principal axis (z-axis) is given by

$$I(z) = \sin^2 \left( \frac{r_n (\pi p \lambda)}{2 \sqrt{r_1^2 - z^2}} \right) = \sin^2 x. \quad (2)$$

In Eq. (2), $z$ when $x = 0$ gives the focal length of the FZP, i.e.,

$$f = r_1^2 / (\lambda p), \quad p = \pm 1. \quad (3)$$

This means it has a real and a virtual focus. The depth of focus, $\delta z$, is given as follows by letting $x = \varepsilon$ when $\sin^2 x = \varepsilon$ (0 < $\varepsilon$ < 1) in Eq. (2):

$$\delta z = \pm \alpha_n [f / (2r_n)]^{3/2}, \quad (4)$$

where
The diameter of the focal spot, $\delta_n$, on the Rayleigh criterion, is given by

$$\delta_n = \beta_n \lambda / \left(2\pi \Sigma \right), \tag{6}$$

where $\beta_n$ is a function of the number of zones $n$ and depends on whether the FZP is positive or negative$^{10,11}$. Since there are no general terms for $\alpha_n$ and $\beta_n$, they are called a "lateral resolution factor" and a "longitudinal resolution factor" respectively in this paper for convenience.

3. Coding, Decoding and Spatial Resolution

Figure 2 shows a diagramatic explanation of the coding system. This system is defined by reference to three parallel planes spaced at distances $u$, $v$, the last plane being taken as the plane of observation (recording plane). The planes are characterized by coordinates $r'$, $r''$, $r$ and the function $i(r')$, $g^x(r'')$. The template having an F-shaped opening is irradiated by diffuse white light from behind. Since an object can be considered as an assembly of numerous point sources, the shadow (shadow hologram) cast by an FPZ intercepting radiation from the object is an overlap of numerous FZP-like patterns (see Figs. 5 and 7) and contains information of the object. For the reason above, the coded image, $S^x(r)$, is basically a convolution of the aperture transparency, $g^x(r'')$, with the object distribution, $i(r')$. That is

$$S^x(r) = i(r') \otimes g^x(r'') = \pm \frac{1}{\pi j} \int \left( \frac{-u}{v} r \right)^{2} \exp \left\{ j \frac{x p}{r^2} \right\} \left[ \left( \frac{-u}{u+v} \right)^{2} r^2 \right] + c.c. + D.C. \ \text{term}, \tag{7}$$

where the symbol $\otimes$ denotes a convolution,

$$\alpha_n = 4/(\pi/(2\pi) - 1/n). \tag{5}$$

By similar calculation, the longitudinal spatial resolution is given as$^{6}$

$$dz = \frac{\alpha_n}{8n} u \left(1 + \frac{u}{v}\right) \tag{9}$$

The same results with Eqs. (8) and (9) are obtained even if the reduction rate of the shadow hologram is taken into account in the above calculation.

4. Experiment and Result

The coding system used is shown in Fig. 2. Templates were used as objects, masking a diffuse white light source and letting a portion of light
pass through their openings. A positive FZP of diameter $2r_{19} = 8.7 \text{ cm}$ and first zone radius $r_1 = 1.0 \text{ cm}$ and a negative FZP $2r_{14} = 8.5 \text{ cm}$ and $r_1 = 1.0 \text{ cm}$ were used for coding. Photographic paper was placed on the recording plane, then appropriately exposed and developed. From the prints obtained in this way, reduced-size holograms of about 1 cm in diameter were prepared and they were used in the decoding system as shown in Fig. 3 to reconstruct the image. Since the image reconstructed on the plane of $p = -1$ was very small, a magnifying lens was used to obtain the enlarged image on the screen for observation or photographing. Figure 4 shows $d$ (in mm) vs. $u$ (in cm) curves calculated using Eq. (8) for the cases of FZPs of $n=18$ and 19. For both cases the lateral resolution factor, $\beta = 1.2$ is used because of $\beta_{18} = 1.16$ and $\beta_{19} = 1.25^{(9,11)}$.

The lateral spatial resolution test using the positive FZP is shown in Fig. 5. Figure 5 (A) shows a template used as a two point source, the diameter of each point being 6.0 mm. Figure 5 (B) to (F) are shadow holograms and reconstructed images at $v = 5.0 \text{ cm}$ and $u$ varied.
grams and reconstructed images of this two point source. The lateral spatial resolution values, \( d \) as captioned are obtained from Fig. 4 for \( v = 5.0 \) cm and \( u = 5.0, 10, 15, 20, 25 \) cm. The \( S \) values are ratios of the two-point sources' center-to-center distance (8.5 mm) to the respective \( d \) values. The \( S' \) values are ratios of the minimum distance between the two point sources (5.5 mm) to the respective \( d \) values.

From Fig. 5, it is seen that:

1. Greater the distance \( u \), the hologram is clearer and so is the reconstructed image.
2. Each reconstructed image in \( (F) (S=1.0) \) and \( (D) (S'=0.98=1.0) \) represents the limit that the two point sources are at least distinguishable. This suggests a good agreement of the theoretical calculation with the experimental result with respect to the lateral spatial resolution (Rayleigh's criterion for resolution).

Equation (9) gives an indication of the tomographic effect (longitudinal spatial reso-
lution). In Eqs. (4) and (5), where the half-value width \( \xi = 0.5 \) considered, \( \alpha_{18} = 3.74 \), \( \alpha_{19} = 3.73 \) and \( \alpha = 3.56 \) if \( \eta \to \infty \). For the 80% half-value width \( \xi = 0.8 \), this will be \( \alpha = 2.1 \). Born and Wolf\(^{12}\) use \( \alpha = 2 \) for the lens and Barrett, et al.\(^4\) suggest \( \alpha = 3 \) for the FZP. However, as shown by the experimental results given in this section, it is recommended to use \( \alpha = 3.6 \) to 3.7 in FZP imaging. Figure 6 shows \( dz \) (in mm) vs. \( u \) (in cm) curves where \( n = 18 \) or 19 and \( v = 5.0 \) and 25 cm.

The longitudinal spatial resolution test using the negative FZP is shown in Fig. 7. In this case, the reconstructed image is dark ones. The reason for this, as understood from the nature of the negative FZP, is because the D.C. light (undiffracted light) and the image light are \( \pi \)-out of phase and resulting destructive interference produces a dark image. To observe the tomographic effect, three templates, each containing a 5.0mm-thick punched letter \( "F" \), \( "Z" \) and \( "P" \), were used; these punched letters are shown in Fig. 7(A). Figure 7 (B) shows a shadow hologram of three objects \( "F" \), \( "Z" \) and \( "P" \), which was obtained by triple exposures on a photographic paper with \( v = 5.0 \) cm and placing one of the objects each time at the corresponding distance \( u \) from the negative FZP as listed in Table 1. The distance \( u \) for each object was so selected that the relative distance between two adjacent objects would not result in overlapping of the theoretical half-value widths of their images reconstructed; the relative arrangement of the three is given in Fig. 8. Figure 7 (C) is a reconstructed image from the hologram (B) with \( "F" \) in focus in which a weak image hardly recognizable as \( "Z" \) is also seen at the position of \( "Z" \), but none is present at the position of \( "P" \); (D) is that when \( "Z" \) is in focus in which something vague is recognizable at the position of each of \( "F" \) and \( "P" \); (E) is that when \( "P" \) is in focus in which something is present at the position of \( "Z" \), but almost none at the position of \( "F" \).

5. Conclusions

From many experimental results, of which some are discussed above, the authors conclude that:

(1) The lateral resolution factor, \( \beta = 1.2 \) agrees well with the experimental results and the lateral spatial resolution is about 5 mm.

(2) It is appropriate to use the half-value width with the longitudinal resolution factor \( \alpha = 3.7 \) as a guide to know the tomographic effect.

These results may be extended for imaging incoherent gamma-ray sources. According to our experiments, radioactivity of several tens of a millicurie is necessary for imaging with \( ^{99m}Tc \), when using X-ray film as a detector. Therefore, it is important to improve the detector in order to apply to the human organ imaging generally.

Acknowledgement

The authors wish to thank Professor Kin-ichi Hisada for encouragement throughout this work.

References


2) H.J. Einighammer: Optik, 23, Heft 7, 627 (1965/66)

5) W.L. Rogers, K.S. Han, L.W. Jones and W.H. Beierwaltes: *J. Nucl. Med.*, 13, 612 (1972)
6) W.L. Rogers, L.W. Jones and W.H. Beierwaltes: *Optical Engineering*, 12, 13 (1973)

要　旨

フレネル ゾーン ブレートによる像形成の分解能

板屋源清, 小島一彦
金沢大学医療技術短期大学部
920 金沢市小立野5-11-80

インコーヒーレントな光源あるいは放射線源での像形成において、フレネル ゾーン ブレート (FZP) が、大きな物体を符号化するための開口として使用できる。本報では、ゾーン数18または19のFZPを使って、縦と横方向の分解能について検討した。横方向の空間分解能は約 5mm で、縦方向の空間分解能は約1cmであった。断層効果を知る目安として、この像形成系では、縦方向の分解能係数 3.7が適当と考えられた。

これらの結果はインコーヒーレントな7線線源の像形成に拡張できる。99mTc での像形成では、検出器としてX線フィルムを使用するため、数10mCi の放射能が必要である。それゆえ、広く人間の臓器の像形成に応用するためには、検出器を改良することが重要である。