1. INTRODUCTION

Constitutive equations are often checked by comparing their predictions with experimental data obtained in simple shear and/or elongational flows.1 This is rather surprising realizing the fact that most flows of practical interest are complex in nature and involve both shear and elongational deformations. This means that the information gathered by fitting the predictions of constitutive equations to simple flows may not be adequate to judge the performance of any given constitutive equation. Thus, use should preferably be made of complex flows in order to test the efficiency of any proposed constitutive equation. Among the variety of complex (or non-viscometric) flows available, those for which the boundary conditions can be defined unambiguously, and, at the same time, provide ease of experimental measurements (e.g., flow visualizations) are obviously more appropriate. Confined swirling flow (i.e., flow in an enclosed cylindrical vessel with a rotating lid, as shown in Fig. 1), provides a good test case for checking the performance of constitutive equations and/or CFD codes. This is perhaps why the flow has been given so much attention in the past, in both experimental and theoretical domains alike.2-17)

Figure 1 shows a schematic of the flow geometry. It is seen to consist of a fixed circular cylinder of radius R and height H with the fluid enclosed between two upper and lower lids. The swirling flow is set in motion due to the no-slip condition by the rotation of the lower lid (at a constant angular velocity $\Omega$) while the upper lid is held stationary. In spite of its geometrical simplicity, the flow exhibits a variety of complicated phenomena. Hill et al.17 showed that the fluid motion consists of a tangential primary flow and a weak superimposed toroidal secondary flow whose direction of rotation depends upon the elastic properties of the fluid. That is, while Newtonian fluids are driven outwards along the lid by the secondary flow and expelled up the side walls then followed by moving down the central axis to maintain continuity, for viscoelastic fluids the direction of this toroidal fluid movement was found to be completely the opposite. The secondary flow itself has been

Confined Swirling Flows of Simplified Phan-Thien-Tanner (SPTT) Fluids: a Numerical Study

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Confined swirling flow of polymer solutions is investigated numerically using the finite volume method combined with a collocated mesh. The Simplified Phan-Thien-Tanner (SPTT) rheological model will be used as the constitutive equation of the fluid. It is assumed that the flow is steady, laminar, and axisymmetric. The effect of different parameters such as fluid’s elasticity, mobility factor, retardation ratio, and the channel aspect ratio will be investigated on the velocity profiles and vortex structure within the cylinder and above the rotating disk. Numerical results are shown to be in good qualitative agreement with experimental data available for certain polymer solutions.

Key Words: Confined swirling flow / Finite volume method / Simplified phan-thien-tanner model
shown\(^{1,4}\) to break down into a multiple of double or triple recirculation bubbles depending on the channel aspect ratio, the Reynolds number, and the Weissenberg number. Escudier and Cullen\(^9\) showed that the size and shape of these vortices are affected by the degree of the shear-thinning behavior of the fluid. In a series of experiments conducted on Boger fluids (i.e., highly elastic fluids with nearly constant shear viscosity) Stokes et al\(^{10-12}\) demonstrated that fluid’s elasticity plays a more important role in dictating the occurrence of vortex breakdown and also the flow reversal phenomena.

Parallel to experimental investigations, much study has been carried out in the past in the theoretical domain to better understand swirling flows of viscoelastic fluids. Due to the complicated nature of swirling flows, most of these studies have relied on numerical techniques for simulating the flow.\(^{13-16}\) A variety of simple rheological models have been used in these studies. One can mention, for example, the use of second-order model, Walters’ B model, Criminale-Ericksen-Filbey (CEF) model, and Wagner model in these studies.\(^{13-16}\) These viscoelastic fluid models were able to predict the reversal in the direction of the secondary flow provided that the Weissenberg number was sufficiently small. But predicting some other aspects of Hill’s observations (e.g., the vortex breakdown phenomenon) was found to be a real challenge for these models, particularly at high Weissenberg numbers or in channels with large aspect ratio. For better agreement with experimental observations (particularly at high Weissenberg numbers) use should preferably be made of more robust rheological models such as Gieskus model, Phan-Thien Tanner model, Larson model, Leonov model, and the like.\(^9\) Indeed, in a recent numerical work, Itoh et al\(^9\) showed that results obtained using Gieskus model are in much better agreement with Hill’s experimental data, both qualitatively and quantitatively. They also demonstrated the strong influence of the relaxation time on the vortex structure.

In another interesting work in this area, Xue et al\(^{17}\) showed that the Phan-Thien Tanner (PTT) model is well capable of predicting the formation of double-cell vortex structures observed by Esudier and Cullen\(^9\) in swirling flow of certain polymer solutions. Surprisingly, however, none of these works addressed the effects of important parameters such as Weissenberg number, mobility factor, aspect ratio, and the retardation time on the velocity profiles or vortex structures within the enclosure. In the present work, we are going to investigate, to the best of our knowledge for the first time, the effects of the afore-mentioned parameters on the flow characteristics of PTT fluids in confined swirling flows. For ease of analysis, and also to be able to compare our numerical results with available experimental data, only the case of steady, laminar, axisymmetric swirling flow will be investigated in the present work.

### 2. MATHEMATICAL FORMULATION

The equations governing the flow of any incompressible fluid, whether Newtonian or non-Newtonian, can be written as:

\[
\nabla \mathbf{u} = 0
\]

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\sigma}
\]

where \(\mathbf{\sigma}\) is the deviatoric stress tensor. In relation to aqueous polymer solutions, this stress tensor can be splitted into a Newtonian part due to the solvent molecules and a polymeric part due to the polymer molecules contribution, that is:

\[
\mathbf{\sigma} = \eta_s \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) + \mathbf{\tau}
\]

where \(\eta_s\) is the viscosity of the base Newtonian solvent. In the present work, the polymeric part of the stress tensor is assumed to obey the Simplified Phan-Thien Tanner model\(^{1,10}\), that is:

\[
\frac{\partial (\hat{\lambda} \mathbf{u})}{\partial t} + \nabla \cdot (\hat{\lambda} \rho \mathbf{u} \mathbf{u}) = \eta_p \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)
\]

\[
+ \hat{\lambda} \left[ (\nabla \mathbf{u})^T \cdot \mathbf{\tau} + \mathbf{\tau} \cdot \nabla \mathbf{u} \right] - \left( 1 + \frac{\hat{\lambda} \lambda}{\eta_p} \right) \mathbf{\tau}
\]

where \(\hat{\lambda}\) is the relaxation time, \(\hat{\epsilon}\) is the mobility factor, and \(\eta_p\) is the viscosity contributed by the polymer chains at zero shear rate. It is worth mentioning that the Maxwell model can be obtained from the SPTT model by simply setting \(\hat{\epsilon} = 0\). For polymer solutions, the viscosity at zero shear-rate, \(\eta_p\), can be written as the sum of \(\eta_p = \eta_n + \eta_m\) where \(\eta_n\) is the viscosity contributed by the Newtonian solvent.

At this stage, and in order to improve numerical stability, we have decided to add the diffusive term \(-\eta_p \nabla^2 \mathbf{u}\) to both sides of the momentum equation (Eq. 2). The momentum equation then becomes:

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) - \eta_p \nabla^2 \mathbf{u} = -\nabla p + \nabla \cdot \mathbf{\sigma} - \eta_p \nabla^2 \mathbf{u}
\]

Before proceeding any further, we have to non-dimensionalize the governing equations. To that end, we introduce \(R\) and \(l/\Omega\) as the characteristic length and time. The velocity, stress, and pressure can then be scaled by \(R\Omega\), \(\eta_p\Omega\), and \(\rho(R\Omega)^2\), respectively. The dimensionless forms of the governing equations are:
\[ \nabla \cdot \mathbf{u} = 0 \quad (6) \]

\[ \frac{\partial (\mathbf{u})}{\partial t} + \nabla (\mathbf{uu}) - \frac{1}{Re} \nabla^2 \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla \sigma - \frac{\beta}{Re} \nabla^2 \mathbf{u} \quad (7) \]

\[ \frac{\partial (We\mathbf{u})}{\partial t} + \nabla (We\mathbf{u}\mathbf{u}) = \beta \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \\
+ We \left[ (\nabla \mathbf{u})^T \cdot \mathbf{r} + \mathbf{r} \cdot \nabla \mathbf{u} \right] - \left( 1 + \frac{\epsilon We}{\beta} tr(\mathbf{r}) \right) \mathbf{r} \quad (8) \]

where \( \mathbf{u} \) is the velocity vector, and \( \beta \) is the retardation factor defined by

\[ \beta = \frac{\eta_p}{\eta_o} \quad (9) \]

where \( \beta = 0 \) refers to the Newtonian solvent itself. In Eq. 8, \( Re \) and \( We \) are the Reynolds number and the Weissenberg number. They are defined, respectively, by:

\[ Re = \frac{\rho R^2 \Omega}{\eta_o} \quad (10) \]

\[ We = \lambda \Omega \quad (11) \]

### 3. NUMERICAL METHOD

The conditions of incompressibility together with the assumption of the flow being steady and axisymmetric enables the governing equations to be recast in the following generic form:

\[ \nabla \cdot (\zeta \mathbf{r} \mathbf{u} \phi) - \nabla (\gamma \nabla \phi) = S_\phi \quad (12) \]

where \( \phi \) represents any physical variable, and \( \zeta \) and \( \gamma \) are coefficients depending on the equation governing this particular variable (see Table I).

It is to be noted that the operator \( \nabla \) appearing in Eq. 12 is a gradient operator defined in the Cartesian coordinate system by:

\[ \nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} \quad (13) \]

where the coordinates \( x \) and \( y \) correspond to the coordinates \( r \) and \( z \) in our particular flow geometry. The term \( S_\phi \) at the right-hand-side of Eq. 12 is a source term; it is deemed to include any extra term which could not be categorized either as a convective term or as a diffusive term.

In the finite-volume formulation utilized in the present work, the conservative equations are first volume-integrated and then the Gauss’ theorem is used to convert them into surface integrals. For example, the volume integral of the convective terms can be written as:

\[ \int \nabla \cdot (\zeta \mathbf{r} \mathbf{u} \phi) dV = \int \zeta \mathbf{r} \mathbf{u} \phi \cdot \mathbf{n} dA \]

\[ = F_\phi \phi - F_\phi \phi_a + F_\phi \phi_s - F_\phi \phi_s \quad (14) \]

where the term \( F_\phi \) is the mass flux defined as:

\[ F_\phi = \zeta \mathbf{r} \mathbf{u} \phi_i, \quad i = e, w, n, s \quad (15) \]

where the indices \( e, w, n, \) and \( s \) refer to east, west, north, and south faces of an elemental volume. The notation \( \mathbf{u}_i \) refers to the notion that the velocity term in the mass flux equation are to be computed using the Rhie and Chow’s interpolation method. The quantity \( A_i \) is the area of the cell face normal to the velocity component \( \mathbf{u}_i \). The value of the dependent variable \( \phi \) are approximated at each face using a third-order interpolation method:

\[ \phi_i \equiv (\frac{x_{U} - x_p}{x_{U} - x_p} \frac{x_{U} - x_s}{x_{U} - x_s}) \phi_D + (\frac{x_{D} - x_p}{x_{D} - x_D} \frac{x_{D} - x_s}{x_{D} - x_s}) \phi_U \\
+ (\frac{x_{D} - x_p}{x_{D} - x_D} \frac{x_{U} - x_s}{x_{U} - x_s}) \phi_D + (\frac{x_{U} - x_p}{x_{U} - x_U} \frac{x_{U} - x_s}{x_{U} - x_s}) \phi_U \quad (16) \]

where \( \phi_i \) refers to the face \( i \) and \( \phi_D \) refers to the downstream face, \( D \) of \( D \) of the control volume, \( D \) and \( U \) are the upstream face and the downstream face, respectively (see Fig. 2). As shown in Fig. 2, the terms \( D \), \( U \), and \( DU \) are equivalent to \( E \), \( P \), and \( W \) when \( u_r \) is positive, and to \( EE \), \( E \), and \( W \) when \( u_r \) is negative. The third-order approximation as given by Eq. 16 can be recast in terms of three first-order approximations plus a correction term:

\[ \phi_i \equiv \phi_i + \Delta \phi_i \quad (17) \]

where \( \Delta \phi_i \) is the correction term which can be expressed as:

\[ \Delta \phi_i \equiv \frac{x_{U} - x_p}{x_{U} - x_D} \frac{x_{U} - x_s}{x_{U} - x_s} \phi_D \\
+ \frac{x_{D} - x_p}{x_{D} - x_D} \frac{x_{D} - x_s}{x_{D} - x_s} \phi_D \\
- \frac{x_{U} - x_p}{x_{U} - x_D} \frac{x_{U} - x_s}{x_{U} - x_s} \phi_U \\
+ \frac{x_{D} - x_p}{x_{D} - x_D} \frac{x_{D} - x_s}{x_{D} - x_s} \phi_U \quad (18) \]

<table>
<thead>
<tr>
<th>Pertinent Equation</th>
<th>( \phi )</th>
<th>( \zeta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Momentum</td>
<td>Velocity components ( u_x, u_y, u_z )</td>
<td>1</td>
<td>|Re</td>
</tr>
<tr>
<td>Constitutive</td>
<td>Stress components ( \tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz} )</td>
<td>We</td>
<td>0</td>
</tr>
</tbody>
</table>
This means that \( F_e \phi_e \), for example, can be discretized as follows:

\[
F_e \phi_e \equiv \max( F_e,0)(\phi_p + \Delta \phi_p) - \max(- F_e,0)(\phi_E + \Delta \phi_E) \\
\cong \max( F_e,0) \phi_p - \max(- F_e,0) \phi_E \\
+ \max( F_e,0) \Delta \phi_p - \max(- F_e,0) \Delta \phi_E
\]  

(19)

The most important feature of this first-order correction term is that it can be lumped into the source term with the price being that the coefficient matrix of the set of algebraic equations so obtained is now penta-diagonal. In the present work, the original high-order method will be used for simulations related to Newtonian and Maxwellian fluids. As to the SPTT fluid (particularly at high Weissenberg numbers) use will be made of the first-order approximation method just described. Central differencing will be employed to discretize all gradients in the diffusive terms. As to the volume integral of the diffusive terms, it can be written as:

\[
\int (\gamma r \nabla \phi) \ln dA = \left( \gamma r \frac{\partial \phi}{\partial r} A \right)_r - \left( \gamma r \frac{\partial \phi}{\partial r} A \right)_r + \left( \gamma r \frac{\partial \phi}{\partial r} A \right)_s \cong D_e (\phi_e - \phi_p) - D_s (\phi_s - \phi_p)
\]  

(20)

where we have:

\[
D_e = \gamma r_e \frac{A_e}{\Delta x_e} \Delta x_e = x_e - x_p; D_s = \gamma r_s \frac{A_s}{\Delta x_s} \Delta x_s = x_p - x_u
\]

\[
D_n = \gamma r_n \frac{A_n}{\Delta y_n} \Delta y_n = y_n - y_p; D_i = \gamma r_i \frac{A_i}{\Delta y_i} \Delta y_i = y_p - y_s
\]  

(21)

The procedure for discretizing the diffusive terms on the right-hand-side of the momentum equations is the same as other terms but they lag by an iteration level from those at the left-hand-side. To discretize stress divergence terms in the momentum equations, we need the stress value at all faces. These values are obtained by a special interpolation method proposed by Oliveira et al.\(^{(25)}\), which is itself based on the Rhie and Chow’s interpolation method.\(^{(29)}\) After some rearrangements, the discretized form of the volume integral in Eq. 12 is written as:

\[
a_p \phi_p - \Sigma a_{nb} \phi_{nb} = \bar{S}_p
\]  

(22)

where,

\[
a_{nb} = \max(- \text{sgn}(nb) F_{nb},0) + D_{nb}
\]

(23)

\[
a_p = \sum a_{nb} + \sum \text{sgn}(nb) F_{nb}
\]

(24)

\[
\text{sgn}(nb) = \begin{cases} 1 & \text{nb} = e, n \\ -1 & \text{nb} = w, s \\ \end{cases}
\]

\[Nb = E, W, N, S \text{ and } \text{nb} = e, w, n, s\]

(25)

In the present work, use will be made of the SIMPLE algorithm, developed by Patankar, to solve the constitutive equation as well as the momentum and continuity equations. To that end, at each iteration, the constitutive equation will be solved first. The updated values of the extra stress components will then be used in the stress divergence source terms in the momentum equations to find out the velocity components such that the continuity equation will be satisfied.\(^{(22)}\)

\section{RESULTS AND DISCUSSION}

The code developed in the present work\(^{(23)}\) had to be verified first before being applied to our fluid of interest, i.e., the SPTT fluid. To that end, we have decided to compare our numerical results with experimental data available in the literature. Figure 3 shows a comparison between our numerical results with experimental data known for Newtonian fluids at a typical Reynolds number of 1800 in a confined cylinder having a dimensionless radius of 0.6 and an aspect ratio of one.\(^{(26)}\) As can be seen in Fig. 3 our code works remarkably well as far as velocity profiles are concerned. Figure 4 presents our numerical results showing the effect of the Reynolds number on the vortex breakdown phenomenon for Newtonian fluids in a channel having an aspect ratio of 2.5. These figures are virtually the same as the numerical results reported recently by Lopez.\(^{(7)}\) Evidently, the code is well capable of dealing with swirling flows of Newtonian fluids.

To check the efficiency of the code developed in the present work in its dealing with viscoelastic fluids, we have decided to compare our numerical results for Maxwellian fluids with
those reported recently by Xue et al. Figure 5 shows the effect of the Weissenberg number on the vortex structure at a Reynolds number of 0.32 in a channel having an aspect ratio of one. The results presented in this figure are virtually the same as those given in Ref. 17. As can be seen in this figure, the vortex reversal phenomenon (a well-established effect for polymer solutions) can be predicted remarkably well by our code. That is, by an increase in the Weissenberg number, the “inertial” vortex retreats and is gradually replaced by the “elastic” vortex (as evident in the lower right corner in Fig. 5b). The latter vortex has a direction of rotation opposite to that of the “inertial” one. In fact, as can be inferred from Fig. 5h, at sufficiently large Weissenberg numbers the “inertial” vortex completely disappears.

Figure 6 shows the effect of the fluid’s elasticity on delaying the vortex breakdown phenomenon. The results obtained for swirling flows of UCM fluids at a Reynolds number of 1500, a Weissenberg number of 0.1, and in a channel with an aspect ratio of two. The suppressing effect of fluid’s elasticity on vortex breakdown is evident in this figure. This effect has previously been observed experimentally by stokes et al., and Xue et al. It is interesting to note that, the flow pattern within the cylinder is, generally speaking, almost the same for both Newtonian and viscoelastic fluids. This is not surprising realizing the fact the ratio of the Weissenberg number to the Reynolds number is less than 0.0001 so that elastic stresses are quite weak.

Having validated our code from different perspectives, we are now at a position to present our brand new numerical results obtained for swirling flow of SPTT fluids. An important feature of the SPTT model is that it correctly incorporates the shear-thinning effect as observed in dilute polymer solutions. This effect is represented by the mobility factor ε. Figures 7 and 8 show the effects of the Weissenberg number on the circumferential velocity component of SPTT fluids. It is interesting to note that, the flow pattern within the cylinder is, generally speaking, almost the same for both Newtonian and viscoelastic fluids. This is not surprising realizing the fact the ratio of the Weissenberg number to the Reynolds number is less than 0.0001 so that elastic stresses are quite weak.

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fluids at a Reynolds number of 50 and two different aspect ratios of 0.3 and 2. In our numerical simulations, as suggested by Escudier and Cullen, we have taken \( \varepsilon = 0.12 \) and \( \beta = 0.99 \) for the fluids under investigation. The strong effect of the Weissenberg number is evident in these figures. From Figs. 7 and 8 it can be inferred that the SPTT model does a remarkable job in predicting the experimental data of Itoh’s et al\(^7\) and this is particularly so the smaller the aspect ratio and/or the Weissenberg number. Based on the results presented in Fig. 8, the performance of SPTT model appears to be quite good, qualitatively. Quantitatively, however, the comparisons are not so great, particularly at large aspect ratios or high Weissenberg numbers. In our view, this is the best that can be expected from a single mode constitutive equation. For better quantitative agreement, particularly when dealing with the flow of polymeric fluids at high Weissenberg numbers, one should preferably rely on multi-mode constitutive equations so that the spectrum of the relaxation times can be taken into account. As a matter of fact, Xue et al\(^6\) have shown that the relaxation time has a strong influence on swirling flow of polymer solutions.

In Fig. 9 we have shown the effect of the Weissenberg number on the vortex structure above the rotating disk. As shown in this figure, by increasing the \( \text{We} \) number the elastic vortex grows in size and is expected to eventually overtake the inertial (Newtonian) vortex.

Figures 10 and 11 show the strong effect of aspect ratio on the vortex structure and circumferential velocity. Figure 10 suggests that the streamline separating elastic vortex from the inertial vortex shifts towards the rotating disk when the aspect ratio is increased. The aspect ratio is also predicted to have a weakening effect on inertia terms; that is, the circumferential velocity drops when the aspect ratio is increased (see Fig. 11). From these figures it can be concluded that the region over which elastic effects become more dominant is enlarged by an increase in the aspect ratio. This is not surprising realizing the fact that swirling flow is basically a shear flow. Therefore, a local reduction of the viscosity in regions of high shear (but with no or little evidence of elastic behavior) should boost the influence of inertia. These numerical predictions are
in line with some of Escudier et al. observations on CMC solutions. However, it should be conceded there are some other observations made by Escudier et al. which cannot be captured by a single-mode SPTT model using a two-dimensional analysis. For example, under certain conditions, Escudier et al. observed an upflowing wavy jet along the axis of rotation which simply cannot be predicted by the SPTT model. Obviously, further work is needed to predict such three-dimensional, chaotic behavior.

Figures 12 and 13 show the effect of the mobility factor \( \varepsilon \) on the vortex structure and circumferential velocity profiles, respectively. As can be seen in Fig. 12, the size of the elastic vortex formed on the upper right corner of the cylinder becomes smaller when the mobility factor is increased. This is not surprising realizing the fact that the mobility factor controls the degree of shear-thinning of the fluid; that is, shear-thinning starts at lower shear rates the higher the mobility factor (see Fig. 14). Therefore, by an increase in the mobility factor the size of the inertial vortex should increase, as can be seen in Fig. 12. The same argument can be used to explain the effect of the mobility factor on the circumferential velocity. As can be seen in Fig. 13, this velocity component decreases by an increase in the mobility factor such that the flow above the rotating lid exhibits characteristics of boundary layers. Interestingly, a comparison between Fig. 5 and Fig. 12 shows that the vortex structure is completely different between Maxwell and SPTT fluids. That is, for \( \varepsilon = 0 \) (i.e., for Maxwell fluids) the elastic vortex starts to form at the lower-left corner whereas for \( \varepsilon \neq 0 \) (i.e., for shear-thinning fluids) this vortex starts to form at the upper-right corner of the cylinder. This can again be attributed to a competition between inertia and elastic stresses with the inertia becoming more important the higher the mobility factor—an effect which can be attributed to the fact unlike Maxwell fluids, PTT fluids are shear-thinning with shear-thinning becoming more severe the higher the mobility factor.

Figures 15 and 16 show the effect of the retardation factor \( \beta \) on the vortex structure and circumferential velocity profiles at a typical Reynolds number of 50. A comparison between Figs. 12 and 13 with Figs. 15 and 16 suggest that, qualitatively, a decrease in \( \beta \) has an effect similar to increasing \( \varepsilon \) on the flow kinematics. This is not surprising realizing the fact that the solution viscosity is reduced when the retardation factor is decreased (see Fig. 17). The easiest way to reduce the retardation parameter, \( \beta \), is through the use of more viscous solvents. Another means of lowering \( \beta \) is obviously by reducing the polymer concentration. By so doing the shear-thinning effect of the solution is reduced while at the same time the overall viscosity of the solution, \( \eta_{s0} \), is increased. An increase in the solution viscosity means that to keep the

![Fig. 11](image1.png)  
**Fig. 11.** The effect of aspect ratio on circumferential velocity for SPTT fluids (\( \varepsilon = 0.12 \) and \( \beta = 0.99 \)) at a Reynolds number of 50 and a Weissenberg number of 4.8 at two different radii: a) \( r = 0.5 \), b) \( r = 0.8 \).

![Fig. 12](image2.png)  
**Fig. 12.** The effect of the mobility factor on the vortex pattern in confined swirling flows of SPTT fluids (\( \beta = 0.99 \)) at a Reynolds number of 50, a Weissenberg number of 5, and an aspect ratio of one: a) \( \varepsilon = 0.12 \), b) \( \varepsilon = 0.18 \), c) \( \varepsilon = 0.25 \), d) \( \varepsilon = 0.50 \).

![Fig. 13](image3.png)  
**Fig. 13.** The effect of the mobility factor on the circumferential velocity of SPTT fluids (\( \beta = 0.99 \)) at a Reynolds number of 50, an aspect ratio of one, and a Weissenberg number of 4.8: a) \( r = 0.5 \), b) \( r = 0.8 \).

![Fig. 14](image4.png)  
**Fig. 14.** Effect of the mobility factor on the shear viscosity (\( \beta = 0.99 \)).
Reynolds number constant one should increase the rotational speed. It is also to be noted that a drop in $\beta$ increases the contribution of the Newtonian viscosity by lowering the relative importance of elastic effects. That is to say that, inertial effects become more and more important when $\beta$ is decreased. Thus it is expected that the inertial vortex should eventually suppress the elastic vortex, as can be seen in Fig. 15. A similar argument can be used to explain the effect of $\beta$ on the circumferential velocity component. It is to be noted that for a given a decrease in $\beta$ is equivalent to a decrease in the concentration of the polymer in a given solution. This means that the total viscosity is increased when $\beta$ is increased (see Fig. 17) so that the solution viscosity will approach a constant value, the Newtonian value, when $\beta$ approaches zero.

Fig. 15. Effect of the retardation factor, $\beta$, on the vortex structure in confined swirling flows of SPTT fluids ($\varepsilon = 0.12$) at a Reynolds number of 50, a Weissenberg number of 5 and an aspect ratio of one: a) $\beta = 0.33$, b) $\beta = 0.66$, c) $\beta = 0.90$, d) $\beta = 0.99$.

Fig. 16. The effect of the retardation ratio on the circumferential velocity profiles of SPTT fluids ($\varepsilon = 0.12$) at a Reynolds number of 50, an aspect ratio of one, a Weissenberg number of 5 and an aspect ratio of one at two different radii: a) $r = 0.5$, b) $r = 0.8$.

5. CONCLUDING REMARKS

Based on the results obtained in the present work it can be concluded that the SPTT model can well capture many of the facets of the confined swirling flows of polymeric liquids, at least qualitatively. The results obtained in the present work suggest that like the relaxation time which has been addressed recently by Itoh et al [see Ref. 16], the elastic normal stresses (as represented by the Weissenberg number), the degree of the shear-thinning (as represented by the mobility factor), the polymer concentration (as represented by the retardation parameter), and also the channel aspect ratio all have a profound effect on the flow kinematics and vortex structure within the cylinder.

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