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Concept of an Evaluation Technique for Planar Elongational Stress and Relaxation Time Using Hoop Stress in Swirling Flow

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The paper reports a novel idea of an experimental approach to estimate the apparent planar elongational viscosity and the relaxation time of low viscosity fluids. A two-dimensional swirling contraction channel is designed to maintain a constant elongation rate to the flow direction. The hydrodynamic focusing creates a stable planar elongation flow field. The flow is injected into the center of the channel and a sheath flow is injected through both sides. The sample fluid is stretched in the flow field, and then forms the ribbon-shaped filament. In the swirling contraction part, a static force balance between the hoop force caused by the planar elongational stress and the centrifugal force determines the radial position of the filament. The distribution of the planar elongational stress can be estimated by observing this position, and then the apparent planar elongational viscosity was calculated using developed simplified expressions. On the other hand, when a channel with a constant width swirling part is used, the elongational stress generated in the straight contraction part gradually decreases passing through the constant width part. The sample filament position is then shifted based on the stress relaxation. The relaxation time can be evaluated by observing this variation.

Key Words: Swirling contraction flow / Hydrodynamic focusing / Planar elongational stress / Relaxation time / Rheometry

1. INTRODUCTION

The planar elongation flow is often seen in film casting¹⁻³⁾ and blow molding⁴⁾, and also applied to control of molecules or particles orientation⁵⁻⁸⁾. It is necessary for the industry to understand the property of the planar elongation flow. In order to analyze its flow field, it is required to clarify the planar elongational viscosity by an experiment. There are several flow cells which evaluate the planar elongational stress and viscosity for low viscous fluids, such as the 4-roll mill flow cell⁹⁻¹¹⁾, the two-dimensional opposing flow cell^{12, 13)}, and the two-dimensional contraction channel^{14, 15)}. Furthermore, the optimized shape cross-slot extensional rheometer (OSCER)¹⁶⁾, which is one of a micro-rheometry platform, is a good design to evaluate the planar elongation property in case of the transparent fluids with strong intrinsic birefringence. These methods used the birefringence technique^{17, 18)}. It can be performed without affecting the flow field and can obtain the stress distribution by a multipoint measurement, however, it is only effective for clear samples in which the stress optic rule has been established. Therefore, this evaluation technique does not have a broad utility. In

addition, a pure planar elongation flow occurs only on the stagnation line or the center of the flow channel in the previously described flow cells due to the wall shear effect.

As an alternative, we focus on the hydrodynamic focusing principle mainly used in microchannels¹⁹⁻²²⁾ to create a planar elongation flow field that is not affected on the wall shear. This flow field consists of a sample flow and a sheath flow. The sheath flow dominates the flow field, and it drives a sample fluid injected into the center of the sheath flow. The sample flow maintains very stable even in very low viscosity fluids, because the wall shear effect can be reduced due to the sheath flow. If the sheath flow can be controlled to induce a steady elongation flow, we can get a steady elongational filament of the sample flow. In this case, unfortunately, it is not easy to measure the elongational stress acting on the sample flow directly, because any force or pressure transducers cannot be installed into the sample flow. The measurement is limited by the non-contact methods.

Therefore, this study uses a ribbon-shaped filament position of the sample flow in the Two-Dimensional Swirling Contraction Channel, whose schematic diagram is shown in Fig. 1(a). If the shear stress and the surface tension acting on the boundary between the sheath and sample flows were negligible, only the elongational stress acts on the sample flow. This elongational stress induces a force to the radial direction

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due to the arc shape of the sample flow in Region II, called the hoop stress. On the other hand, the centrifugal force is also acting on the sample flow when the density of the sample fluid is not the same to the sheath fluid. The filament position of the sample flow is then shifted from the center line due to the balance of these forces. Therefore, the elongational stress acting on the filament would be evaluated measuring the filament position observed from the images of the flow visualization.

We also develop an evaluation method for the relaxation time of the planar elongational stress using a similar technique explained above. Although there is an interesting result that Giudice *et al.*²³⁾ measured relaxation times of dilute polymer solutions in the extensional flow in a microfluidic optimized cross-slot configuration based on the onset of the flow-induced birefringence, there are still few reports on the relaxation behavior in the extensional flow. In this case, the Two-Dimensional Swirling Parallel Channel, which has a constant width swirling part (Region II), is used. The schematic diagram is shown in Fig. 2(a). The elongational stress generated in the straight contraction part gradually disappears passing through the swirling part, and then the filament position of the sample fluid is shifted based on this stress distribution. Therefore, the stress relaxation behavior can be evaluated measuring the radial position of the filament.

In this study, we first introduce the concept of this new method using the swirling / hydrodynamic focusing design to evaluate the apparent planar elongational viscosity of low viscosity fluids. The theory and the assumptions introduced in the calculation are explained. The results of the trial tests are shown.

2. DESIGN OF FLOW CHANNELS WITH SWIRLING / HYDRODYNAMIC FOCUSING

2.1 Outline of Two-Dimensional Swirling Contraction Channel

Figures 1(a) and (b) show a schematic diagram of the Two-Dimensional Swirling Contraction Channel (2-DSCC) and one of the visualized photograph of the flow field using the swirling / hydrodynamic focusing design, respectively. The channel consists of three regions, which are named Region I, II and III along the flow direction. 2-DSCC has a constant depth of 5 mm with varying channel width. The bottom of Region I is the inlet of this channel, and it is connected to a flow straightener. In Region I and II, the channel width is reduced with a function which generates a constant elongation rate along the flow direction. The details of the channel

Fig. 1 (a) Dimensions of the Two-Dimensional Swirling Contraction Channel (2-DSCC) for evaluation of the planar elongational stress. 2-DSCC is designed to have $\dot{\epsilon} = 4 \text{ s}^{-1}$ when $\bar{v}_0 = 10 \text{ mm/s}$. The depth of the geometry is 5 mm. (b) A visualized photograph of the flow field using the hydrodynamic focusing in 2-DSCC. The sample fluid is PAA0.01 wt%.

width function will be described in section 2.1.1 and 2.1.2. The center line of the channel width is straight in Region I, and then becomes a half-circle in Region II. These two regions are connected smoothly, and a constant elongation rate acts on the elongated filament of the sample flow in both regions. The radial position of the ribbon-shaped filament of the sample flow is measured in Region II. Region III has a constant width of 1 mm. It is connected to the outlet.

2.1.1 Two-dimensional straight contraction part

The channel width of the two-dimensional straight contraction part (Region I as shown in Fig. 1(a)) was designed so that the planar elongation rate $\dot{\epsilon}$ becomes constant. The x -direction is defined as the flow direction. The equation of continuity between the cross-section area A_0 at the reference position and A_x at an arbitrary position x can be written as

$$\begin{aligned} A_0 \bar{v}_0 &= A_x \bar{v}_x \\ h w_0 \bar{v}_0 &= h w_x \bar{v}_x \end{aligned} \quad (1)$$

with $\bar{v}_0$ and $\bar{v}_x$ being the cross-sectional mean flow velocity at

the initial position $x = 0$ mm and an arbitrary position, respectively. w_0 and w_x denote the channel width. h is a constant value of the channel depth. The channel width w_x at an arbitrary position x becomes

$$w_x = \frac{w_0 \bar{v}_0}{\bar{v}_x} \quad (2)$$

The wall shear effect due to the narrow gap is ignored as the first step of this study. It is assumed that the cross-sectional average flow velocity $\bar{v}_x$ increases in proportion to the flow direction, because the planar elongation rate $\dot{\epsilon}$ is constant. $\dot{\epsilon}$ is written as Eq. (3).

$$\frac{\partial \bar{v}_x}{\partial x} = \dot{\epsilon} = \text{const.} \quad (3)$$

Eq. (3) is integrated over x . $\bar{v}_x$ is then denoted by Eq. (4).

$$\bar{v}_x = \dot{\epsilon}x + C \quad (4)$$

C denotes the integration constant. Since $x = 0$ is $\bar{v}_x = \bar{v}_0$, $\bar{v}_x$ is written as Eq. (5).

$$\bar{v}_x = \dot{\epsilon}x + \bar{v}_0 \quad (5)$$

Therefore, the channel width w_x of the two-dimensional straight contraction part is described by Eq. (6).

$$w_x = \frac{w_0 \bar{v}_0}{\dot{\epsilon}x + \bar{v}_0} = \frac{w_0}{\frac{\dot{\epsilon}}{\bar{v}_0}x + 1} \quad (6)$$

2.1.2 Two-dimensional swirling contraction part

The previously described straight contraction part is smoothly connected to the two-dimensional swirling contraction part, which has a constant channel center radius $R = 10$ mm. This part is Region II as shown in Fig. 1(a). The planar elongation rates of both regions are the same. The channel width at the swirling contraction part is described by Eq. (7).

$$w_\theta = \frac{w_1}{\frac{\dot{\epsilon}}{\bar{v}_1}R\theta + 1} \quad (7)$$

w_1 and $\bar{v}_1$ denote the channel width and the cross-sectional mean flow velocity at the entry of Region II. The radius of the inner and the outer wall can be expressed by Eqs. (8) and (9) using the polar coordinate system (r, θ) .

$$r_{inner} = R - \frac{w_\theta}{2} = R - \frac{1}{2} \frac{w_1}{\frac{\dot{\epsilon}}{\bar{v}_1}R\theta + 1} \quad (8)$$

$$r_{outer} = R + \frac{w_\theta}{2} = R + \frac{1}{2} \frac{w_1}{\frac{\dot{\epsilon}}{\bar{v}_1}R\theta + 1} \quad (9)$$

2.2 Outline of Two-Dimensional Swirling Parallel Channel

Figures 2(a) and (b) show a schematic diagram of a Two-Dimensional Swirling Parallel Channel (2-DSPC) and one of the visualized photograph of the flow field using the hydrodynamic focusing in this channel. 2-DSPC also has a constant depth is 5 mm. The two-dimensional straight contraction part is smoothly connected to the two-dimensional swirling parallel part with a constant width is 2.7 mm. These parts are called the Region I and II, respectively. The planar elongation stress generated in Region I gradually relaxes in Region II. In this case, the position of the ribbon-shaped filament of the sample fluid should move outward, because the hoop force becomes smaller than the centrifugal force. Therefore, we can evaluate the stress relaxation behavior by observing its radial position.

(a) Two-Dimensional Swirling Parallel Channel

(b) Visualized photograph of PAA0.01wt% in 2-DSPC

Fig. 2 (a) Dimensions of the Two-Dimensional Swirling Parallel Channel (2-DSPC) for evaluation of the relaxation time after steady planar elongation. 2-DSPC is designed to have $\dot{\epsilon} = 4 \text{ s}^{-1}$ when $\bar{v}_0 = 10 \text{ mm/s}$. The depth of the geometry is 5 mm. (b) A visualized photograph of the flow field using the hydrodynamic focusing in 2-DSPC. The sample fluid is PAA0.01 wt%.

2.3 Flow straightener

The flow straightener is installed the upstream part of the region I in order to guide the undisturbed ribbon-shaped filament of the sample fluid to 2-DSCC and 2-DSPC (see Fig. 3). In the flow straightener, the planar elongation rate increases in proportion in the initial part, and the cross-sectional average flow velocity $\bar{v}_x$ at an arbitrary position x is written as

$$\bar{v}_x = \frac{1}{2}ax^2 + \bar{v}_0 \quad (10)$$

with a being a constant of proportion. Therefore, the channel width w_x is shown as

$$w_x = \frac{w_0 \bar{v}_0}{\bar{v}_x} = \frac{w_0}{\frac{1}{2}ax^2 + 1} \quad (11)$$

The planar elongation rate $\dot{\epsilon}$ then becomes constant at $x > x_1$ in the middle part. The channel width w_x is written as

$$w_x = \frac{w_{x1}}{\frac{\dot{\epsilon}_{x1}}{\bar{v}_{x1}} + 1} \quad (12)$$

The channel width of the flow straightener finally becomes a constant that is 5 mm. We set $w_0 = 30$ mm, $a = 0.05$ (mm·s)⁻¹, $x_1 = 20$ mm and depth is 20 mm. The flow straightener is designed to have $\dot{\epsilon}_{x1} = 1$ s⁻¹ when $\bar{v}_0 = 5$ mm/s.

3. SAMPLES AND METHOD OF TRIAL TEST

3.1 Method of trial test

Figure 4 shows a schematic diagram of an experimental apparatus. The sheath fluid flows out of the tank due to the water head difference, and then its flow rate is adjusted by a needle valve attached to the tube. A syringe pump driven by a linear motor supplies a sample fluid. These fluids flow into the test channel after passing through the flow straightener. Figure 5 shows an experimental setup for the flow visualization in Region II. An optical microscope (CM-10L, NIKON) with a high-speed camera (FASTCAM-NEO, PHOTRON) and a telecentric lens is used to observe a ribbon-shaped filament of the sample fluid. A halogen lamp is used as a light source. The sample flow velocity is evaluated by the particle tracking velocimetry method (PTV). In this case, nylon particles are dispersed in the sample fluid at a concentration of 0.01 wt%. The average diameter of particles is 73 μm. The total flow rate, that is the sum of the sample and sheath flow, is calculated by the weight method using a load cell. The experimental condition is as follows. The sample flow rate

Fig. 3 Setup of a test channel and a flow straightener.

Fig. 4 Schematic diagram of the experimental apparatus. A syringe pump driven by a linear motor supplies a sample fluid. The sheath fluid flows out of the tank due to the water head difference.

Fig. 5 Schematic diagram of the experimental setup for the flow visualization in the swirling part of the test channel.

Q_{sample} was $71 \text{ mm}^3/\text{s}$, and the sheath flow rates Q_{sheath} were 600, 1500, 2700 and $3600 \text{ mm}^3/\text{s}$. The room temperature was $25 \pm 1^\circ\text{C}$.

3.2 Samples

The sheath fluid was tap water. The viscoelastic aqueous fluid was used as the sample fluid, and we also used tap water as our sample. Polyacrylamide aqueous solutions with a mass concentration of 0.01, 0.025 and 0.10 wt% were used as the viscoelastic fluid. They are named PAA0.01 wt%, PAA0.025 wt% and PAA0.10 wt%, respectively. Polyacrylamide is AL310P manufactured by Sanyo Chemical Industries, Ltd., and its molecular weight is about 10 million. These sample fluids were colored by a black Indian ink in order to visualize their radial position. Table I shows the density ρ of the sheath and sample fluids. The density was measured using a pycnometer with a volume of 50 ml and an electronic balance. There is a relative error of -0.8% comparing the density of water between the literature value²⁴⁾ and the measurement value. The pycnometer, however, showed the qualitative tendency that the density of PAA aqueous solution increases with the increasing concentration. Furthermore, the important value in this study is the difference in the density. Therefore, we used measured values to analyze.

4. RESULTS AND DISCUSSION

4.1 Observation of flow field

Figures 6 and 7 show the visualized images of the sample fluid filament at nine positions in the swirling part of 2-DSCC and 2-DSPC, respectively. The experimental conditions were $Q_{sample} = 71 \text{ mm}^3/\text{s}$ and $Q_{sheath} = 2700 \text{ mm}^3/\text{s}$. In order to evaluate a precise position of the sample fluid filament, the magnified photographs at the nine positions are taken separately unlike the overall images taken by one shot as shown in Fig. 1(b). The sample fluid, which looks the black filament, flows in the center. The clear sheath fluid that is just tap water flows on both sides. These flows were very stable and the position of the filament did not change with the passage of time during each experiment. In Region II, a static force balance between the hoop force due to the planar elongational stress and the centrifugal force determines the radial position of the filament. Figure 6 indicates a magnitude of the planar elongational stress acting on the sample fluid filament in the swirling contraction part of 2-DSCC, and Fig. 7 shows a relaxation behavior of the planar elongational stress in the swirling parallel part of 2-DSPC.

We then estimated the position of the filament

Table I Densities of the sheath and sample fluids measured using a pycnometer with a volume of 50 ml. The room temperature was $25 \pm 1^\circ\text{C}$. An asterisk symbol means the sample fluid. The sample fluid contains a black Indian ink in a low dose.

Fluids	$\rho \text{ [kg/m}^3\text{]}$
Tap water	989.2
Tap water*	989.4
PAA0.01wt%*	989.2
PAA0.025wt%*	989.4
PAA0.10wt%*	989.6

* Colored by a black Indian ink

Fig. 6 Visualized images of the sample fluid filament at nine positions in the swirling contraction part of 2-DSCC. The experimental conditions were $Q_{sample} = 71 \text{ mm}^3/\text{s}$ and $Q_{sheath} = 2700 \text{ mm}^3/\text{s}$. The filament position indicates a magnitude of the planar elongational stress acting on the sample flow.

quantitatively based on visualized images. Figure 8 shows the radius r_s and the width w_s of the sample fluid filament. The velocity v of the sample flow was estimated using the PTV method. The width w_s was measured from end-to-end of the filament, and the radius r_s donated the distance from the center of the channel radius R to the center of the width. The shift of the filament position from the center line of the swirling part Δr , which is described by $\Delta r = r_s - R$, was then calculated. Figure 9(a) shows the Δr as a function of angle θ in 2-DSCC. The filament of tap water that contains a black Indian ink gradually shifted to the outer wall with the

Fig. 7 Visualized images of the sample fluid filament at nine positions in the swirling parallel part of 2-DSPC. The experimental conditions were $Q_{\text{sample}} = 71 \text{ mm}^3/\text{s}$ and $Q_{\text{sheath}} = 2700 \text{ mm}^3/\text{s}$. The filament position indicates a relaxation behavior of the planar elongational stress acting on the sample flow.

increasing angles, because the centrifugal force was stronger than the hoop force. On the other hand, the filament of PAA0.025 wt% and PAA0.10 wt% shifted to the inner wall ($\theta < 45 \text{ deg.}$), because the hoop force became stronger than

Fig. 8 Quantitative evaluation of the filament position. The width w_s was measured from end to end of the filament, and the radius r_s donated the distance from the center of the channel radius R to the center of the width. The sample flow velocity v was measured using the PTV method.

Fig. 9 Shift of the filament position from the center line Δr in (a) 2-DSCC and (b) 2-DSPC as a function of angle θ . The experimental conditions were $Q_{\text{sample}} = 71 \text{ mm}^3/\text{s}$ and $Q_{\text{sheath}} = 2700 \text{ mm}^3/\text{s}$.

the centrifugal force. These samples then flowed at the fixed position from the inner wall surface. It is considered that the wall and the sheath flow affect the position of the filament. In the case of 2-DSPC as shown in Fig. 9(b), the filament of tap water shifted to the outer wall immediately. It is suggested that its relaxation time is short. The filament of PAA0.025 wt% shifted to the inner wall ($\theta < 70$ deg.), and then moved to the outer wall. It is predicted that the hoop force caused by the planar elongational stress gradually became weaker than the centrifugal force, because the planar elongational stress generated in the straight contraction part gradually relaxed in Region II. On the other hand, the filament of PAA0.10 wt% remained moving to the inner wall ($\theta < 160$ deg.). It is shown that the relaxation time of the polyacrylamide aqueous solution becomes long with the increasing concentration rate.

4.2 Evaluation of planar elongation rate

The planar elongation rate $\dot{\epsilon}$ evaluated by the PTV method is a value estimated from the elongational direction, and $\dot{\epsilon}$ based on the distribution of the filament width is a value calculated from the shrinking direction (see Fig. 10).

In the case of the planar elongation flow, these evaluated planar elongation rates must be equal due to its two-dimensionality. As shown in Fig. 10(a) and (b), $\dot{\epsilon}$ evaluated by the two methods of PAA0.01 wt% and PAA0.025 wt% were relatively consistent. Furthermore, they also coincided with the predicted value calculated from the channel width and the total flow rate. Based on these results, it is considered that PAA0.01 wt% and PAA0.025 wt% approximately maintained the two-dimensionality in the swirling contraction part. On the other hand, the results evaluated by two kinds of methods of tap water and PAA0.10 wt% were different. Especially, $\dot{\epsilon}$ were larger than the predicted value. It is suggested that tap water and PAA0.10 wt% shrank not only in the radial direction but also in the depth direction. In other words, it means that a uniaxial elongation flow was generated instead of the planar elongation flow. Therefore, this study evaluated the planar elongational stress and its relaxation time of PAA0.025 wt%, because it maintained the planar elongation flow and exhibited a clear relaxation behavior (see Fig. 9(b)).

4.3 Evaluation of apparent planar elongational stress and viscosity

The planar elongational stress acting on the sample fluid was estimated based on the radial position of the sample fluid filament in the swirling part of 2-DSCC. As the first step of this study, we will analyze simply from the balance of

Fig. 10 Evaluated planar elongation rate $\dot{\epsilon}$ as a function of the sheath flow rate Q_{sheath} in 2-DSCC at $Q_{sample} = 71 \text{ mm}^3/\text{s}$. (a) and (b) show the $\dot{\epsilon}$ evaluated by the PTV method and the distribution of the filament width in the swirling part, respectively.

centrifugal force and hoop force, without considering the surface tension and shear stress acting on the interface and the influence of the wall shear and the secondary flow. The balance of forces acting on the control volume of the filament with a small angle $d\theta$ was considered (see Fig. 11). The hoop force T , which is a resultant force of tensile forces caused by the planar elongational stress $\sigma_{\theta\theta}$, can be written as

$$T = 2\sigma_{\theta\theta}w_s dz \sin \frac{d\theta}{2} \quad (13)$$

with dz being a small thickness in the depth direction. Equation (13) can be rewritten as Eq. (14), because $d\theta$ is very small.

$$T = \sigma_{\theta\theta}w_s d\theta dz \quad (14)$$

The centrifugal force F_c caused by the density difference $\Delta\rho = \rho_{sample} - \rho_{sheath}$ is described by Eq. (15).

Fig. 11 Balance of the forces acting on the control volume of the filament with a small angle $d\theta$. T denotes a hoop force, which is a resultant force of the tensile force caused by the planar elongational stress. F_c is shown as a centrifugal force due to the density difference between the sample and sheath fluid.

$$F_c = \Delta m \frac{v_\theta^2}{r_s} = \Delta \rho w_s r_s d\theta dz \frac{v_\theta^2}{r_s} \quad (15)$$

Where, v_θ denotes the velocity of the θ -direction. Therefore, the balance of forces in the r -direction can be expressed by Eq. (16).

$$\begin{aligned} \rho_{\text{sample}} w_s r_s d\theta \frac{\partial v_r}{\partial t} &= F_c - T \\ \rho_{\text{sample}} r_s \frac{\partial v_r}{\partial t} &= \Delta \rho v_\theta^2 - \sigma_{\theta\theta} \end{aligned} \quad (16)$$

Where, v_r denotes the velocity of the r -direction. If it is assumed that the channel width is sufficiently smaller than the channel radius R , a small arc of the control volume $r_s d\theta$ can be rewritten as $R d\theta$, that is $r_s d\theta = R d\theta$. Furthermore, the velocity v_θ is shown as Eq. (17), because the planar elongation rate $\dot{\epsilon}$ is constant in the two-dimensional swirling contraction part.

$$v_\theta = v_0 \exp(\dot{\epsilon} t) \quad (17)$$

with v_0 being the velocity at the entry of Region II, that is $\theta = 0$. Thus Eq. (16) can be rewritten as Eq. (18).

$$\frac{\partial v_r}{\partial t} = \frac{\partial^2 r_s}{\partial t^2} = \frac{1}{\rho_{\text{sample}} R} \{ \Delta \rho v_0^2 \exp(2\dot{\epsilon} t) - \sigma_{\theta\theta} \} \quad (18)$$

An initial condition is assumed as follows: $r_s = r_0$, $v_r = 0$ when $t = 0$. Therefore, the radius of the filament r_s can be shown as Eq. (19) by integrating Eq. (18) twice with respect to the time t .

$$r_s = \frac{1}{\rho_{\text{sample}} R} \left\{ \frac{\Delta \rho v_0^2}{4\dot{\epsilon}^2} \exp(2\dot{\epsilon} t) - \frac{1}{2} \sigma_{\theta\theta} t^2 - \frac{\Delta \rho v_0^2}{2\dot{\epsilon}} t - \frac{\Delta \rho v_0^2}{4\dot{\epsilon}^2} \right\} + r_0 \quad (19)$$

The values of $\dot{\epsilon}$, v_0 and r_0 , which were evaluated based on the

quantitative observation of the sample fluid filament, were substituted into Eq. (19). The apparent planar elongational stress $\sigma_{\theta\theta}$ was then able to estimate by fitting Eq. (19) to the experimental result of the radius of the filament r_s .

Figure 12 shows a comparison between the experimental result and a calculation of tap water and PAA0.025 wt% in 2-DSCC. The experimental conditions were $Q_{\text{sample}} = 71 \text{ mm}^3/\text{s}$ and $Q_{\text{sheath}} = 2700 \text{ mm}^3/\text{s}$. As shown in Fig. 12(a), tap water that contains a black Indian ink showed a large difference between the calculation and the experiment in the downstream region. Therefore, we introduced the correction coefficient C instead of factors that were not considered in the calculation, and then Eq. (19) can be rewritten as

$$r_s = \frac{1}{\rho_{\text{sample}} R} \left\{ \frac{\Delta \rho v_0^2}{4\dot{\epsilon}^2} \exp(2\dot{\epsilon} t) - \frac{1}{2} \sigma_{\theta\theta} t^2 - \frac{\Delta \rho v_0^2}{2\dot{\epsilon}} t - \frac{\Delta \rho v_0^2}{4\dot{\epsilon}^2} \right\} + r_0 + \frac{1}{2} C t^2 \quad (20)$$

In this study, we assumed that the correction coefficient C corresponded to the initial acceleration to the r -direction, and it varies with the quadratic function with respect to the sheath flow rate Q_{sheath} (see Fig. 13). The calculation introducing the correction coefficient C (gray solid line on Fig. 12) was able to represent the filament position of tap water approximately.

Fig. 12 Comparison of the experimental data (circles) and a calculation of the filament position in 2-DSCC. The black and gray solid line were calculated by Eq. (19) and Eq. (20), respectively. The experimental conditions were $Q_{\text{sample}} = 71 \text{ mm}^3/\text{s}$ and $Q_{\text{sheath}} = 2700 \text{ mm}^3/\text{s}$.

In the case of PAA0.025 wt%, the calculation introducing C coincided with the experimental result in $t < 0.02$ s (see Fig. 12(b)). In this way, we approximated each experimental result of PAA0.025 wt% by Eq. (20), and estimated the apparent planar elongational stress $\sigma_{\theta\theta}$. The apparent planar elongational viscosity η_{pe} was then evaluated using Eq. (21).

$$\eta_{pe} = \frac{\sigma_{\theta\theta}}{\dot{\epsilon}} \quad (21)$$

Figure 14 shows the apparent planar elongational stress $\sigma_{\theta\theta}$ and viscosity η_{pe} of PAA0.025 wt%, which are described by solid symbols. Open symbols show the shear stress τ and shear viscosity η measured by the rotary rheometer. The apparent planar elongational viscosity η_{pe} of PAA0.025 wt% showed the elongation thickening²⁵⁻²⁸. This behavior is characterized by an increasing viscosity with the increasing planar elongation rate. Although the sample fluid was different, Ober *et al.*²⁹ reported that the elongation thickening of polyethylene oxide solution (3000 ppm polyethylene oxide in

34:66 wt% water:glycerol) using a micro fluidic device with a hyperbolic contraction geometry. η_{pe} of PAA0.025 wt% was on the order of 10^0 Pa s around $\dot{\epsilon} = 10$ s⁻¹. Trouton ratio $Tr(= \eta_{pe}/\eta)$ was approximately 15 to 150. There are a few papers that reported the experimental result of the planar elongational viscosity of such a low viscous liquid²⁹⁻³², and Shirakashi *et al.*³³ reported the apparent planar elongational viscosity of the polyacrylamide aqueous solution with the different concentration using the slit entry flow of Hele-Shaw cell. They showed Trouton ratio was about 200, and it is close to our result.

4.4 Evaluation of relaxation time

The relaxation time of the planar elongational stress was estimated based on the relaxation behavior of the sample fluid filament in the constant width swirling part of 2-DSPC. In this study, we assumed that the apparent planar elongational stress $\sigma_{\theta\theta}$ decreased with the exponential function. The stress relaxation behavior can be written as

$$\sigma_{\theta\theta}(t) = \sigma_0 \exp\left(-\frac{t}{\lambda_{pe}}\right) \quad (22)$$

Where, σ_0 denotes the planar elongational stress at $\theta = 0$, and λ_{pe} is the relaxation time. The balance of forces in the r -direction can be shown as Eq. (23).

$$\frac{\partial v_r}{\partial t} = \frac{1}{\rho_{sample} R} \left\{ \Delta \rho v_{\theta}^2 - \sigma_0 \exp\left(-\frac{t}{\lambda_{pe}}\right) \right\} \quad (23)$$

Therefore, the radius position of the sample fluid filament r_s in the relaxation process can be estimated by Eq. (24).

$$r_s = \frac{1}{\rho_{sample} R} \left\{ \frac{1}{2} \Delta \rho v_{\theta}^2 t^2 - \lambda_{pe}^2 \sigma_0 \exp\left(-\frac{t}{\lambda_{pe}}\right) - \lambda_{pe} \sigma_0 t + \lambda_{pe}^2 \sigma_0 \right\} + r_0 \quad (24)$$

Furthermore, we also introduced the correction coefficient C , and then Eq. (25) can be rewritten as

$$r_s = \frac{1}{\rho_{sample} R} \left\{ \frac{1}{2} \Delta \rho v_{\theta}^2 t^2 - \lambda_{pe}^2 \sigma_0 \exp\left(-\frac{t}{\lambda_{pe}}\right) - \lambda_{pe} \sigma_0 t + \lambda_{pe}^2 \sigma_0 \right\} + r_0 + \frac{1}{2} C t^2 \quad (25)$$

Figure 15 shows a comparison between the experimental result and a calculation of tap water and PAA0.025 wt% in 2-DSPC. The experimental conditions were $Q_{sample} = 71$ mm³/s and $Q_{sheath} = 2700$ mm³/s. The previously described results of $\sigma_{\theta\theta}$, which were estimated using 2-DSCC, were used for σ_0 , because the planar elongation rate in Region I of 2-DSCC and 2-DSPC are the same. The calculation introducing the correction coefficient C could describe the experimental result. In the case of PAA0.025 wt%, the calculation introducing C matched the experimental result, that is the filament

Fig. 13 Correction coefficient C as a function of the sheath flow rate Q_{sheath} .

Fig. 14 Solid symbols show the evaluated planar elongational stress $\sigma_{\theta\theta}$ and viscosity η_{pe} of PAA0.025 wt%, and open symbols show the shear stress τ and shear viscosity η .

Fig. 15 Comparison of the experimental data (circles) and a calculation of the filament position of the stress relaxation behavior in 2-DSPC. The black and gray solid line were calculated by Eq. (24) and Eq. (25), respectively. The experimental conditions were $Q_{\text{sample}} = 71 \text{ mm}^3/\text{s}$ and $Q_{\text{sheath}} = 2700 \text{ mm}^3/\text{s}$.

moves to the inner wall based on the hoop stress at the entry part and then moves to the outer wall with the stress relaxation (see Fig. 15 (b)). In this way, we approximated each experimental result of PAA0.025 wt% by Eq. (25), and then evaluated the relaxation time λ_{pe} . These results are shown in Fig. 16. The solid line shows the relaxation time obtained using an assumption that is the result of a frequency sweep test of PAA0.025 wt% can be approximated by the Maxwell model. λ_{pe} of PAA0.025 wt% was 0.0075 to 0.042 s, and these results were one tenth to one hundredth smaller than λ .

Although this study could not fully verify the accuracy of this technique using the swirling / hydrodynamic focusing design, the potential of this novel concept could be demonstrated. Through this trial experiment, we got some points to be improved. The experimental accuracy should be improved by selecting a suitable sheath fluid such as glycerol/water mixtures, which has an almost same shear viscosity of the sample fluid and can be arbitrarily adjusted in the density. The two-dimensionality will be improved by increasing the thickness of the channel with a good treatment of the stability of the inlet flow. The calculation would be brushed up with

Fig. 16 Evaluated relaxation time of the planar elongational stress λ_{pe} of PAA0.025 wt%. λ shows the result obtained from a frequency sweep test using the rotary rheometer.

considering the other factors ignored in this study. Future works investigate this method more deeply based on the above items.

5. CONCLUSIONS

This study demonstrated a novel technique to evaluate the apparent planar elongational stress and its relaxation time using a hoop stress in the swirling flow. This technique using the swirling / hydrodynamic focusing design needs only a photograph of a visualized ribbon-shaped filament in 2-DSCC and 2-DSPC. The radial position of the sample fluid filament in the swirling part is determined based on the static force balance of the centrifugal force and the hoop stress. The shape and position of the filament are analyzed theoretically with some assumptions. Therefore, the apparent planar elongational stress acting on the filament and the relaxation time can be evaluated from the visualized photographs. This swirling / hydrodynamic focusing design is suitable for the general liquids, which are opaque and not applicable the stress-optic rule. It requires a sheath fluid that is clearly less density than the sample fluid. The apparent planar elongational viscosity and the relaxation time of the dilute polymer solutions was estimated by this technique. Evaluated Trouton ratio of PAA0.025 wt% was approximately 15 to 150, and it was similar to the published data measured by Hele-Shaw flow method³³⁾. At present, the elongation rate range is on the order of 10^0 to 10^1 s^{-1} for this new design, while OSCER¹⁶⁾ is on the order of 10^0 to 10^2 s^{-1} . In addition, the swirling / hydrodynamic focusing design evaluated the elongational property of some samples with a shear viscosity on the order of 10^{-3} to 10^1 Pa s . On the other hand, this method has room for improvement in the sheath fluid selection, the channel

two-dimensionality, and the analysis method. We hope this concept being recognized and brushed up through discussions.

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