The Contact Force between Pantograph and Contact Wire
—An Estimation Method Using the Inversion Technique—

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To measure the contact force between a pantograph and contact wire, the author is proposing to use a new inversion method, the distinct feature of which is to reduce restraints on the sensor arrangement to facilitate contact force measurement. The contact force is calculated using the convolution between the sensor output and impulse response function, which is defined as the relationship between the former and the contact force. To achieve sufficient levels of precision, high order vibration modes of the panhead have to be taken into consideration by applying a pseudo-inverse matrix. The author has confirmed the accuracy of this method by numerical calculations and an excitation test on a currently-used pantograph.

Keywords: contact wire, pantograph, contact force, measurement, inversion technique

1. Introduction

Measurement of the contact force between a pantograph and contact wire plays an important role in the evaluation and maintenance of overhead contact lines. Contact force measurement methods are based on two principles as shown in Fig. 1. The first principle is the dynamic equilibrium of panhead (the equilibrium method\(^{1,2}\)). The other, which is the subject of this paper, involves the inversion analysis of the dynamic behavior of the panhead (the inversion method).

In the case of the equilibrium method, the contact force is estimated by measuring the internal force applied to the panhead by an articulated frame and measuring the inertial force of the panhead, based on the contact force being dynamically in equilibrium with these forces. As this method is simple and does not require complex calculations, it has been applied to many types of pantograph.

In contrast, the inversion method is derived from the response relation of the pantograph. From this point of view, the pantograph is regarded as a linear system. The input and the output of this system correspond with the contact force and the dynamic response of the pantograph, respectively. Hence, if the response relation between the contact force and the dynamic response of the pantograph is given, we can calculate the contact force, corresponding to solving an inverse problem, by measuring the dynamic response of the pantograph.

The inversion method is advantageous in that sensors can be easily installed on the pantograph, restrictions on their location being less than is the case with the equilibrium method. However, in order to express the response relation numerically, the positions on which each input is exerted must be discretely defined on the panhead, while the contact point with the contact wire and the panhead always varies according to the zig-zag of the contact wire and would cause large errors in calculation. Hence, the author developed an improved inversion method that can estimate the contact force accurately even if the contact point varies.

2. Principles of the inversion method

2.1 Outline

Estimation of effects by observing causes is called the direct problem. With the inversion problem, causes are revealed by observing their effects. In the case of the dynamic behavior of a pantograph, estimation of the dynamic response from the time history of the contact force corresponds to a direct problem, and estimation of the contact force by observing the time history of the dynamic response of the pantograph corresponds to an inverse problem. In this sense, the contact force estimation method proposed in this paper is named the inversion method, an outline of which will be described below.
If the pantograph is regarded as a linear system with \( n \) inputs and \( n \) outputs, the response relation between dynamic response (displacement, velocity or acceleration) and external forces (contact force) may be written as the following equation by using frequency response functions.

\[
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n 
\end{bmatrix} = 
\begin{bmatrix}
D_{11} & D_{12} & \cdots & D_{1n} \\
D_{21} & D_{22} & \cdots & D_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
D_{n1} & D_{n2} & \cdots & D_{nn}
\end{bmatrix} 
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_n
\end{bmatrix}
\]

(1)

where \( F_i \) is the force applied to the \( i \)-th input node; \( X_j \) is the dynamic response of the pantograph at the \( j \)-th output node, and \( D_{ij} \) denotes the transfer function giving the external force on the \( i \)-th node due to the dynamic response on the \( j \)-th node. \( F_i, X_j \) and \( D_{ij} \) are expressed in the frequency domain. The input and output nodes, which need not be in the same position, are discretely defined on the pantograph. One thing to mention here is that it is assumed that the influence of the vehicle vibration on the pantograph behavior can be ignored.

The value we want to obtain here is the external force, i.e., the contact force. Letting \( F_{\text{imp}} \) denote the total contact force, it can be written as the sum of external forces applied to all input nodes as:

\[
F_{\text{imp}} = \sum_{i=1}^{n} F_i
\]

(2)

By substituting (1) into (2), we obtain

\[
F_{\text{imp}} = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} D_{ij}X_j
\]

(3)

\[
= \sum_{j=1}^{n} X_j \sum_{i=1}^{n} D_{ij}
\]

Therefore, we can estimate \( F_{\text{imp}} \) from the outputs \( X_j \) (\( j = 1, 2, \ldots, n \)) and the transfer function \( D_{ij} \), which should be identified in advance.

### 2.2 Reduction of the number of input nodes in consideration of vibration properties

If we want to express the dynamic response characteristics of the pantograph exactly, there should be as many input nodes as possible. It is because the contact point between the contact force and the panhead is constantly changing due to the zig-zag of the contact wire. In order to save measurement costs, however, the author attempted to reduce the number of output nodes, as shown in Fig. 2.

In general, we can express the dynamic behavior of the pantograph by superposing modes of vibration. If the mode \( p \)-th or less is dominant in the frequency range in which the contact force is to be measured, the dynamic behavior can be expressed by using the mode below at least \( p \)-th. Therefore, we can estimate the contact force by obtaining independent physical information of at least \( p \). The detail of this point will be described below.

The transfer function is expressed by superposing the transfer function written by \( p \) modes.

\[
D_{ij} = \sum_{r=1}^{p} D'_{ijr}
\]

(4)

\[
\text{Figu} \:
\begin{array}{c}
\text{Displacement, acceleration, etc.} \\
\text{F} \quad \text{W} \\
\text{Measurable dynamic response} \\
\text{Panhead} \\
\text{F} \quad \text{W} \\
\text{Contact force (unknown)}
\end{array}
\]

\[
\text{Fig. 2 Reducing the number of output nodes}
\]

where \( D'_{ijr} \) is the contribution of the \( r \)-th mode to the transfer function giving \( F_i \) due to \( X_j \). We assume that \( n \) external forces are expressed by the dynamic responses on \( p \) output nodes.

\[
F_i = \sum_{r=1}^{p} W_{ir} D_{ir} X_j
\]

(5)

where \( W \) denotes the weight coefficient and \( n \geq p \). Hence, the following equation shall be satisfied.

\[
F_i = \sum_{r=1}^{p} D_{ir} X_j = \sum_{r=1}^{p} W_{ir} D_{ir} X_j
\]

(6)

Furthermore, when the pantograph can be regarded as either a proportional damping or a hysteretic damping system, \( X_j \) can be expressed in the form

\[
X_j = \sum_{i=1}^{n} \xi_i \phi_i
\]

(7)

where \( \phi_{ij} \) is the \( i \)-\( j \)-th element of the modal matrix and \( \xi_i \) is the \( i \)-th modal displacement. Substituting equations (4) and (7) into equation (6), we obtain

\[
\sum_{r=1}^{p} \xi_i \sum_{i=1}^{n} \phi_{ij} \sum_{j=1}^{n} \sum_{r=1}^{p} W_{ir} D'_{ir} \phi_{ij} = 0
\]

(8)

Therefore, for equation (8) to hold for any \( \xi_i \), the following equations must be identical.

\[
\sum_{i=1}^{p} \phi_{ij} \sum_{r=1}^{p} D'_{ir} = \sum_{i=1}^{n} W_{ir} \sum_{r=1}^{p} D'_{ir} \quad (i = 1, \ldots, p)
\]

(9)

where \( W_{ir} \) are unknown variables and the remaining parameters should be determined by the response relation of the system unambiguously. Therefore, equation (9) is \( p \times n \), a simultaneous linear equation that can be solved analytically.

These analyses indicate that the contact force can be estimated accurately by measuring the dynamic response of \( p \) in the frequency range where \( p \) modes of vibration dominate.

### 3. Practical procedure to apply the inversion method

In Chapter 2, the theoretical derivation of the inversion method for contact force estimation is described. However, this method requires a more practical procedure for actual application, so this chapter describes an experimental procedure to obtain the transfer functions.

A pantograph excitation test should be carried out first. The external force is applied to one of the input nodes, and the dynamic responses of the panhead on all \( p \)
nodes measured. Here, $\rho$ must not be less than the number of dominant modes of vibration in the frequency range where we want to obtain the contact force. The frequency response functions $H_{ij}$ giving a response on the $i$-th output node due to the external force on the $j$-th input node can be obtained by the excitation on the $j$-th input node. Then we can obtain

$$\begin{bmatrix} X_1 \\ X_2 \\ X_r \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{1q} \\ H_{21} & H_{22} & H_{2q} \\ H_{r1} & H_{r2} & H_{rq} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_r \end{bmatrix}$$

(10)

However, we want to obtain an equation that gives the external forces by dynamic response, that is

$$\begin{bmatrix} F_1 \\ F_2 \\ F_r \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{1p} \\ D_{21} & D_{22} & D_{2p} \\ D_{r1} & D_{r2} & D_{rp} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_r \end{bmatrix}$$

(11)

Here, we will lead this equation by using the Moore-Penrose pseudo-matrix of a coefficient matrix of equation (10). To identify the pseudo-matrix $B$ for $A$ is equivalent to find out the matrix $B$ which holds the following equation for a rectangular matrix $A$.

$$E = AB$$

(12)

where $E$ is a unit matrix. If $A$ is a square matrix, $B$ corresponds to the inverse matrix of $A$. If $A$ is not a square matrix, it should be noted that the following relation holds

$$E \neq BA$$

(13)

After $D_{ij}$ is obtained, the contact force can be evaluated by equation (11).

$$F_{c_{xy}} = \sum_{j} \sum_{i} D_{ij} X_j$$

(14)

The time history of the contact force can be obtained by transforming equation (14) in the frequency domain to the one in the time domain as follows

$$f_{c_{xy}} = \sum \sum h_{ij}(t) \bar{X}_i(t) - \tau d\tau$$

(15)

where $h_{ij}(t)$ is an impulse response function of $D_{ij}$.

4. Validation of the inversion method using numerical calculation

4.1 Validation method

In this chapter, the author verifies the availability of the inversion method by numerical calculations using a simple model to simulate the panhead. This model consists of an elastic beam supported by two springs at both ends, as shown in Fig. 3. This is sufficient to consider the influence of at least the first bending mode of the panhead. The equation of motion for this model is described as

$$EI \frac{\partial^4 y}{\partial x^4} + \rho \frac{\partial^2 y}{\partial t^2} = \delta(x-l_c)F_c$$

(16)

where $y(x,t)$ is the displacement of the panhead at the position $x$; $F_c$ is the external force applied to the panhead; $EI$ is the flexural rigidity of the beam; $l_c$ is the location of the external force in the $x$-directional coordinate; $\delta(\ )$ is a Dirac’s distribution; and $k$ denotes the stiffness constant of the spring supporting the beam. From the boundary condition, the eigenfunctions of the beam can be obtained. The transfer function giving the dynamic response due to the external force can be expressed by the eigenfunctions, taking orthogonality into account.

The author performed numerical calculations to verify estimation accuracy of the inversion method by using this model. The validation was performed using the following procedure.

1. Define $q$ input nodes to which the external force is applied and $p$ output nodes on which dynamic responses are measured.
2. Calculate the transfer function $H_{ij}$ defined by equation (10). Then obtain the transfer function $D_{ij}$ defined by equation (11) by using the pseudo-inverse matrix of $H$.
3. Calculate the dynamic response on each output node when the unit force is applied to an arbitrary position.
4. Estimate the external force by equation (14).
5. Evaluate estimation accuracy by comparing the estimated result and the unit force.

The constants used for this calculation are shown in Table 1. The transfer function was calculated by using up to the sixth eigenfunction. The natural frequencies of these modes are: 7.0Hz (translational mode), 12.3Hz (rotational mode), 94.5Hz (first bending mode), 258Hz (second bending mode), 505Hz (third bending mode) and 829Hz (fourth bending mode), respectively.

<table>
<thead>
<tr>
<th>Table 1 Constants used in numerical calculation</th>
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<tbody>
<tr>
<td>Linear density of beam $\rho$</td>
</tr>
<tr>
<td>Flexural rigidity of beam $EI$</td>
</tr>
<tr>
<td>Length of beam $l$</td>
</tr>
<tr>
<td>Spring constant of spring $k$</td>
</tr>
</tbody>
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### 4.2 Validation results

1. **In the case of $q=p=1$**

   Figure 4 indicates the force estimation accuracy of the inversion method by using only one accelerometer installed at the center of the panhead ($x=0.5$), where only one input node is defined at the same position. In case of $l=0.5$, i.e. when the position of the external force is equal to the input node, the estimation is of course extremely accurate. However, if the external force is not exerted on a predetermined input node, it is difficult to maintain such accuracy. In particular, measurement accuracy rapidly deteriorates at frequencies over 20Hz, because the third mode (first bending mode) affects accuracy over 20Hz. It is notable that measurement accuracy is not affected by the secondary mode (rotational mode), whose natural frequency is 12.3Hz as mentioned above, because the position of the accelerometer corresponds to the nodal point of this mode. From this it was proven that the measurement sensors should be installed as near as possible to the nodal points of disregarded modes.

2. **In the case of $q=p=3$**

   When response measuring points are placed at $x=0.16$, 0.5 and 0.84m, and input nodes for the external force at $x=0.4$, 0.5 and 0.6m, the external force estimation accuracy is as shown in Fig. 5. In this case, as only three modes of vibration are dominant up to 100Hz, the external force can be obtained accurately up to 100Hz wherever the external force is applied.

3. **In the case of $q=5$ and $p=3$**

   When the response measurement points are placed at $x=0.16$, 0.5 and 0.84m, as in the preceding example, and input nodes at $x=0.3$, 0.4, 0.5, 0.6 and 0.7m, the force estimation accuracy is as shown in Fig. 6. We can measure the external force accurately up to 100Hz, also in the same way as before, but it should be noticed that the measurement accuracy is not seriously affected in the vicinity of 250Hz where the fourth mode becomes dominant.

This is because increasing the input nodes is equivalent to considering the influence of ignored high order modes of vibration for identification of the transfer functions, or the residual stiffness of this system. In this way, the measurable frequency range can be set up more widely by redundantly determining the input nodes.

4. **Cases of interference by electrical noise**

   Generally, nonparametric identification techniques like this method are prone to signal contamination from electrical noise. Therefore, the author investigated the effect of noise mixed in with the measured dynamic response signals on the force estimation accuracy.

   Let us consider the case where a 2% error, made by a uniform random number generator, is intentionally mixed in the identified transfer functions used in the force estimation. Figure 7 shows the estimation accuracy in the case of $q=p=3$. Response measuring points (output nodes) and the input nodes are defined at the same positions as example (2). Deterioration of estimation accuracy due to contamination by the error is conspicuous even up to...
On the other hand, Fig. 8 indicates the estimation accuracy in case of \( q=5 \) and \( p=3 \). The deterioration of the measurement accuracy can be observed only in the vicinity of 94Hz that corresponds to the natural frequency of the third mode of vibration. This result shows that the force estimation accuracy can be maintained despite noise contamination from measured dynamic response signals by using redundantly determined sets of input nodes. This offers benefits for the measurement on line tests where sensor output signals are often affected by electrical noise.

5. Application to a currently-used pantograph

In order to verify the suitability of the inversion method for an actual pantograph, the author carried out an excitation test of the currently-used single-arm type pantograph, as shown in Fig. 9. The exciting force loaded on the panhead by a shaker can be regarded as the contact force, which was directly measured by a load cell installed on the tip of the shaker. The author estimated the force measurement accuracy of the inversion method by comparing the contact force estimated by the inversion method with that directly measured by the load cell. Furthermore, the author estimated the contact force by the equilibrium method, as described in Chapter 1.

The sensors installed on the pantograph consisted of three accelerometers attached inside the panhead and two sets of strain gauges placed on the springs supporting the panhead to measure the spring forces. These sensors are commonly used for measurement by the inversion and equilibrium methods, the measurement results obtained by each methods being compared.

Figure 10 (a) indicates the force estimation accuracy of the inversion method in the case of nine input nodes defined at the center of the panhead, \( \pm 100\text{mm}, \pm 200\text{mm}, \pm 300\text{mm} \) and \( \pm 400\text{mm} \) distant from the center. The external force was exerted at the center of the panhead. In contrast, Fig. 10 (b) shows the force estimation accuracy of the equilibrium method under the same condition. These figures indicate that we can evaluate the contact force up to 100Hz by the inversion method and up to 40Hz by the equilibrium method. The natural frequencies of the third and fourth modes of vibration are 45Hz and 60Hz, respectively. Therefore, in the case of the equilibrium method using three accelerometers, large errors occur above 40Hz where the fourth mode becomes dominant. In the case of the inversion method, all five sensors entirely contribute to compensating for the influence of high order modes of vibration. Therefore the force can be measured accurately up to the frequency where the fifth mode is dominant.

Figure 11 shows an estimation result of the contact forces in the time domain by the inversion and equilibrium methods when the force was applied to the center of the panhead. Figure 11(a) indicates the estimated forces that are given through a 40Hz low-pass filter. In this case, the two evaluated forces are almost equal to the contact force measured by the load cell directly. Figure
(b) Equilibrium method

**Fig. 10 Comparison of force estimation accuracy**

11 (b) indicates the measured contact forces given through a 100Hz low-pass filter, this figure showing that the inversion method can evaluate more accurately and in a wider frequency range than the equilibrium method.

### 6. Conclusions

In this paper, the author proposes a new method to estimate contact forces based on the inversion technique by which the contact force is estimated using transfer functions. This provides the forces on discretely defined input nodes due to the dynamic response on discretely defined output nodes. The key feature of the method is to use the output nodes which are the same or larger in number than the dominant modes in the target frequency range and use the input nodes which are larger in number than the output nodes. By this method, we can evaluate the contact force in a wide frequency range even if sensor output signals are contaminated by electric noise.

Since the inversion method does not require the sensor output signals to be in exact equilibrium, there is a greater degree of freedom in sensor installation than in the case of the equilibrium method and force estimation can be performed comparatively easily, even if the sensors have to be installed in confined space. Therefore, this method can be applied to the estimation of the contact force in the front-back direction.

However, when significant noise levels are mixed in with the sensor output signals, deterioration of the estimation accuracy cannot be avoided completely, this being one of the problems with the inversion method that has yet to be resolved. On the contrary, the equilibrium method is resistant to contamination by noise signals and the data processing algorithm used to calculate the contact force is very simple with this method. Therefore, it is advisable to choose the method most suited to the measurement conditions.

### References
