Verification of Wavelength-Fixing Mechanism for Rail Corrugation Caused by Multiple-Wheel Interaction

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In this paper the author proposes a hypothesis to explain a wavelength-fixing mechanism for rail corrugation. Corrugation growth was predicted using a theoretical model that took into account multiple-wheel interaction and the Doppler effect of rail vibration caused by wheel movement. The wheel-rail contact forces due to virtual sinusoidal irregularities on the rail head were calculated, and the corrugation growth at an arbitrary wavelength was predicted. To verify the hypothesis, the author conducted a precise investigation into the corrugation wavelengths on various track sections using vertical acceleration of the axle box. Even on a specific section of track, large-amplitude corrugation was not generally found at a specific wavelength but rather in a few wide-band ranges. Moreover, in each range where significant corrugation occurred, several sharp peaks were observed at constant frequency intervals. These peaks were found to appear at regular intervals approximately equal to fractions of a wheelbase length, and the theoretical prediction showed good agreement with measured corrugation characteristics. It was concluded that various kinds of corrugation are the result of rail vibration interference excited by multiple wheels.

Keywords: corrugation, rail, multiple-wheel interaction, wave interference, wheelbase, interaction

1. Introduction

On railways, it has long been known that there is a tendency for rail heads to develop a periodic wear pattern known as corrugation. Because of the harmful effect this can exert on the wheel/rail system and the environment, a wide range of studies have been carried out to address the problem. Types of corrugation and the relevant mechanisms reported before 1990 were reviewed by Grassie and Kalousek 1), who classified corrugation into six categories according to the two basic mechanisms of wavelength-fixing and amplitude growth. Sato et al. 2) reviewed a history of studies made on corrugation since 1890, described recent research and occurrences of corrugation in Japan. The review mentioned the study by Suda, Matsumoto and others on short-pitch corrugation occurring on subway lines. Suda et al. insisted that the phenomenon is a result of stick-slip vibration with vertical force fluctuation caused by the rails’ natural lateral vibration. The review also reported on the three types of corrugation often observed in Japan, (1) short pitch-type on the low rails of sharply curved tracks; (2) short pitch-type on tangent tracks, and (3) intermediate pitch-type on the high rails of curved tracks. The author presents another type of corrugation with a wavelength of approximately 1.2m observed in an undersea Shinkansen tunnel as reported by Sunaga et al. 3), who also verified that axlebox acceleration waveforms were very similar to those of corrugation.

Fredrick 4), Hempelmann et al. 5), Tassilly and Vincent 6), Igeland 7), and Wu and Thompson 8) respectively, have all carried out various research projects in recent years. The three former works found that corrugation was the result of vertical or lateral wheel/rail system dynamics and, apart from differences in detail, their viewpoints on the cause of corrugation generally concentrated on the concept that corrugation wavelength depends on the natural frequencies of wheelsets and rails. In other words, they suggested by implication that corrugation wavelength did not depend on a trains’ wheel arrangement.

By contrast, the two latter papers and a previous paper by the author 9) addressed the effect of multiple wheels on corrugation formation. The idea was originally mooted by Igeland, who applied a model consisting of a track system and two wheels in her research. Numerical analytical investigation showed a close correlation between the corrugation wavelength and the wheelbase, but the wavelength-fixing mechanism was not clearly explained in detail. Wu and Thompson studied the interaction between four wheels on two bogies and a rail and showed possible corrugation at more frequencies than in the case of a single wheel. However, two important factors were ignored in their model: the Doppler effect of rail vibration caused by wheel movement and the effect of vibration waves excited by one wheel on the other wheels, referred to in their paper as passive force. The author considered these two factors essential for gaining a clear understanding of the wavelength-fixing mechanism, as described below.

2. Rail vibration interference caused by multiple wheels

2.1 Theoretical model

Figure 1 shows a theoretical model for analyzing corrugation growth and specifies the mathematical symbols used. The rail was modeled as an Euler-Bernoulli beam of infinite length with a continuous elastic layer, and the
two wheels as two masses, each representing a wheel. On the rail surface, there is an initial virtual sinusoidal irregularity with arbitrary wave numbers and unit amplitude. The two masses, spaced at interval of D, move along the rail surface at constant velocity V.

Although actual tracks have a more complex structure composed of ties, a ballast bed and a ground base, a simpler model was introduced to reproduce pure characteristics capable of carrying flexural waves. The linear density of the beam was set between 100 kg/m and 200 kg/m for the total equivalent mass of rail and sleepers, and a wheelbase of 2.1m (typical for meter-gauged vehicles) introduced. The stiffness of the elastic layer was set so that rail displacement due to the axle load was close to the actual observed value.

2.2 Equation of motion and solution

The vertical displacement of a rail y excited by n traveling wheels is governed by

$$\rho \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} + Ky = \sum_{j=1}^{n} \left(P_j + F_j \exp(i\omega t)\right) \delta(x-x_j-V t)$$

(1)

where \(P_j\), \(F_j\), and \(x_j\) are the mean and fluctuating contact force components and the initial location of wheel \(j\), respectively. The delta in the equation represents the function known as Dirac’s delta function. In the coordinate system moving with train, Equation (1) can be reduced to

$$\rho \frac{\partial^2 y}{\partial t^2}\bigl(2\pi V\bigr)^2 + EI \frac{\partial^4 y}{\partial x^4} + Ky = \sum_{j=1}^{n} \left(P_j + F_j \exp(i\omega t)\right) \delta(x-x_j)$$

(2)

The solution to Equation (2) can be expressed as a linear summation of \(F_j\)-weighted responses to a unit force (\(\exp(i\omega t)\)), which are easily obtained through a number of operations. The rail’s vertical displacement and rail irregularities at each mass location can be written as follows.

$$Y_j \exp(i\omega t)$$

(3)

$$2\pi V \frac{\partial}{\partial x}\left(Y_j + V t\right)$$

(4)

Therefore, the following relation is derived:

$$
\begin{bmatrix}
Y_1 \\
\vdots \\
Y_n \\
\end{bmatrix} = 
\begin{bmatrix}
\alpha_1 & \ldots & \alpha_n \\
\vdots & \ddots & \vdots \\
\alpha_1 & \ldots & \alpha_n \\
\end{bmatrix}
\begin{bmatrix}
F_1 \\
\vdots \\
F_n \\
\end{bmatrix}
$$

(5)

where \(\alpha_i\) is the influence coefficient from force \(j\) to displacement \(i\).

The exciting force on the right-hand side of Equation (2) is equal to the mass inertial force, which can be expressed in another form as

$$F_j = M \omega^2 \left(y_j + \exp(i\omega t)\right)$$

(6)

where

$$k = \frac{2\pi}{\lambda}, \omega = kV$$

(7)

The complex amplitude of contact force and displacement at each mass location can be determined from Equations (5) and (6).

2.3 Contact force and corrugation promoting force

Actual corrugation shows a number of substantially different growth mechanisms such as wear, plastic flow, rolling contact fatigue, etc. It can be generally assumed that stronger forces produce greater deformation, irrespective of the growth mechanism. It can therefore be assumed that decreases in rail height are proportional to the summation of the contact forces working on a specific rail location. The assumption allows corrugation growth to be predicted using the force component in phase with the initial corrugation. Then, the sum of the force components with the same phase as the initial corrugation can be used as a measure to predict corrugation growth, referred to below as the corrugation-promoting force (CPF). This is defined by

$$CPF = \sum_j \text{Re}\left\{F_j \exp(i\omega t)\right\}$$

(8)

If the contact force has the same phase as the corrugation, the CPF shows a positive value representing a growth in corrugation amplitude, and vice versa. Figure 2 shows the calculated results where the linear density and flexural rigidity of the beam are 170 kg/m and 4.0 × 10^6 Nm² respectively, at vehicle speeds of (a) 20m/s and (b) 30m/s. The figure illustrates cases of two wheels compared with a single wheel.

The points plotted on the left in Fig. 2 show the contact force amplitudes per unit corrugation magnitude versus wave number. In the case of a single wheel, the contact force amplitude increases steadily with the wave number. However, the contact force values of the preceding and succeeding wheels in a bogie vary periodically with the wave number and generally differ from each other. The increases and decreases repeat periodically at intervals of one wave number in a wheelbase. Generally speaking, there are a few areas where the contact force amplitudes increase significantly compared with the single wheel, and at the center of these areas the forces of the preceding and succeeding wheel equalize. As shown later, the centers of these areas are considered identical to the natural frequencies of the track model clamped at both ends of a wheelbase length.

The points plotted on the right of Fig. 2 depict the CPF. To enable an equivalent comparison in the case of a bogie, the CPF of a case involving a single wheel was doubled. In the case of a single wheel the CPF also increases steadily with wave number, but a bogie value shows periodic increases and decreases at intervals of one wave number in a wheelbase. Specifically, the peaks
in the lowest wave number area are situated at wave number integer values in a wheelbase, but those in the next area are located at integer-plus-a-half values. Comparing the CPF at 20m/s with that at 30m/s, it is evident that the wave numbers of areas with larger amplitudes decrease with train velocity and then occur at an almost constant frequency. In Fig.2, three peaks are shown located almost equally to the first three natural frequencies of a wheelbase-long model track clamped at both ends.

3. Vibration of rail excited by two forces

3.1 Model and analysis

To provide an even simpler explanation, two point forces representing two wheels were considered, as outlined below. A rail was modeled as a free beam of infinite length, and the rail surface was assumed to have pre-existing sinusoidal corrugation. Two forces were introduced, representing the fluctuating vertical forces between the wheel and rail caused by the virtual corrugation with unit amplitude having the same phases as the corrugation. Figure 3 shows the model, which, although too simple to give a quantitatively correct corrugation prediction, was introduced to clarify the principle of the interaction of two wheels. The rail’s response to the two forces was considered useful in understanding corrugated-rail vibration.

Fig. 2 Theoretical dependence of contact force and corrugation promoting force on irregularity wavenumber in a wheelbase

(a) V=20 m/s

(b) V=30 m/s

Fig. 3 Schematic view of rail vibration due to multiple forces (forward-propagating wave)

The symbols below were set as: the distance between two forces =D, traveling velocity of force =V, wavelength of virtual corrugation =λ, and wave propagating velocity in the moving coordinate =C. Exciting frequency f, vibration period T and the time difference between the two exciting point forces t₅ are as follows:

\[ f = \frac{V}{\lambda}, \quad T = \frac{1}{f}, \quad t₅ = \frac{D}{V} \]  \hspace{1cm} (9)

The conditions under which the vibration amplitudes of the rail diminish at the front and rear parts of the exciting forces can then be expressed as
where (+) means the front part and (−) the rear part of the two forces. The left-hand side of Equation (10) represents the time difference between the wave arrival from one force and that from the other excited by the corrugation at every loading point. Then, Equation (10) means a condition whereby the time difference is equal to odd-number multiples of half a period, and the two vibrations have exactly opposite phases. Under this condition, the rail vibration amplitude in the external section of the two forces will become very small.

Figure 3 shows a schematic view of rail vibration interaction for the forward propagating wave. The figure shows a case in which rail vibration waves excited by preceding and succeeding forces have the same amplitude and exactly opposite phases. The summation of the two waves results in zero wave propagation forward of the two forces, and the rail between two forces only vibrates.

When Equation (9) is substituted into Equation (10), the following condition is obtained:

\[ \lambda^* = \frac{2D}{2n-1} \left( 1 \pm \frac{V}{C^*} \right) \]

(11)

If there is corrugation on a rail head with a wavelength equal to Equation (11), the vertical vibration magnitudes of the rail at two exciting points become almost zero. Because of the Doppler effect and the force’s direction of travel, the wavelengths in Equation (11) generally differ according to the wave directions.

If a rail is modeled as an Euler-Bernoulli beam, the phase velocity of the flexural wave in the rail in the moving coordinate is almost equal to

\[ C^* = \sqrt{\frac{EI}{\rho} \left( \frac{V}{C^*} \right)^2} \]

(12)

where \( \rho \) and \( EI \) are the linear density and flexural rigidity of the rail, respectively. By substituting Equation (12), the wavelength in Equation (11) can be calculated.

### 3.2 Wave interaction results

The wavelengths expressed in Equation (11) for 60 kg rail parameters were transformed into wave numbers in a wheelbase, and are shown in Fig. 4 against train velocity.

In Fig. 4, the lines with a positive gradient represent the wave number in the rear part of the two forces \( (D/\lambda^-) \), while those with a negative gradient correspond to the wave number in the front part \( (D/\lambda^+) \). All lines start from wave numbers \( (2n-1)/2 \) (here, \( n \) is an arbitrary integer) at zero velocity, representing a situation with inverse excitation at the two points. Two groups of lines with opposite gradients cross each other, forming cross point areas that appear lighter than other areas. In these areas, waves propagating both forward and backward almost vanish from the whole section except between the two forces. Then, under these conditions, the vertical motion of the rail at both loading points becomes much smaller than that made by a single-force excitation.

Generally speaking, the contact force amplitude caused by surface irregularities on the rail head is proportional to the parallel mechanical impedance of rail and wheel. If the impedance of the rail is much smaller than that of the wheel, which is true for single-force excitation at rather high frequencies such as corrugation, rail motion is pronounced and the contact force becomes small.

Conversely, under cross-point area conditions, the impedance of the rail becomes greater as a result of the interaction of the two forces, and the amplitude of contact force is likely to increase. The contact force phase is also likely to agree with that of corrugation. These circumstances result in the growth of corrugation at the cross points shown in Fig. 4.

The lighter areas in Fig. 4 seem to be distributed along four nearly hyperbolic lines corresponding to constant frequencies. For a constant velocity, cross points are situated at an interval of one wave number in a wheelbase. The characteristics of the cross points correspond closely to the sharp peaks in CPF, as shown in Fig. 2. It is therefore considered that these peaks occur as a result of rail vibration interference excited by two wheels.

At the cross points, \( \lambda'(n+m) \) is equal to \( \lambda'(n) \) in Equation (11) for any integer of \( n \) and \( m \). When the train velocity is assumed to be much smaller than the phase velocity of the flexural rail vibration, cross point frequency is approximated as

\[ f = \frac{\gamma_n^2}{2\pi D^2} \sqrt{\frac{EI}{\rho}} \gamma_n = \frac{2m+1}{2} \]

(13)

In other words, the frequency is regarded as the \( m \)-th order of natural frequency in a \( D \)-long beam clamped at both ends.

As mentioned above, the author proposed a hypothesis, which can be summarized as follows:

Rail corrugation may have wavelengths equal to unit fractions of double the wheelbase as a result of the interference of waves excited by two wheels. Corrugation frequencies may be almost equal to the natural frequencies of a wheelbase-long track clamped at both ends.
4. Corrugation wavelength characteristics

4.1 Measurement method

For a precise investigation of actual rail corrugation wavelengths, the vertical acceleration of axle boxes on Japan Railway Group companies’ commercial lines was measured. The total length of the measured lines was approximately 900 km in terms of single track. Axle-box acceleration was generally assumed to correspond to rail-head irregularities. Therefore, although analysis of axle-box acceleration could not accurately indicate corrugation amplitudes, it would give detailed information on the wavelengths involved.

4.2 Time histories of axle-box acceleration

A typical example of data measured in a section of successive sharp curves is shown in Fig. 5. The figure plots time histories of the left-wheel load and the vertical acceleration of the left and right axle-boxes. The duration of the data sample was 60 seconds, roughly corresponding to a track length of 1,500 m. The curved sections are easily identified from the wheel-load profile, and the quasi-periodic pulses at approximately one-second intervals in the acceleration data represent rail-joint impacts. The relatively larger amplitudes of high-frequency acceleration (circled in Fig. 5) were observed only on circular curves, especially on the low-rail side, but were unnoticeable on transition curves. Larger acceleration was considered to occur as a result of corrugation on the low-rail head, with frequencies varying from 160 Hz to 190 Hz, equivalent to the wavelengths of 0.12 m to 0.15 m of corrugation as directly observed. Although the degree of intensity is lower, axle-box acceleration on the high-rail side has the same components as that on the low-rail side.

4.3 Frequency analysis of axle-box acceleration

A number of time histories (as measured on curved or tangent sections of meter-gauged lines and standard-gauged Shinkansen track) were selected, and the power spectral densities (PSDs) of axle-box acceleration calculated. Typical PSD examples are shown in Figs. 6, 7 and 8 along with the track curvatures and speeds of the measuring train. The accelerations were not calibrated as the relative intensities were sufficient to detect corrugation wavelengths. In the figures, the two lines represent the PSDs of both axle-box accelerations in a wheelset, and a line of black squares is marked in the upper part of the chart. The abscissas of these black squares correspond to the frequencies determined as a ratio of the train velocity to the unit fractions of the wheelbase. For simplicity, the frequencies are referred to below as wheelbase frequencies (WBFs), and are defined as

\[ \text{WBF} = \frac{nV}{D} \quad \text{for } n: \text{natural number} \]  

In this study, a 2.1-m wheelbase was used to represent a typical vehicle running on a meter-gauged lines, and a 2.5-m wheelbase for a vehicle running on a standard-gauged Shinkansen track.

It is apparent that the PSDs indicate higher intensities in some frequency ranges (circled in Figs. 6 and 7) where the rail is subject to severe corrugation at the same frequency. Figures 6 to 8 indicate some unique corrugation wavelength characteristics that seem essential for understanding the corrugation occurrence mechanism. The interesting characteristics are as follows:

(1) Axle-box acceleration shows higher intensity in a few fairly wide-banded frequency ranges. As there are two or more ranges under 300 Hz in most cases, it is understood that corrugation in a specific section is likely to occur not at a specific wavelength but rather in a few wide-band ranges.

(2) The corrugation ranges consist primarily of many

\begin{align*}
\text{low rail} & : 0.19-0.12 \text{m} \\
\text{high rail} & : 0.7-0.3 \text{m} \\
\text{WBF} & : 0.05 \text{m} \\
\end{align*}

\begin{align*}
\text{PSD of axle-box acceleration} \\
\text{Frequency(Hz)}
\end{align*}

\begin{align*}
\text{Wheel load (left)} \\
\text{Axle-box acceleration (left)} \\
\text{Axle-box acceleration (right)}
\end{align*}

\begin{align*}
\text{Fig. 5} \quad \text{Axle-box acceleration measured on section with successive curves}
\end{align*}

\begin{align*}
\text{Wheel load (left)} \\
\text{Axle-box acceleration (left)} \\
\text{Axle-box acceleration (right)}
\end{align*}

\begin{align*}
\text{Fig. 6} \quad \text{PSD of axle-box acceleration on curved track with radius of 400 m at V=22.5 m/s}
\end{align*}

\begin{align*}
\text{Wheel load (left)} \\
\text{Axle-box acceleration (left)} \\
\text{Axle-box acceleration (right)}
\end{align*}

\begin{align*}
\text{Fig. 7} \quad \text{PSD of axle-box acceleration on tangent track of meter-gauged line at V=24.4 m/s}
\end{align*}
sharp peaks distributed at a constant pitch equal to WBF intervals. In other words, corrugation can be considered to grow in line with these intervals.

(3) On curved tracks, corrugation with a relatively short wavelength is predominant on low rails, but a longer wavelength is dominant on high rails, as confirmed by visual inspection. Although the corrugation intensities on both rails differ, their wavelengths seem to be identical.

(4) The Shinkansen track example in Fig. 8 highlights the continuous extent of sharp impulses with wavelengths from 0.8m to 0.05m or less. The frequencies of the peaks also seem to move from the WBF point to the median of subsequent WBFs and then vice versa.

The characteristics described above indicate a close correlation between the theoretical prediction shown in Fig. 2 and the cross-point distribution in Fig. 4. This can be considered to support the wavelength-fixing mechanism proposed here.

5. Conclusion

The author carried out an investigation into rail corrugation wavelengths occurring under various conditions through the analysis of axle-box acceleration. Corrugation growth was also predicted theoretically using a fairly simple model. From the results of the study, the wavelength-fixing mechanism for corrugation with wavelengths between approximately 0.05 m and 1.0 m generated on both meter-gauged lines and standard-gauged Shinkansen track was considered to be the interaction of rail vibrations excited by plural successive running wheels.

The fundamental mechanism is the increase in contact forces and the agreement of their phases with those of corrugation, which occurs as a result of the interference of rail vibration waves excited by multiple wheels. The corrugation wavelengths based on this mechanism are generally predicted to be approximately equal to fractions of a wheelbase length unit. The frequency ranges of significant corrugation are considered to correspond to the natural frequencies of a wheelbase-long track clamped at both ends. Corrugation does not occur at a specific wavelength but rather in a few wide-band ranges.

Based on the mechanism outlined above, the following countermeasures are recommended to prevent rail corrugation:

(a) Reducing the number of train wheels
(b) Lengthening the spaces between wheels
(c) Increasing track damping
(d) Using materials and/or lubricants to minimize rail wear and deformation

References