Evaluation Method of Robustness for Train Schedules

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The aim of this study is to examine the qualitative and quantitative characteristics that robust train schedules should have. The method we propose to evaluate the robustness of train schedules has two main characteristics; one is a focus on passenger disutility, and the other involves the development of a probabilistic model for the statistical evaluation of train operations. To calculate the robustness indices, we applied a Monte Carlo simulation method that performs iterative train-passerger simulations. We then carried out simulation experiments using actual data to calculate the robustness indices for two train schedules, and compared the results.

Keywords: train schedule, robustness, disutility, stochastic estimation, Monte Carlo simulation

1. Introduction

Although trains are operated according to predefined schedules, unexpected incidents sometimes occur and trains inevitably experience delays. An example of such a delay is when dwell time increases due to an object caught between the doors; this leads to a build-up of passengers on the platform, who then rush onto the train when the doors are reopened to release the object. Train delays may also occur due to various meteorological conditions such as rain, snow or fog, and can eventually affect other trains. This paper looks at the development of a robust train schedule under which stable transportation services can be provided regardless of unexpected incidents.

In order to consider a robust train schedule, we need to define a measure of robustness.

Generally, train delays lead to a reduced level of transportation service, and References 1) - 3) have suggested robustness indices based on train delays. Such a concept, however, does not apply from the viewpoint of passengers as individual train delays do not reflect the reduced level of transportation service that each one suffers. As an example, it is more important for passengers who transfer trains several times to be able to make their connections as planned.

We have therefore proposed a new concept in Reference 4) involving a robustness index based on passenger convenience. We calculate the specific robustness index for train schedules using the level of passenger disutility derived from the traveling time, congestion rates, the number of transfer lines and the waiting time.

Any attempt to calculate robustness indices deterministically is inappropriate, since it is impossible to know the frequency, location, timing and length of delays in advance. It is possible, though, to anticipate such data probabilistically based on measurement surveys. In light of this, we introduce a method to define robustness indices probabilistically. Specifically, we determine that the robustness index should be calculated using a value that represents the expected increase in passenger disutility when delays occur. We calculated the robustness index by applying a Monte Carlo simulation method iteratively to estimate train operation from passenger behavior.

In Section 2, we explain passenger disutility and define the robustness index, and Section 3 illustrates the calculation method for the robustness index using the simulator developed and Monte Carlo simulation. In Section 4, we calculate robustness indices for an actual train schedule and a modified one, and give a comparison of the results. Section 5 outlines the conclusions reached.

2. The definition of the robustness index

To determine the level of transportation service from a passenger viewpoint, we define the robustness index using the disutility of all passengers. In this section, we illustrate passenger disutility with an example and explain the definition of the robustness index.

2.1 Passenger disutility

Passenger disutility is represented by a discomfort...
index for each passenger on a train. In Reference 5), for example, this index signifies the degree of passenger discomfort experienced from congestion, the number of transfers and the waiting time, which is converted to a traveling time and multiplied by a certain coefficient. Specifically, passenger disutility is calculated by Equation (1):

\[ \alpha \sum_i \left( \frac{C_i}{100} \right) + \gamma N + \delta T_w + T_r \]  

(1),

where \( t_i \) [min] represents the running time for section \( j \) between stations, \( C_i \) [%] represents the congestion rate of the same section, \( N \) represents the number of transfers, \( T_w \) [min] represents the total waiting time at stations, \( T_r \) [min] represents the boarding time, and \( \alpha, \beta, \gamma \) and \( \delta \) are the parameters.

A passenger’s disutility represents the level of transportation service for him/her. A reduced level of transportation service due to train delays is therefore expressed by an increase in passenger disutility.

Illustrated here is an example of the transition of a transfer passenger’s disutility when a certain delay occurs. The case concerns connection between trains as shown in Fig. 1. Passenger (p) departs from Station (X), makes a connection at Station (Y) and moves to Station (W). We assume that the passenger selects the fastest train, boarding Trains (1) and (4) when the train diagrams are as outlined in Fig. 2.

If a train delay then occurs as shown in Fig. 3. Passenger (p) must wait for Train (5) at Station (Y). Therefore, the term of \( T_w \) in Equation (1) increases, and we can represent the reduced level of transportation service for Passenger (p).

2.2 Definition of the robustness index

If train delays are given, the total increase in overall passenger disutility can be used to represent the decreased level of transportation service. However, to calculate the robustness of train schedules, we must consider a range of incidents rather than only a specific nature.

It is, though, impossible to know in advance where and how long delays will be, and for this reason we assume that it is possible to obtain a probability distribution of the location, frequency and length of delays. This is a reasonable assumption, as we can estimate the data through the analysis of train operation results and measurement surveys. We label this assumption as the probabilistic representation of incidents. We can then define the robustness index of a train schedule as the expected value of the total increase in overall passenger disutility. The robustness index is obtained from:

\[ (Robustness \ Index) = E \left[ \sum_{i=1}^{N} D(X) \right] \]  

(2),

where \( X \) represents a random variable of the probabilistic representation of incidents, \( D(X) \) represents a random variable of the increase in passenger (ii)‘s disutility under \( X \), \( N \) represents a random variable of the total number of passengers, and \( E(Y) \) represents the expected value of a random variable \( Y \). It should be noted that the amount of passenger disutility is not included if it decreases when incidents occur.

3. Method of calculating the robustness index

3.1 A simulator that estimates train operation from passenger behavior

To calculate passenger disutility, train operation results and details of all passenger routes are needed. Recently, a number of railway companies have introduced PRC (Programmed Route Control) systems, meaning that actual train operation results can be obtained. However, as it is impossible to estimate every passenger’s route from these results, we have developed a simulator that can predict the results of train operation by tracing the behavior of passengers. These simulation results enable
The flow of the simulator to predict the results of train operation (Shaded procedures represent passenger behavior)

calculation of passenger disutility.

The required functions of this simulator are:

(1) It can accurately estimate train operation results in a short period of time.

(2) It includes a passenger behavior model where all passengers decide which trains they board.

To implement function (1), we modeled train operation using the PERT network to provide high-speed computing. Boarding and alighting times were also calculated using the number of boarding and alighting passengers and the train congestion rate. For function (2), the model was made in such a way that passengers select their own behavior based on the information supplied by railway companies. The flow of the simulator is shown in Fig. 4.

3.2 Method of calculating the robustness index using Monte Carlo simulation

To analytically calculate the robustness index defined by Equation (2), we need the probability distribution of $D_i$ that represents the increase in passenger $(i)$’s disutility. However, it is difficult to obtain this value as it is affected by train congestion and the behavior of other passengers.

For this reason, we estimate the expected value of $D_i$ by applying a Monte Carlo simulation method that runs the developed simulator iteratively. Delays are generated using random numbers according to the probabilistic representation of incidents, and the disutility of all passengers is calculated in the simulator. Figure 5 shows the flow of the calculation. As a result, the formula for computing the robustness index is rewritten as Equation (3), where $u_i$ represents passenger $(i)$’s usual disutility, $d_{i,j}^{(j)}$ represents passenger $(i)$’s disutility when incident $(j)$ occurs, $m$ represents the repeat count of Monte Carlo simulation and $n_i^{(j)}$ represents the total number of passengers when incident $(j)$ occurs.

$$\text{(Robustness Index)} = \sum_{j=1}^{m} \frac{1}{n_i^{(j)}} \sum_{i=1}^{N} \max\{d_{i,j}^{(j)} - u_i, 0\}$$  \hspace{1cm} (3)

4. Results of experiments using actual data

The actual train schedule satisfies a great number of physical constraints such as vehicle schedules, crew schedules, various station facilities, signaling systems and other related areas. As rush-hour schedules can already be prepared to leverage all facilities to the maximum extent, it is unrealistic to substantially modify the actual train schedule, as this would require the improvement of station facilities and signaling systems as well as the introduction of high-performance vehicles. On the other hand, it is preferable for transfer passengers that arrival/departure times at stations with connections between trains should be determined in consideration of other lines’ train diagrams.

Even if it is difficult to substantially modify the train diagram as described above, it may be possible to prepare more robust train schedules by changing arrival/departure times slightly wherever possible. To study such a possibility using actual data, we proposed a train diagram with slight variations in arrival/departure times, calculated the robustness indices and compared the results.

4.1 Experimental parameters

- The experimental line for which the robustness index is calculated

  The line is located in an urban area, and has a track length of approximately 60 km. The number of stations is 39, and only local trains run every couple of minutes during peak times.

- The stations that have connections between trains

  We consider the waiting time at two stations (A and B) where a large number of passengers transfer from the experimental line. Passenger disutility is calculated by including the waiting times at Stations (A) and (B).

- The passenger OD data

  We assume that passengers arrive randomly at stations without checking train diagrams, and that passenger arrival times occur according to Poisson distribution for every OD. Arrival rates and the ratio of transfer passengers at Stations (A) and (B) are prepared using $\lambda$, and passenger arrival times are then generated using random numbers.

  Passenger disutility

  Passenger disutility is calculated using Equation (1).

- Actual train diagram

  We examined 27 up-line trains and 36 down-line trains during the morning rush hour. Figure 6 shows a part of
the train diagram of the experimental line and other lines that have connections between trains at Stations (A) and (B).

- Proposed train diagram

In order to propose a possible train diagram without improvements to the infrastructure, we focus on the margin time that is added to running times and allocate this time somewhere else. Specifically, we modify the down-line arrival/departure times at Station (D) and the arrival times at Station (B) to be 10 seconds earlier than actual. To do this, the running time from Station (C) to Station (D) is shortened by 10 seconds (Fig. 7).

- Passenger boarding and alighting times

It is thought that passenger boarding and alighting times can be determined by the number of passengers boarding/alighting and the congestion rate on a train. This time value can therefore be calculated using Equation (4) \[ \text{(boarding and alighting time)} \]

\[ f(r) = f(r_{\text{on}})C_{\text{on}}n_{\text{on}} + f(r_{\text{off}})C_{\text{off}}n_{\text{off}}, \]

where \( r_{\text{off}} / r_{\text{on}} \) represents the congestion rate during alighting/boarding, \( C_{\text{off}} / C_{\text{on}} \) represents the alighting/boarding time for a single passenger when nobody else is on the train, and \( \alpha \) represents the parameter estimated from measurement surveys.

\[ f(r) = ar^2 + 1.0a = 0.083 \] (4)

- The frequency of incidents and the duration of delay times

In this case study, we introduce increases in the boarding/alighting time as incidents; alighting passengers positioned further inside the train may move more slowly than others, or the platform may be narrow. We assume such increases conform to normal distribution \( N(\mu(n), \sigma(n)) \), where \( n \) represents the total number of boarding and alighting passengers, \( \mu(n) \) and \( \sigma(n) \) and are calculated by Equations (5) and (6) estimated from measurement surveys.

\[ \mu(n) = 0.6223n(\text{sec}) \] (5)

\[ \sigma(n) = 0.2355n(\text{sec}) \] (6)

- The number of iterations for Monte Carlo simulation

The number of iterations is 100.

- Other assumptions in this case study

- All departure times on the two lines that connect trains at Stations (A) and (B) are on time.
- All passengers select the fastest available train route.
- Arrivals/departures that are earlier than planned are deemed to be on time.

4.2 Results of experiments

We calculated the following two different robustness indices (for the actual diagram and the proposed diagram) by iterating simulations and using Equation (3), and the results are shown in Fig. 8. The proposed diagram is confirmed as being more robust than the actual version.

![Proposed train diagram and changing points from the actual version](image)

![Fig. 6 Actual train diagrams](image)

![Fig. 7 Proposed train diagram and changing points from the actual version](image)

![Fig. 8 Results of robustness index experiments](image)
Table 1  Expected decreases in passenger disutility
(Shaded cells indicate the lower value)

<table>
<thead>
<tr>
<th>Expected increase in passenger disutility (mins)</th>
<th>0 ~ 2</th>
<th>2 ~ 4</th>
<th>4 ~ 6</th>
<th>6 ~ 8</th>
<th>8 ~ 10</th>
<th>10 ~ 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of passengers Actual diagram</td>
<td>72,690</td>
<td>9,110</td>
<td>8,490</td>
<td>10,600</td>
<td>4,520</td>
<td>490</td>
</tr>
<tr>
<td>Proposed diagram</td>
<td>69,490</td>
<td>8,980</td>
<td>8,810</td>
<td>11,540</td>
<td>3,480</td>
<td>240</td>
</tr>
</tbody>
</table>

Fig. 9  Expected decreases in passenger disutility

4.3 Discussions

Table 1 and Fig. 9 show the expected increase in disutility for all passengers. The findings are as follows:

- Passengers with a disutility increase in the range of 0 to 2 mins.
  ➢ The number of passengers in this range is large; approximately 70% of the total falls into this category, both for the actual diagram and the proposed version.

- Passengers with a disutility increase in the range of 10 to 12 mins.
  ➢ The number of passengers in the proposed diagram falls by 3,200 in comparison to the actual version, showing that the proposed one is more robust.

Table 2 also shows the expected number of passengers able to transfer according to the schedule. The number in the proposed diagram is 59 more than in the actual version.

<table>
<thead>
<tr>
<th>Table 2  Results of transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected number of passengers able to transfer trains as planned</td>
</tr>
<tr>
<td>Actual diagram</td>
</tr>
<tr>
<td>Proposed diagram</td>
</tr>
</tbody>
</table>

5. Conclusions

We have suggested a robustness index based on the viewpoint of passengers in order to evaluate the level of transportation services when unexpected incidents occur. Specifically, we define the robustness index as the expected increase in all passenger disutility under the assumption that it is possible to obtain the probability distribution of the frequency, location and length of delays. We have also calculated the robustness indices for the actual train diagram and the proposed version with arrival/departure times differing slightly from the planned schedule. The results confirm the proposed version as being more robust without the need for infrastructure improvements.

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References