Prediction of Strong Ground Motion over Wide Areas by Coupling the Stiffness Matrix Method and FEM Analysis

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Railway structures are often built near active faults in the Japanese Archipelago. In order to improve the safety of trains running on railway structures, therefore, it is important to predict earthquake ground motion taking into account the effect of such faults. We first investigated the characteristics of earthquake ground motion propagation over wide areas with active faults using the stiffness matrix method, and then developed a technique to predict near-fault ground motion by coupling the results of the stiffness matrix method with 2D FEM analysis.

**Keywords:** near-fault ground motion, stiffness matrix method, FEM analysis, coupling

1. Introduction

Many active seismic faults are located on the Japanese Archipelago, and railway structures are often built near these faults. To improve the safety of the overall railway system in such areas, therefore, it is important to predict earthquake ground motion with rational consideration of the effects of faults.

In order to clarify near-fault ground motion, an analytical method capable of executing the following two points is required:

1) Investigation of the propagation characteristics of earthquake ground motion over a wide bedrock area containing a fault.
2) Investigation of complicated wave propagation caused by soil non-linearity and irregularities in the surface ground.

It is very difficult, however, to develop an analytical method capable of executing both these points. In this study, the characteristics of wave propagation over a wide bedrock area were first clarified through 3D analysis using the stiffness matrix method, and the characteristics of seismic behavior in the surface ground were then investigated using 2D FEM analysis. Finally, near-fault ground motion was calculated by coupling the results of the stiffness matrix method with those of the 2D FEM analysis.

2. Investigation of wave propagation over a wide area containing a fault

2.1 Analytical method

In order to improve the safety of the railway system overall (i.e. the combination of ground-structure-vehicle), it is necessary to repeat the simulation of earthquake ground motion while changing various parameters. This study therefore required an analytical method with fast computation and high stability. For this purpose, we used the stiffness matrix method developed by Harada and Wang [1] to clarify the characteristics of wave propagation over a wide area containing a fault. This method can be applied only to horizontally layered ground. Considering the difficulty of obtaining the topographic features and ground properties of the bedrock area in the present geographic investigation, we assumed the bedrock area containing the fault as horizontally layered ground. This assumption seems sufficiently practical.

As an example, we assume the horizontally layered ground shown in Fig. 1. Using the transmission matrix formulated in the field of seismology, the relationship between the displacement and the stress values in the $i$-th layer is defined as follows:

$$
\begin{pmatrix}
u^{ii}(z_i) \\
\tau^{ii}(z_i)
\end{pmatrix} = \begin{pmatrix}
P_{11}^{ii} & P_{12}^{ii} \ \\
P_{21}^{ii} & P_{22}^{ii}
\end{pmatrix} \begin{pmatrix}
u^{ii-1}(z_{i-1}) \\
\tau^{ii-1}(z_{i-1})
\end{pmatrix}
$$

(1)

where $\mathbf{u}$ expresses the displacement vector; $\mathbf{t}$, the stress vector; and $\mathbf{P}$, the transmission matrix. $z_i$ and $z_{i-1}$ are the depths of the lower and upper surface in the $i$-th layer respectively. By formulating the relationships between the displacement and stress values in all layers in the same way and solving the equations, we can calculate the displacement values in the surface layer.

In the stiffness matrix method used in this study, on the other hand, the relationship between the displacement and stress values in each layer is formulated using stiffness matrix $\mathbf{K}$. At first, by transforming (1), the relationships between displacement and stress values in the
- The displacement of the boundary surface in each layer is therefore continuous as follows:

\[ \tau^i(z_i) = \tau^{i+1}(z_{i+1}) \]

(5)

2. The displacement of the boundary surface in each layer is continuous as follows:

\[ u(z_i) = u^i(z_i) = u^{i+1}(z_{i+1}) \]

(6)

where \( u(z_i) \) expresses the displacement in the depth of \( z_i \).

By superposing (2), (3) and (4) under the conditions shown in (5) and (6), the stiffness equation is obtained as follows:

\[
-\mathbf{K}_{m} \mathbf{u}^m(z_{m+1}) = \mathbf{K}_{s} \mathbf{u}^s(z_i) + \mathbf{K}_{s} \mathbf{u}^s(z_{i+1}) + \mathbf{q}^m(z_{m+1}) - \mathbf{q}^m(z_i) - \mathbf{q}^m(z_{i+1})
\]

(7)

where the external force vectors \( \mathbf{q} \) are determined by the parameters of the fault, and the stiffness matrices \( \mathbf{K} \) are obtained by the parameters of each soil layer. The displacement in each layer can therefore be calculated by solving (7). The stiffness equation is formulated in the frequency-wave number domain [2], so time histories of earthquake ground motion at arbitrary points can be obtained by applying triple-Fourier transformation to the solutions of (7).

In order to obtain the external force vector \( \mathbf{q} \), it is necessary to define a slip time function on a fault. In this paper, we used an approximate expression of the slip velocity time function taking into account the complicated process of slip on the fault developed by Nakamura and Miyatake [3]. Faults generally have asperities where slip displacements are larger than those in other parts of the fault. In this paper, however, we assume that slip displacements are the same for all fault parts.

### 2.2 Numerical simulation

We simulated sample earthquake ground motion for the ground model with the strike-slip fault shown in Fig. 3. The slip-velocity time function of the fault used in this simulation is shown in Fig. 4.

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**Fig. 2** Model of horizontally layered ground

**Fig. 3** Ground model

**Fig. 4** Slip time function used in simulation
Figure 5 shows snapshots of the velocity on the ground surface. As the slip of the fault progresses, clockwise and counterclockwise vortices are generated on the ground surface and propagate in the direction of the fault slip. We can therefore see that the velocities at two points within a few hundred meters are in opposite directions, which illustrates the complex characteristics of near-fault ground motion propagation.

3. Coupling of the stiffness matrix method and FEM analysis

3.1 Analytical method

The characteristics of earthquake ground motion over a wide area containing a fault were clarified using the stiffness matrix method outlined in Chapter 2. Conversely, local variations of earthquake ground motion in surface ground can be estimated using FEM analysis, which can take into consideration the non-linearity of soil and irregularities in the surface ground. In this chapter, we propose a method to evaluate not only the characteristics of wave propagation from the fault but also the properties of the surface ground by coupling the results of the stiffness matrix method with those of FEM analysis.

As the stiffness matrix method involves 3D analysis, it is preferable that the ground used in FEM analysis is expressed by a 3D model. In order to evaluate railway system safety, however, a ground model of about one kilometer in length would be needed. It is very difficult, therefore, to conduct 3D FEM analysis due to the limited capacity of computers. Additionally, considering that soil investigation is very rarely conducted at right angles to railway structures, it is not usually possible to ascertain sufficient properties to construct a 3D ground model. In this paper, therefore, we conducted 2D FEM analysis to clarify the characteristics of wave propagation in surface ground. In the following section, we describe the method of coupling the results of the stiffness matrix method with those of 2D FEM analysis.

3.1.1 Boundary conditions

On the assumption that a 2D FEM model is put into the 3D ground model used in the stiffness matrix method, it is necessary to take into account the input motion not only from the bottom of the FEM area but also from both sides. In cases where surface waves predominate in particular, it is problematic that the influence of surface waves from both sides of the FEM model cannot be taken into account in the present seismic response analysis [4]. Such analysis is, however, generally conducted using only the input motion from the bottom of the model. According to this practice, we set the boundary conditions using the method shown in Fig. 6. The accelerations obtained using the stiffness matrix method were set as the boundary condition at the bottom of the FEM model, and the simple slide boundary or isodisplacement boundary condition was used on both sides of the model.

3.1.2 Rotational transformation of earthquake ground motion

The direction of the coordinate system in the stiffness matrix method is generally different from that in the 2D FEM model as shown in Fig. 7. In this paper, we calculated the input motion in 2D FEM analysis by transforming the earthquake ground motion obtained using the stiffness matrix method as follows:
3.1.3 Interpolation of earthquake ground motion

The size of each mesh in the 2D FEM model is generally about one meter. However, the output interval of ground motion in the stiffness matrix method is usually greater than the size of the mesh in the 2D FEM model because computer capacity restrictions make it very difficult to calculate ground motion at intervals of one meter in the stiffness matrix method.

In order to conduct 2D FEM analysis, therefore, it is necessary to set the input motion by interpolating the waves calculated using (8). In this paper, we interpolated the earthquake ground motion in the frequency domain using the method proposed by Kawakami et al. [5].

We assume that the point at \( X = x' \) is put between two output points at \( X = 0, L \) in the stiffness matrix method as shown in Fig. 8. The Fourier transformations \( C_d(\omega) \), \( C_l(\omega) \) of time histories at the output points \( X = 0, L \) are expressed as follows:

\[
C_d(\omega) = |C_d(\omega)| \exp \{i \phi_d(\omega)\} \\
C_l(\omega) = |C_l(\omega)| \exp \{i \phi_l(\omega)\}
\]  

(9)

(10)

where \( |C_d(\omega)|, |C_l(\omega)| \) express the Fourier amplitudes, and \( \phi_d(\omega), \phi_l(\omega) \) are the Fourier phase spectra. Then, the Fourier transformation \( C_r(\omega) \) of the time history at the point of \( X = x' \) is expressed using \( C_d(\omega) \), \( C_l(\omega) \) as follows:

\[
C_r(\omega) = \frac{1}{L} \left \{ - \exp \left [ i \left ( \phi_d(\omega) - \phi_l(\omega) \right ) \right ] \right \} \frac{L}{\omega} \left \{ - \exp \left [ i \left ( \phi_d(\omega) - \phi_l(\omega) \right ) \right ] \right \}
\]  

(11)

The time history \( f_r(t) \) at the point of \( X = x' \) is obtained by the Fourier inverse transformation of \( C_r(\omega) \). However, the value of \( \phi_d(\omega) - \phi_l(\omega) \) in (11) is not determined uniquely due to the periodicity of phase spectra. In this paper, we decided the value of \( \phi_d(\omega) - \phi_l(\omega) \) using (12) shown below, so that the phase velocity obtained from the variation of phase, \( \phi_d(\omega) - \phi_l(\omega) \), is almost the same as the apparent phase velocity \( c \) calculated using the time-history cross-correlation function of time histories \( f_d(t), f_l(t) \).

\[
-c \frac{\omega L}{\omega - \phi_l(\omega) - \phi_l(\omega) \leq \frac{\omega L}{c} + \pi}
\]  

(12)

3.2 Numerical simulation

We assume that soft ground with a non-flat base is put into the 3D ground model containing a fault as shown in Fig. 3. Figure 9 shows the relationship between the fault and the soft ground with a non-flat base. In this section, we outline 2D dynamic FEM analysis in the out-of-plane direction for the following cases (shown in Fig. 10):

(Case 1) The earthquake ground motion calculated using the method shown in 3.1 is input at each point at the bottom of the non-flat-base soft ground.

(Case 2) The earthquake ground motion at the point of \( X = 0 \) in Case 1 is input uniformly at all points at the bottom of the non-flat-base soft ground.

We then conducted analysis, taking into account the non-linearity of the soil in the soft ground with a non-flat base. The RO model was used as the nonlinear constitutive law of soil.

Figures 11 and 12 compare the acceleration distributions for Cases 1 and 2. For Case 2, we see a wave generated in the base slant area that propagates in the horizontal direction. Due to these wave propagation characteristics, the earthquake ground motion amplifies around the base slant area. Away from this area, however, the accelerations at each point are almost the same. For Case 1, on the other hand, the arrival time of earthquake ground motion is different at each point on the horizon-
tally layered ground due to the time lag of input motion. In the base slant area, the characteristics of wave propagation then become more complicated under the influence of horizontally propagating waves generated around the area in addition to the time lag of input motion.

Figure 13 shows the acceleration time histories at five points on the ground surface for Case 1. Comparing the waves at \(X = -400\) (m) and \(X = 0\) (m) shows that the arrival time of the first pulse is more delayed as the distance from the fault becomes greater. These characteristics occur because of waves propagating from the fault. At the place on the soft surface layer \((X > 0)\), the arrival time of the first pulse is then delayed by about 0.3 sec compared to that at \(X = -400\) (m) due to the slow secondary wave velocity of the soft surface layer in addition to the time lag of waves propagating from the fault. As time passes, the direction of acceleration at the place on the soft surface layer \((X > 0)\) becomes opposite to that of the base area \((X < 0)\). If railway structures are built around
this area, the difference in displacement between two structures may become large.

To examine the safety of railway systems near faults, therefore, it is essential to take into account the influence of not only the wave propagation from the fault but also the properties of the surface ground.

4. Conclusions

In this paper, we have clarified the characteristics of wave propagation over a wide area containing a fault using the stiffness matrix method, and have proposed a technique to couple the stiffness matrix method with 2D FEM analysis. We then conducted numerical simulation of near-fault ground motion using the proposed method. The results are as follows:

1) As the slip of the fault progresses, clockwise and counterclockwise vortices are generated on the ground surface and propagate in the direction of fault slip, according to analysis using the stiffness matrix method. As a result, the characteristics of wave propagation near the fault become complicated.

2) According to analysis using the proposed method, on soft ground with a non-flat base near the fault, the arrival time of earthquake ground motion is different at each point on the horizontally layered ground due to the time lag of input motion. In the base slant area, the characteristics of wave propagation become more complicated under the influence of horizontally propagating waves generated around the area in addition to the time lag of input motion.

3) To discuss the safety of railway systems near faults, therefore, it is important to consider the influence of not only wave propagation from the fault but also the properties of the surface ground.

The method of setting the boundary conditions in the proposed technique, however, should be improved further. We plan to continue investigation of these issues in future studies.

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