Modeling the Phase Spectrum Characteristics of Ground Motion Considering Source, Propagation Path and Local Site Effect Amplification

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Modeling the phase characteristics of earthquake ground motion is important in synthesizing a design earthquake motion consistent with a given set of response spectra. We assume that earthquake ground motion can be expressed by a convolution of the three time functions of source, path and site effect. This paper presents a new methodology to model the phase characteristics of earthquake motion using the concept of group delay time. The group delay times of source effects caused by rupture propagation on the fault plane are theoretically calculated, while those of the path and site effects are empirically modeled from observed records using the inversion technique. We also demonstrate that earthquake motion can be synthesized based on our newly developed phase model.

Keywords: phase spectrum, group delay time, earthquake motion synthesis

1. Introduction

The design earthquake motion is often defined by response spectra. To check the seismic capacity of designed structures, however, it is necessary to perform dynamic analysis using earthquake motion (time history) which is compatible with the design spectra. In this regard, the phase characteristic of earthquake motion is one of the most important issues in addition to that of the amplification characteristic.

Early-stage research has been performed to simulate design earthquake motion. One rudimentary method is to use an envelope function [1] and multiply it to the stationary wave form simulated using a random phase criterion. Another approach is to use the phase spectrum of a particular instance of observed earthquake motion [2]. However, these methods do not clarify the phase characteristics of earthquake motion.

We have previously pointed out that the phase characteristic of earthquake motion strongly controls its non-stationary nature, and have developed methods to model the phase characteristics of earthquake motion [3, 4]. One of these is an empirical approach by which regression equations for the mean and standard deviation of group delay times of earthquake motion have been derived as functions of earthquake magnitude and epicentral distance [3]. Another one is a theoretical approach through which we have developed a method to model the group delay time for near-source earthquake motion by assuming that the rupture process of an earthquake fault can be expressed by a train of impulses and that the minimum phase concept is effective in evaluating phase shift caused by wave propagation [4].

In the empirical method, regression equations were introduced based on the assumption that the group delay time of earthquake motion can be represented as a product of the source, the path and the local site effects. However, it subsequently became clear that this assumption did not hold for the mechanism of earthquakes. In addition, the equations could not be used to estimate phase

spectra in near-source regions because the assumption of a single-source-mechanism is adopted. The theoretical method used only direct S-wave motion, and it was concluded that this approach is not suitable for taking the effect of surface wave motion on the group delay time into account.

We therefore developed a new method to model phase spectra in order to consider the earthquake mechanism and offer a wider range of applicability than our past methods. We also demonstrated its efficiency in synthesizing earthquake motion.

2. Outline of earthquake ground motion phase spectrum modeling

It is well known that a time history of earthquake ground motion $O(t)$ can be described by a convolution of the time functions of source effects, $S(t)$, transmission path effects, $P(t)$, and local site amplification effects, $L(t)$. The Fourier transform of $O(t)$ is obtained from:

$$O(\omega) = S(\omega) \cdot P(\omega) \cdot L(\omega) \cdot \exp[i(\phi^S(\omega) + \phi^P(\omega) + \phi^L(\omega))] \quad (1)$$

where $S(\omega)$, $P(\omega)$ and $L(\omega)$ are the Fourier amplitude spectra of each time function and $\phi^S(\omega)$, $\phi^P(\omega)$ and $\phi^L(\omega)$ are the Fourier phase spectra of each time function. Equation (1) shows that the phase spectrum of earthquake ground motion, $\phi(\omega)$, can be represented as a linear summation of these three phase spectra.

The group delay time is defined by the derivative of the Fourier phase spectrum $\phi(\omega)$ with respect to circular frequency ω [5]:

$$t_{gr}(\omega) = \frac{d\phi(\omega)}{d\omega} \quad (2)$$

The average value of the group delay time within a certain frequency band for the central frequency ω expresses the arrival time of a wave component with frequency ω . The distribution width of the group delay time is related to the duration of the time history of the wave

component [6]. Because of these characteristics of group delay time, its modeling is much easier than direct modeling of the phase spectrum.

Since the group delay time is a derivative of the Fourier phase spectrum, the linear relationships of the phase are saved and the group delay time of earthquake ground motion $t_{gr}^O(\omega)$ is expressed by:

$$t_{gr}^O(\omega) = t_{gr}^S(\omega) + t_{gr}^P(\omega) + t_{gr}^L(\omega) \quad (3)$$

where $t_{gr}^S(\omega)$, $t_{gr}^P(\omega)$ and $t_{gr}^L(\omega)$ are the group delay time functions of the source, the path and the local site characterized functions, respectively. Each group delay time function is modeled as described below.

The source group delay time function, $t_{gr}^S(\omega)$, is obtained theoretically by modeling the rupture process of an earthquake fault as a train of impulses [4]. The path effects on the group delay time function, $t_{gr}^P(\omega)$, and the local site effects on the group delay time function, $t_{gr}^L(\omega)$, are modeled empirically based on regression analysis of earthquake records.

3. Modeling of the group delay time function

3.1 Group delay time function of source effects

The Haskell-type fault model is assumed. The fault plane is divided into $n_L \times n_W$ elements, each of which corresponds to the area of a small event. The motion of a large event, $g_{0L}(t)$, at an observation point is expressed by taking into account the time delay of each small event occurrence, $g_{0S}(t)$ (see Fig.1) [7].

$$g_{0L}(t) = \sum_{i=1}^{n_L} \sum_{j=1}^{n_W} g_{0S}(t - t_{ij}) + \sum_{i=1}^{n_L} \sum_{j=1}^{n_W} \sum_{k=1}^{(n_D-1)n'} \frac{1}{n'} g_{0S}(t - t_{ijk}) \quad (4)$$

in which

$$t_{ij} = \frac{(R_{ij} - R_0)}{V_s} + \frac{\xi_{ij}}{V_r}, \quad t_{ijk} = t_{ij} + \frac{k\tau}{(n_D - 1)n'} \quad (5)$$

where τ is the rise time, V_s is the shear wave velocity of the media, V_r is the rupture velocity, ξ_{ij} is the distance between the starting point of rupture and small elements (i, j) , the values of n_L , n_W and n_D represent the similarity ratios of fault length, width and rise time between large and small events, R_{ij} is the distance between the representative point of the small event and the observation point, and n' is an arbitrary integer used to eliminate artificial periods caused by rise time subdivision.

Equation (4) can be deconvoluted with respect to the source time function and the train of impulses $p(t)$. The train of impulses $p(t)$ expresses the rupture process of the earthquake fault. The group delay time from the source effects, $t_{gr}^S(\omega)$, is obtained by differentiating the phase spectrum of $p(t)$ with respect to the circular frequency ω :

$$t_{gr}^S(\omega) = \frac{-\sum_{i=1}^N a_i^2 t_i - \sum_{i=1}^{N-1} \sum_{j=i+1}^N a_i a_j (t_j + t_i) \cos\{\omega(t_j - t_i)\}}{\sum_{i=1}^N a_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N a_i a_j \cos\{\omega(t_j - t_i)\}} \quad (6)$$

where a is the amplitude of impulse and N is the total number of impulses.

Fig. 1 Schematic figure of fault model

3.2 Group delay time function of path effects

Equation (3) indicates that the average group delay time and its standard deviation can be expressed by (7).

$$\mu_{igr}^O(\omega) = \mu_{igr}^S(\omega) + \mu_{igr}^P(\omega) + \mu_{igr}^L(\omega) \quad (7a)$$

$$(\sigma_{igr}^O(\omega))^2 = (\sigma_{igr}^S(\omega))^2 + (\sigma_{igr}^P(\omega))^2 + (\sigma_{igr}^L(\omega))^2 \quad (7b)$$

When the earthquake magnitude is small, the assumption of the single source mechanism can be adopted to model the group delay time. In this case, the first term on the right of (7), which expresses the source effects, is negligible. When the observation point is located on base rock with a shear velocity close to 3,000 m/s, the third term on the right of (7) may be small enough to ignore. Only the path effects are therefore included in earthquake records that satisfy the above two conditions:

$$\mu_{igr}^O(\omega) = \mu_{igr}^P(\omega) \quad (8a)$$

$$(\sigma_{igr}^O(\omega))^2 = (\sigma_{igr}^P(\omega))^2 \quad (8b)$$

The group delay time function with the path effects, $t_{gr}^P(\omega)$, is related to the arrival time delay and the expansion of the duration time of waves due to wave propagation from the epicenter. This suggests that the value of $t_{gr}^P(\omega)$ strongly depends on the epicentral distance [8, 9]. It can therefore be assumed that the mean value $\mu_{igr}^P(\omega)$ and its variation $(\sigma_{igr}^P(\omega))^2$ are proportional to the square of the epicentral distance R .

$$\mu_{igr}^P(\omega) = \beta_1(\omega) \cdot R^2 \quad (9a)$$

$$(\sigma_{igr}^P(\omega))^2 = \beta_2^2(\omega) \cdot R^2 \quad (9b)$$

where $\beta_1(\omega)$ and $\beta_2(\omega)$ are coefficients. Earthquake records with small events (magnitude) observed at the base rock were collected, and least-squares analysis was conducted to determine the coefficients $\beta_1(\omega)$ and $\beta_2(\omega)$. The relationships between the magnitude and the epicentral distance in the data used are shown in Fig. 2 (a). The relationship between the shear wave velocity at the observation point and the epicentral distance is also shown in Fig. 2 (b). Figure 3 shows the values of coefficients $\beta_1(\omega)$ and $\beta_2(\omega)$, and clarifies their strong dependence on frequency ω ; they are large in the low-frequency range

Fig. 2 Epicentral distance, earthquake magnitude and shear velocity of observation points of data used in the analysis

Fig. 3 Regression coefficients $\beta_1(\omega)$ and $\beta_2(\omega)$

and become smaller in the higher-frequency range. This is because the dispersion of propagating waves is large in the low-frequency range, and as a result their arrival time and duration are extended.

3.3 Group delay time function of local site conditions

In a case when the magnitude of an earthquake is small (i.e., the assumption of the single source mechanism can be adopted), the group delay time function related to local site amplification can be separated from observed records by:

$$\mu_{igr}^L(\omega) = \mu_{igr}^O(\omega) - \beta_1(\omega) \cdot R^2 \quad (10a)$$

$$(\sigma_{igr}^L(\omega))^2 = (\sigma_{igr}^O(\omega))^2 - \beta_2^2(\omega) \cdot R^2 \quad (10b)$$

The characteristics of the group delay time function in relation to local site conditions are calculated from observed earthquake records by using (10). Seismic records observed using the underground seismograph of the KiK-NET system were used in this study in order to exclude the influence of topography and soil nonlinearity. The data used are listed in Table 1.

$\mu_{igr}^L(\omega)$ and $(\sigma_{igr}^L(\omega))^2$, which signify the characteristics of the group delay time in relation to local site effects, are modeled by (11) as it is thought that these values are related to the shear velocity of the soil where the seismograph is located.

$$\mu_{igr}^L(\omega) = \gamma_1(\omega) \cdot (3000 - V_s)^2 \quad (11a)$$

$$(\sigma_{igr}^L(\omega))^2 = \gamma_2^2(\omega) \cdot (3000 - V_s)^2 \quad (11b)$$

The relationship between shear velocity V_s and the average group delay time $\mu_{igr}^L(\omega)$ and its variation $(\sigma_{igr}^L(\omega))^2$

Table 1 Observation data used in the analysis

Observation station	Shear wave velocity of the layer where underground seismometer is set	Numbers of used data
AOMH03	620	3
AOMH12	820	3
AOMH13	430	8
AOMH16	770	4
IWTH01	1300	2
IWTH11	1080	3
IWTH12	1130	3
IWTH15	680	5
IWTH16	1160	4
MYGH05	690	4
MYGH07	740	5
YMTH01	600	1
YMTH02	510	5
YMTH06	1090	3
KNGH10	1250	4
KNGH20	1400	1
TKYH12	1800	1
NARH04	1807	2
NARH06	1791	2
SIGH02	1500	2
HYGH01	826	1
KYTH08	2770	1
KGSH05	680	2
KGSH07	710	2
Total numbers of used data N		71
NS and EW components: $2N$		142

at a representative frequency are shown in Figs. 4 and 5. Regression analysis using (11) was conducted, and the results are outlined in these two figures. Figure 6 also shows the regression coefficients $\gamma_1(\omega)$ and $\gamma_2(\omega)$. Using regression equation (11), the average group delay time $\mu_{igr}^L(\omega)$ and its variation $(\sigma_{igr}^L(\omega))^2$ at the seismic base rock (whose shear velocity is 400 m/s) can be calculated, and the results are shown in Fig. 7. We can use the values of Fig. 7 to synthesize earthquake ground motion on seismic base rock.

Even though it is clear that the group delay time function is affected by the structure of sedimentation between the base rock and the observation depth, and that site characteristics are not determined only by the properties of the soil where the seismograph is located, we can say that the shear velocity of the soil where the seismograph is located is a good index for representing site conditions, and that (11) is adequate for expressing local site effects on the group delay time.

4. Simulation of Earthquake Motion

We applied this method to simulate earthquake motion just above a fault with the magnitude of 7.4 on the Japan Meteorological Agency scale and verified its efficiency.

Fig. 4 Relationship between $\mu_{igr}^L(\omega)$ and V_s

4.1 Setting of source parameters

The phase of the source function was obtained theoretically using (6) developed as outlined in Section 3.1. The assumed earthquake was a vertical-fault type with a magnitude of 7.4 on the Japan Meteorological Agency scale. The macroscopic and microscopic parameters of the fault were defined based on “the recipe for predicting strong ground motion”. In parameter studies, we selected several combinations for the starting point of fault rupture and the distribution of asperity, as these values significantly influence the phase characteristics of earthquake motion. There were three types of asperity distribution and two rupture starting points. An example of asperity distribution and the fault rupture starting point is shown in Fig. 8, in which the eight observation points around the fault are considered to define the earthquake motion.

Representative results for trains of impulses expressing the rupture process of the fault are shown in Fig. 9. These results were obtained using the asperity and rupture starting point distribution shown in Fig. 8. Point A is located in the direction of rupture propagation, and Point B is located in the opposite direction. The scattering of the train of impulses is small for the observation point lo-

Fig. 5 Relationship between $\sigma_{igr}^L(\omega)^2$ and V_s

Fig. 6 Regression coefficients $\gamma_1(\omega)$ and $\gamma_2(\omega)$

Fig. 7 Group delay time characteristics with local site amplification at the seismic base rock

Fig. 8 Example of rupture fault and relative position between observation points

Fig. 9 Simulated train of impulses

cated in the direction of rupture propagation (Point A), but becomes large in the opposite direction (Point B). The directivity of rupture propagation is clearly shown in these figures.

4.2 Group delay time with function of transmission path and function of local amplification

The assumed earthquake involved ground motion just above the fault. The group delay time in relation to the transmission path was calculated from (9) with a shortest distance of 3 km. The group delay time caused by local site amplification was stochastically calculated from the average group delay time $\mu_{igr}^L(\omega)$ and its standard deviation $\sigma_{igr}^L(\omega)$ (as shown in Fig. 7), which was defined at the ground surface with a shear velocity of 400 m/s. However, the standard deviation $\sigma_{igr}^L(\omega)$ could be used with some correction.

The group delay time $t_{igr}^P(\omega) + t_{igr}^L(\omega)$ related to the transmission path and local site amplification was simulated by generating random values based on a normal distribution $N(\mu_{igr}^P(\omega) + \mu_{igr}^L(\omega), \sigma_{igr}^P(\omega) + \sigma_{igr}^L(\omega))$. The total group delay time $t_{gr}(\omega)$ was then calculated using (3). The phase spectrum $\phi(\omega)$ was obtained by integrating $t_{gr}(\omega)$ with respect to ω .

4.3 Synthesizing earthquake time history

We simulated earthquake motion compatible with a response spectrum $S_{DA}(\omega)$ defined at just above the fault as shown in Fig. 10 [10] using the phase spectrum $\phi(\omega)$ obtained as outlined in Section 4.2. The response spectrum was defined with a damping coefficient of 5%. To simulate earthquake motion compatible with the response spectrum, we need an initial Fourier amplitude spectrum $A_0(\omega)$. This was assumed to be given by the velocity response spectrum with a zero damping coefficient converted from the acceleration response spectrum $S_{DA}(\omega)$. The time history can be simulated by the inverse Fourier transform (FFT) of the initial Fourier amplitude spectrum $A_0(\omega)$ and the phase spectrum $\phi(\omega)$ obtained as outlined in Section 4.2. As the response spectrum calculated using this earthquake motion did not coincide with the target response spectrum, we modified the initial Fourier amplitude $A_0(\omega)$ at circular frequency ω by multiplying the ratio $r(\omega)$ between the design response spectrum and the calculated response spectrum:

$$r(\omega) = \frac{S_{DA}(\omega)}{S(\omega)} \quad (12)$$

where $S_{DA}(\omega)$ is the target spectrum and $S(\omega)$ is the calculated spectrum. Assuming the calculated $r(\omega) \times A_0(\omega)$ as the new initial Fourier amplitude of earthquake motion and using the original phase spectrum $\phi(\omega)$, we can simulate the earthquake motion of the second step. By repeating this process until the ratio $r(\omega)$ reaches almost 1.0, we can finally simulate earthquake motion compatible with the target response spectrum. The synthesized time history is shown in Fig. 11. The simulated earth-

Fig. 10 Response spectrum of an earthquake just above the fault

Fig. 11 Example of earthquake motion time history synthesized using our phase model

quake motion shows typical near-source motion characteristics such as a short duration time and a pulse-like waveform. This indicates that the newly developed phase model is very effective for synthesized earthquake motion.

5. Conclusions

A method to model the phase spectra of earthquake motion using the concept of group delay time was proposed. In this method, group delay times from source effects, propagation path effects and local site amplification effects are modeled independently. The model satisfied the laws of seismological physics, and its applicability to the synthesis of earthquake motion was confirmed.

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