Evaluation for Flexural-load Capacity of Prestressed Concrete Girders with Broken Tendons

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The use of low-grade material sheath grouting for PC girders has resulted in some PC girders initially constructed in the 1950s having insufficiently filled sheaths. The exposure of PC tendons in the sheath generates corrosion and rupture of the PC tendons, reducing the load-bearing capacity of PC girders. This research focused on the interface bond between PC tendons and grout in the sheath through loading tests in order to explain the loading capacity of deteriorated PC girders.

Keywords: flexural-load capacity, broken tendons, PC girder

1. Introduction

After the construction of the first Daidogawa-bridge in 1954- the first post-tensioned prestressed concrete (PC) girder in Japan- more than 10,000 such post-tensioned PC girders have been used in constructions for Japanese railways. The inside of the sheaths for such PC girders should be filled with grout to protect the PC tendons from corrosion. The grout, however, does not completely fill inside the sheath for various reasons [1]. As a result, cases have been reported of corrosion and ruptured tendons [2]. PC tendons in post-tensioned PC girders are anchored at both ends in order to provide the necessary stress. Corrosion of these tendons therefore drastically changes the mechanical performance of PC girders and reduces the serviceability and safety of the structure. Therefore, the mechanical performance of incompletely-grouted PC girders with broken tendons should be evaluated in order to carry out the appropriate repairs and reinforcement work [3].

If the sheath is partially filled with grout, stress transfers from the tendons to the concrete because of bond between the tendons and the grout after the tendon snaps. This means that the bond between grout and tendons as well as the residual stress of the PC girder is key in reasonable evaluation of the PC girder’s mechanical performance. There are several reports on pull-out tests involving PC tendons from concrete if reference is made to tests performed on reinforcement bars [4]. The method, however, cannot be utilized to duplicate the strain conditions of a broken tendon.

There is growing understanding about non-destructive evaluation systems to assess grout inside the sheath. The corrosion process of PC tendons also has been widely reported on around the world. Integration of this research with the evaluation of the residual stress harbors the potential of leading to direct method for evaluating the mechanical performance of PC girders.

The objective of this paper is to evaluate the residual stress based on the bond between the PC tendon and the grout inside the sheath, and the flexural load capacity of incompletely-grouted PC girders with broken tendons. To meet this goal, a series of PC tendon rupture tests were carried out by using uniaxially prestressed specimens. The bond characteristics of the PC tendon were assumed to be a function of the PC-tendon type and the grout strength. Finally, the paper proposes a flexural load capacity for incompletely-grouted PC girders with broken tendons with verification on the basis of experimental results.

2. Evaluation of the residual prestress of PC-girders with broken tendons

2.1 Rupture test with uniaxially tensioned PC-tendon specimens

Figure 1 is a schematic diagram of the specimen. The rectangular cross-section of the specimen was either 250 × 250 mm or 400 × 400 mm, and a hole was located at the edge of test sample in order to cut the PC tendon before loading. The target compressive strength of the concrete was 50 N/mm².

The grout filling was injected into the steel sheath, and four longitudinal rebars were arranged around it. Stirrups with the nominal diameter of 13 mm were fitted at 100 mm intervals. In addition, the deformed rebars equipped with a series of strain gauges were installed along the sheath of the specimen.

Table 1 lists the parameters of the nine specimens used in this study: the compressive strength of the grout (grout strength) and the type of PC-tendon. According to Korenaga and Watanabe [4], both affect the tendon/grout bond characteristics. Prestressing and cutting of the tendon (as Fig. 2) took place 10 and 30 days after casting, respectively.

2.2 Residual stress ratio

The magnitude of residual stress after cutting the PC tendon was measured by using the gauges fitted to the rebars. Figure 3 shows the residual stress ratio, which was calculated by comparing pre- and post-severance strain on the tendon.

Comparison of the results of experiments 1-2 and 1-3
Table 1 Specimen detail

<table>
<thead>
<tr>
<th>No.</th>
<th>Dimension</th>
<th>PC tendon</th>
<th>Grout</th>
<th>Concrete</th>
<th>Effective prestress force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_1) (mm)</td>
<td>(a_2) (mm)</td>
<td>(b) (mm)</td>
<td>(c) (mm)</td>
<td>Sheath (mm)</td>
</tr>
<tr>
<td>1-1</td>
<td>250</td>
<td>250</td>
<td>3000</td>
<td>280</td>
<td>38</td>
</tr>
<tr>
<td>1-2</td>
<td>250</td>
<td>250</td>
<td>3000</td>
<td>280</td>
<td>38</td>
</tr>
<tr>
<td>1-3</td>
<td>250</td>
<td>250</td>
<td>3000</td>
<td>280</td>
<td>38</td>
</tr>
<tr>
<td>1-4</td>
<td>250</td>
<td>250</td>
<td>3000</td>
<td>280</td>
<td>38</td>
</tr>
<tr>
<td>1-5</td>
<td>250</td>
<td>250</td>
<td>3000</td>
<td>280</td>
<td>38</td>
</tr>
<tr>
<td>1-6</td>
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<td>10000</td>
<td>450</td>
<td>65</td>
</tr>
<tr>
<td>1-7</td>
<td>400</td>
<td>400</td>
<td>10000</td>
<td>450</td>
<td>65</td>
</tr>
<tr>
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<td>250</td>
<td>280</td>
<td>38</td>
<td>12T12.7</td>
</tr>
<tr>
<td>1-9</td>
<td>250</td>
<td>250</td>
<td>280</td>
<td>38</td>
<td>12T12.7</td>
</tr>
</tbody>
</table>

\(\phi\): Diameter of round bar, \(f_r\): Yield strength of PC tendon, \(f'_c\): Compressive strength of grout, \(E_c\): Young's modulus of grout, \(f'_c\): Compressive strength of concrete, \(E_c\): Young's modulus of concrete.

Figure 3(a) illustrates the influence of the grout strength on the residual stress ratio. The figure indicates that as grout strength increases so does tendon/grout bond, and consequently the stress-decreasing region shrank where the ratio fell below 1.0.

Figure 3(b): the results of 1-8 and 1-9, clearly show that grout strength had a greater influence on the size of the stress-decreasing area when the PC tendon was a round rebar. In addition, Fig. 3(c), which compares the result of 1-9 using a round rebar PC tendon and 1-3 using strand-type PC tendon shows that the length of the stress-decreasing region also changes strongly depending on the type of tendon. Figure 3(d) indicates the PC tendon comprising two or more strands increased the length of the stress-decreasing region because the contact between the strands prevented their bond to the grout.
2.3 Calculation of residual stress

Figure 3 implies that in the stress-decreasing region there is a linear relationship between the residual stress and the distance from the breaking position of PC tendon. By extension, therefore, it is possible to assume a constant bond force in the stress-decreasing region. This study calculated the bond strength by dividing the effective stress by the surface area of the PC tendon.

Based on the experimental results, Fig. 4 plots the bond strength as a function of grout strength, which displays a tendency for bond strength to increase with grout strength. With reference to Korenaga and Watanabe [4] this study assumed the latter relationship to be a function of the grout compressive strength. $(f'_c)$

As opposed to existing research by Korenaga and Watanabe [4] which focused on the bond-slip displacement based on pull-out tests of PC tendons from concrete, the phenomenon examined in this research is the generation of bond force in relation to stress released by PC tendons. Even though the two phenomena are different from the point of view of the bond characteristics with grout, in terms of extension or shrinkage of the PC tendon, this study as far as possible employs the functions derived by Korenaga and Watanabe [4].

Results shown in Fig. 3(d) confirm that the area of the stress-decreasing zone differed between single and multi-cable-strand specimens. Therefore, the bond strengths of grout and PC tendon were obtained as a function of $f'_c$ in (1) and (2) respectively, according to the number of cable strands. Calculations yielded through (1) and (2) are plotted in Fig. 4 to allow comparison with the experimental results.

\[ r_1 = 0.55 \sqrt{f'_c} \]  (1)

\[ r_2 = 0.30 \sqrt{f'_c} \]  (2)

The bond-slip relationship was expressed as a bi-linear relationship based on the bond strength obtained with (1) or (2).

Figure 5 shows the pattern diagrams of the strain distribution of concrete and PC tendons after rupture of the tendon in the PC beam. In certain sections the forces are balanced: the accumulation of bonding force between the PC tendon and grout is equal to the decreased force in the tendon in the section, and is derived with the following (3) to (6).

\[ \varepsilon_{s,x} = \varepsilon_s + \left( \frac{\varepsilon_{pp} - \varepsilon_{s,x-1}}{2} \right) \Delta x - \left( \frac{\varepsilon_{pp} + \varepsilon_{s,x-1}}{2} \right) \Delta x \]  (3)

\[ r_{s,x-1} = \frac{E_s (\varepsilon_{s,x-1} - \varepsilon_{s,x}) A_s}{\pi Dx} \]  (4)

\[ r_{pp-1} = \frac{E_p (\varepsilon_{pp-1} - \varepsilon_{pp}) A_p}{\pi Dx} \]  (5)

\[ r_{s,x} = r_{pp} \]  (6)

where,

$P$: Effective prestress force (N),

$\varepsilon_p$: Strain of PC tendon,

$\varepsilon_s$: Strain of concrete,

$s$: Slip displacement (mm),

$x$: Distance from cut-point $\Delta x = x_{s,x-1} - x_{s,x-1}$ (mm),

$A_s$: Cross sectional area of rebars (mm$^2$),

$E_p$: Young’s modulus of the PC tendon (kN/mm$^2$),

$D$: Nominal diameter of rebar (mm),

$A_c$: Cross sectional area of concrete (mm$^2$),

$E_c$: Young’s modulus of the concrete (kN/mm$^2$),
The residual stress ratio in the specimens was calculated with (3) to (6), and plotted in Fig.3. Figure 6 indicates the stress-decreasing region from both experimental and calculation results. The latter confirms that the calculated values can reproduce test results. The stress-decreasing region and the magnitude of stress were calculated by the developed bond-slip relationship ($\tau$-s) and (1) to (6).

3. Evaluation of flexural load capacity of incompletely-grouted pc girders with broken tendons

3.1 Bending test for PC girders with broken tendons

Table 2 lists the parameters of the specimen test and materials used in this study; Fig.7 shows the specimen detail. A single-strand PC tendon was utilized (SWPR7BL: 12T12.7) with a yield strength ($f_{py}$) of 1849 N/mm$^2$, tensile strength of 2020 N/mm$^2$, and Young's modulus ($E_c$) of 194 N/mm$^2$. The longitudinal rebar had a nominal diameter of 12.7 mm, tensile yield strength ($f_{sy}$) of 370 N/mm$^2$, and Young's modulus ($E_s$) of 194 kN/mm$^2$. Stirrups had a nominal diameter of 15.9 mm, tensile yield strength ($f_{sy}$) of 345 N/mm$^2$, and Young's modulus ($E_s$) of 194 kN/mm$^2$.

A steel sheath with a diameter of 65 mm was used. The sheaths were filled with grout (compressive strength of 34.1 N/mm$^2$). Bond between the PC tendon and the grout was removed over 1000 mm from one end of the support beam by wrapping tape around the tendon. Four specimens were prepared: 2-1 with tendons intact and 2-2 to 2-4 with 50 to 75 % of tendons broken. PC tendons in 2-2 and 2-3 were cut before the loading test, and tendons in 2-4 were gradually severed applying a load of 620 kN.

Table 2: Specimen detail

<table>
<thead>
<tr>
<th>No.</th>
<th>Concrete</th>
<th>Effective prestress force (kN)</th>
<th>Cutting ratio (%)</th>
<th>PC-tendon yield load</th>
<th>Flexural capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f'c$ (N/mm$^2$)</td>
<td>$E_c$ (kN/mm$^2$)</td>
<td>$P_y$ (N)</td>
<td>$P_{y1}$ (N)</td>
<td>$P_{y}/P_{y1}$</td>
</tr>
<tr>
<td>2-1</td>
<td>58.1</td>
<td>39.5</td>
<td>1213 0</td>
<td>448 439</td>
<td>1.02</td>
</tr>
<tr>
<td>2-2</td>
<td>1268</td>
<td>50</td>
<td>381 401</td>
<td>0.95</td>
<td>698 594</td>
</tr>
<tr>
<td>2-3</td>
<td>1174</td>
<td>75</td>
<td>355 276</td>
<td>1.28</td>
<td>600 430</td>
</tr>
<tr>
<td>2-4</td>
<td>1217</td>
<td>0 ~ 67</td>
<td>-</td>
<td>356 544</td>
<td>1.13</td>
</tr>
</tbody>
</table>

$f'c$: Compressive strength of concrete. $E_c$: Young's modulus of concrete, $P_y$: Load at longitudinal rebars yielding, $P_{y1}$: Load corresponding to yielding moment ($M_{y1}$) acting on failed cross-section, $P_{max}$: Experimental value of flexural load capacity, $P_{mu}$: Load corresponding to maximum moment ($M_u$) acting on failed cross-section, $P_{max}/P_{mu}$: Experimental value of flexural load capacity.

The residual stress ratio in the specimens was calculated with (3) to (6), and plotted in Fig.3. Figure 6 indicates the stress-decreasing region from both experimental and calculation results. The latter confirms that the calculated values can reproduce test results. The stress-decreasing region and the magnitude of stress were calculated by the developed bond-slip relationship ($\tau$-s) and (1) to (6).

3. Evaluation of flexural load capacity of incompletely-grouted pc girders with broken tendons

Figure 8 shows the crack distribution on the tested specimens. All specimens suffered flexural failure showing concrete spalling at the top-edge. Table 2 summarizes the experiment and calculation results. The flexural cracks were generated under loads of between 252 to 346 kN. The number of cracks increased with the number of severed PC tendons.

According to the crack distribution in 2-2, some flexural cracks were observed on the left side, and concrete spalling appeared in the middle span of the specimen. Exfoliation shifted to the left -1300 mm from left support in specimen 2-3. The number of flexural cracks on the left span in 2-4 rose in line with PC tendon cutting, and between 1200 to 1800 mm from the left support the concrete suffered compression induced spalling.
Figure 9 compares the load-displacement relationship of all specimens. The load-bearing capacity and stiffness depended on the number of tendons. Specimen 2-4 could no longer bear the load beyond 620-kN when 67 % of strands were broken.

### 3.2 Evaluation of flexural load capacity

The flexural capacity of PC beams with broken tendons was calculated assuming constant plane stress and a balance in cross-sectional forces as [5]:

\[
C^* = \int_{x} \sigma'(u) \cdot b(u) du
\]

\[
T_p = \sum A_{ps} \cdot \sigma_{ps}
\]

\[
T_i = A_i \cdot \sigma_i
\]

where, \( C^* \): compressive force carried by concrete (kN), \( T_p \): tensile force carried by PC tendon (kN), \( T_i \): tensile force carried by longitudinal rebars (kN), \( \sigma'(u) \): compressive stress carried by concrete at the height \( u \) from neutral axis (N/mm²), \( b(u) \): width of cross-section at the height \( u \) from neutral axis (mm), \( A_{ps} \): cross-section area of PC-tendon at \( n \) (mm²), \( \sigma_{ps} \): stress of PC-tendon at \( n \) (N/mm²), \( A_i \): cross-section area of tensile rebars (mm²), and \( \sigma_i \): tensile stress at centroid of longitudinal rebars (N/mm²).

The flexural capacity \( M_u \) was calculated by (10) to (12) as:

\[
N'_d = C^* - T_p - T_i
\]

\[
M_u = C'(d_x - \beta_x) + T_i \cdot \epsilon_i + \sum A_{ps} \cdot \sigma_{ps} \cdot \epsilon_{ps}
\]

\[
\beta_x = x \int_0^{\beta_x} \sigma(u) \cdot b(u) u \cdot du
\]

where, \( N'_d \): force in longitudinal direction (kN), \( M_u \): maximum moment (kNm), \( d_x \): distance from compressive edge to the centroid of tensile longitudinal bars (mm), \( \epsilon_i \): distance between centroids of tensile longitudinal bars and cross-section (mm), \( \epsilon_{ps} \): distance between centroids of PC tendon at \( n \) and cross-section (mm), and \( x \): distance between compressive flange and neutral axis (mm). The tensile force carried by the broken PC tendon was calculated by the tensile strain \( (\epsilon_{ps}) \) estimated by considering the bond of PC tendon as shown in Fig. 10.

### 3.3 Verification of calculations through comparison with experimental results

The flexural capacity of partially prestressed concrete beams was calculated by verifying the balance between the action and the resistance moments in all cross-sections of the specimen. The checking position corresponds to the position of failed cross-section sections in the experiment.

Table 2 lists the maximum load capacity in the experiment (\( P_{max} \)) and the calculation (\( P_{mu} \)) according to Section 3.2. Based on the results of 2-4, the load-capacity of the specimen was assumed to be 620-kN when 67 % broken tendons. The calculated results seem to be a similar to the experimental values. The ratios between both \( P_{max} \) and \( P_{mu} \) were 1.13 to 1.40; that increased with further cutting of PC tendons.

One of the reasons why experimental values exceeded calculated results is that the calculation was considered as the influence of the moment action on bond behavior. Even so, the proposed method in this study can be used to evaluate the flexural load capacity of incompletely-grouted PC girders with broken tendons, and is a more reasonable method for determining repairs and reinforcement work on deteriorated PC girders.

### 4. Conclusions

The influence of grout strength and a type of PC tendon on the length of stress-decreasing region was exam-
ined, and a method was proposed for obtaining the residual stress in PC girders. A proposal was then made for a method to evaluate the flexural load capacity of incompletely-grouted PC girders with broken tendons.

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References