Method to Estimate Equivalent Natural Period with Micro-tremor Measurements

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The object of this paper is to propose a method to estimate the equivalent natural period with micro-tremor measurements on the basis of the correlation between the natural frequency and the yield frequency. A method, combining the Eigensystem Realization Algorithm and some empirical hypotheses of vibration modes, was proposed for identifying the first natural vibration modes of structures. A ratio of the yield frequency to the natural frequency was mostly a certain constant value and the average value of the ratio depends on the type of structures. This paper proposes a simple procedure for estimating the yield frequency with micro-tremor measurements.

**Keywords:** micro-tremor measurement, equivalent natural period, natural period, modal identification, Eigensystem Realization Algorithm, nonlinear analysis

1. Introduction

Dynamic characteristics such as natural frequency are important factors in structures which affect their seismic capacity and vehicle running safety during earthquakes. The equivalent natural period is calculated from the equivalent stiffness corresponding to the first yield point on the load-displacement curve line of the whole structure system. The equivalent natural period is an important indicator in seismic diagnosis and maintenance of existing structures as well as for the seismic design of new structures to be constructed in the future[1], [2]. For example, it can give the seismic response of structures as well as for the seismic design of new structures to be constructed in the future[1], [2].

In Japan, the seismic capacity of some existing railway structures has not been calculated because they have been designed on the basis of an elastic design method such as the allowable stress design method. Calculation of the equivalent natural period by nonlinear analysis can provide seismic capacity; however, this calculation is difficult if it is to be performed on multiple structures because of the time and cost involved or in cases where structural information such as reinforcement measures and material properties have not been preserved. There are therefore many reasons for developing an easy method to estimate the equivalent natural period.

This paper focuses on the correlation between the natural frequency and the yield frequency. The natural frequency corresponds to the elastic state; it can therefore be obtained by measurement. The yield frequency is estimated by the following formula.

\[
T_{eq} = 2\pi \sqrt{\frac{W}{K_y}}
\]

where:
- \(T_{eq}\) : Equivalent natural period (s)
- \(W\) : Equivalent load (kN)
- \(K_y\) : Rigidity at yield (kN/m)
- \(\delta_y\) : Displacement at yield (m)

\[
K_y = \frac{K_{sy} \delta_y}{K_{sy}}
\]

where:
- \(K_{sy}\) : Seismic coefficient at yield

\[
\delta_y = 2.0 \sqrt{\frac{\delta_y}{K_{sy}}}
\]

\[
T_{eq} = 2.0\pi \sqrt{\frac{W}{K_{sy}}}
\]

**Fig. 1** Definition of the equivalent natural period
defined as the reciprocal of the equivalent natural period in this paper and corresponds to the first yield state of the structure; therefore, it can be obtained by nonlinear analysis.

Previous studies have already proposed some methods for identifying the natural frequency with measurements. For railway viaducts, there is a method which compares the measured natural period with the standard value for evaluating the level of soundness. Measurement techniques and evaluation indices have been systematically organized [3]. However, these methods involve impractical impact tests.

Although some methods for identifying vibration modes based on micro-tremor measurements have already been proposed, they have only been applied to structures which have a comparatively high micro-tremor level of vibration, such as suspension bridges [4], Langer girder bridges and truss bridges [5]. The latter have not been applied to railway viaducts which have relatively small micro-tremor vibration levels because of the high degree of indeterminacy. The method developed was employed by extending the applicability of previous methods to railway viaducts.

The object of this paper is to propose a method for estimating the equivalent natural period with micro-tremor measurements on the basis of the correlation between the natural frequency and the yield frequency. Specific points examined for proposing the method are as follows:

(i) Effective methods for identifying the first vibration mode of railway structures on the basis of micro-tremor measurements were investigated.
(ii) The correlation between the yield frequency and the measured natural frequency was studied.
(iii) A procedure for estimating the yield frequency with micro-tremor measurements was specified.

2. Outline of measurements, identification and analyses

2.1 Structures to be examined

Tables 1, 2 and 3 show lists of existing structures: RC wall-type piers (hereinafter referred to as piers), RC girder-type rigid frame viaducts (hereinafter referred to as girder viaducts) and RC cantilever-type rigid frame viaducts (hereinafter referred to as cantilever viaducts). Figure 2 shows structures to be examined in this paper.

Piers and girder viaducts were designed based on the Seismic Standard. The foundations of the girder viaducts are made of single piles without footing beams. The pile diameter of piers is around 1200mm.

Cantilever viaducts were designed according to the allowable stress design method. They are of 1 or 2 layers, and of standard viaduct design, with 3 spans of 8m and an overhang at both ends. The structure height ranges from 6.5m to 19m with pile group foundations; pile lengths are from 4m to 14m; and ground classifications are from G1 to G5. With respect to the characteristics of respective railroad sections, all of the railroad sections “A”, some parts of the railroad sections “B” and some parts of the railroad sections “C” contain two-layer viaducts with high columns. The surface ground type of railroad sections “A”, “B” and “E” were G2 and G3, and their ground conditions were relatively good and stable. On the other hand, the ground conditions for railroad sections “C” were G1, G4, and G5 and for “D” were G3 and G4, indicating that ground conditions were relatively weak and unstable.

2.2 Method for Micro-tremor measurements

Figure 2 shows the sensors “A”, “B”, “C” and “D” which were placed at the start and end points of the viaducts, at the middle point on the surface of the ground near the viaduct column, and at a point 5m to 10m away from the viaduct column on the surface of the ground, respectively.
Two horizontal components (transverse and longitudinal components, in relation to the railroad) and one vertical component were recorded 3 times at each measurement point of each structure, at 200Hz of the sampling frequency and 60 seconds of the measurement period. Lesser external disturbances such as trains running on the viaduct, cars running near the viaduct and the wind were also included in the measurement conditions because such disturbances cannot be always anticipated, and furthermore, the kinds of disturbances which may lead to natural vibration excitation have not yet been made clear quantitatively and qualitatively. Figure 3 and Table 4 show the measurement situations and a list of measurement equipment respectively.

### Table 4 Measurement equipment

<table>
<thead>
<tr>
<th>Measurement system</th>
<th>Maker</th>
<th>Product</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUTTA Nservice</td>
<td>GEODAS-15-USB</td>
<td>Sensitivity: 1V/cm/sec Frequency range: 0.5 ~ 18Hz</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4 shows the basic concept behind the representative 2-D elasto-plastic frame analysis model of viaducts to be analyzed. Although these structures were designed according to the elastic design method, they were analyzed in accordance with the nonlinearity based on the new Seismic Standard [1]. The modification factor of the base bearing capacity $\alpha_f$ was set to 1.0 and the modification factor of the material $\rho_m$ was set to 1.2. A static constant loading by using a displacement control was performed on this model without considering train load and snow load, and the first yield point was obtained.

Because cantilever viaducts to be analyzed had been reinforced with a steel plate jacketing method, filling mortar added to the column cross-sectional area was taken into consideration when the material bending property was calculated (since the steel plate does not contribute to the resistance force of the column in the axial direction, it was ignored in the bending property evaluation).

### 2.3 Method for numerical analyses

#### 2.3.1 Analysis model

Figure 4 shows the concept of the lateral subgrade reaction specified in the Seismic Standard. The Seismic Standard specifies a stiffness for ground lateral springs at the strain level of ground springs of about $10^{-3}$; however, micro-tremor measurements or impact tests supposed this to be about $10^{-6}$. The stiffness of ground lateral springs at very small strain level could be obtained from a theoretical formula, but the obtained value would be between 4 and 10 times larger than that given in the Seismic Standard [1]. Because the ground lateral spring was regarded as a factor which had an impact on natural frequency, we performed the static nonlinear analysis where the stiffness of the ground spring was set as a parameter (1 time, 4 times and 10 times as large as that given in the Seismic Standard [1]).

### 2.3.2 Analysis parameter

Figure 5 shows the concept of the lateral subgrade reaction specified in the Seismic Standard. The Seismic Standard specifies a stiffness for ground lateral springs at the strain level of ground springs of about $10^{-3}$; however, micro-tremor measurements or impact tests supposed this to be about $10^{-6}$. The stiffness of ground lateral springs at very small strain level could be obtained from a theoretical formula, but the obtained value would be between 4 and 10 times larger than that given in the Seismic Standard [1]. Because the ground lateral spring was regarded as a factor which had an impact on natural frequency, we performed the static nonlinear analysis where the stiffness of the ground spring was set as a parameter (1 time, 4 times and 10 times as large as that given in the Seismic Standard [1]).
3. Identifying the first natural vibration mode

3.1 Method for identifying the first natural vibration mode

Figure 6 is a flow chart of the method for identifying the first natural vibration mode of structures. This study applied ERA (Eigensystem Realization Algorithm) to identify vibration modes [3]. The input ERA data were autocorrelation functions which were converted to waves of free oscillation by using FFT and IFFT [4].

Vibration modes calculated by ERA sometimes include fake modes generated by measured noises and inappropriate structure system degree. The Stabilization Diagram was therefore applied to some measured structures in order to get the proper structure system degree as shown in Fig.7 [4], and set the structure system degree to 40 after confirming the stability of the identified first natural vibration mode on the basis of the result of this Stabilization Diagram.

The following 4 terms have been conditioned as shown in Fig.6(f) for identifying the first natural vibration mode of structures from lots of vibration modes calculated by ERA: \( MAC_{q} \), gained from (1), the range of the natural frequency and structure height. The strong correlation between the natural frequency and structure height. The figure indicates that the measured first natural frequency of structures and structure height. The figure indicates that the measured first natural frequency of structures and structure height.

3.2 Identification results

The method proposed in 3.1 made it possible to identify the first natural vibration mode of the target structures automatically, in most cases. Although it identifies the first natural vibration mode of the target structure may be recognized by evaluating identified first natural vibration modes of the adjacent structures for comparison. As shown in Fig.7, the identified natural frequency agrees with the maximum peak of the Fourier spectrum of the autocorrelation function in most cases. However, in some cases it doesn’t agree with the peak because of other vibration modes such as those of adjacent structures, torsion modes and second vibration modes.

Figure 8 illustrates the relationship between the measured first natural frequency of structures and structure height. The figure indicates that the measured first natural frequency ranges from 1.5Hz to 5.0Hz and decreases with the increase in height. The strong correlation between the natural frequency and structure height can be confirmed.

Figure 9 illustrates the relationship between the

\[
MAC_{q} = \left[ \frac{|q_i(0)|}{|q_i|} \right]^2
\]

(1)

\[
MAC_{bs} = \left[ \frac{|\varphi_i \cdot \varphi_{bs}^{-1}|}{|\varphi_i| \cdot |\varphi_{bs}|} \right]^2
\]

(2)

where \( q_i \) is the mode shape of No.\( i \) calculated by ERA; \( q_i(0) \), the initial mode deformation of No.\( i \) calculated by ERA; \( \varphi_{bs} \), the mode shape of structures defined in Fig.6(f)
measured first natural frequency of structures and the design natural frequency of the ground surface. The figure demonstrates that the natural frequency of the structure and the ground have a weak positive correlation.

4. Equivalent natural period evaluation method

4.1 Relationship between the yield frequency and the natural frequency

Figure 10 shows the relationship between the yield frequency and the yield frequency of structures obtained from nonlinear analyses (hereinafter referred to as the analyzed yield frequency) and the measured natural frequency. As shown in Fig. 10, a ratio of the analyzed yield frequency to the measured natural frequency is mostly a fixed value for each structural type. The average ratio values ratio are 0.39 for piers, 0.38 for girder viaducts and 0.55 for cantilever viaducts. The difference in ratio among structural types depends on where these structures have plastic hinges.
4.2 The influence of the ground spring scale

The vibration mode gained from the measurements seemed to depend on soil layers at the construction site, soil parameters, the Young’s modulus of the concrete, etc. Understanding these parameters quantitatively through site surveys, such as boring tests, would make it possible to evaluate the effects of these parameters precisely by executing analyses. However, conducting site surveys at every site is difficult; therefore, only the stiffness of the ground spring was selected as a parameter (1 time, 4 times and 10 times larger than that given in the Seismic Standard) for static nonlinear analyses.

Figure 11 shows the relationship between the first natural frequency obtained from the numerical analyses (hereinafter referred to as the analyzed natural frequency) and the ground spring. The figure indicates that the analyzed natural frequency gets closer to the measured natural frequency as ground spring increases. Additionally, the analyzed natural frequency generally agrees with the measured natural frequency where the ground spring ranges from 4 to 10. Therefore, the validity of the proposed method of identification could be confirmed by the comparison between the measured natural frequency and the analyzed natural frequency based on the analysis in the case where the ground spring is larger than the Seismic Standard.

5. Procedure for estimating the yield frequency with micro-tremor measurements

A procedure for estimating the equivalent natural period with micro-tremor measurements for the existing RC cantilever-type rigid frame viaducts is as follows:

(i) measure structure micro-vibration with the method shown in 2.2.
(ii) acquire the measured natural frequency of the structure $f_{s,m}$ by the method shown in 3.1.
(iii) estimate the yield frequency $f_{y,e}$ with Eq. (3).
(iv) calculate the equivalent natural period $T_{eq,e}$ which is the reciprocal of the estimated yield frequency $f_{e,y}$.

$$f_{eq,e} = \begin{cases} 
\text{Wall-type pier} & 0.39 \times f_{s,m} \ (\text{c.v.} \approx 14\%) \\
\text{Girder-type viaduct} & 0.38 \times f_{s,m} \ (\text{c.v.} \approx 10\%) \\
\text{Cantilever-type viaduct} & 0.55 \times f_{s,m} \ (\text{c.v.} \approx 6\%) 
\end{cases}$$

Figure 12 shows an example estimation of yield frequency by the above proposed method for the railroad section 2-A → 2-E. The figure demonstrates that the yield frequency and the natural frequency decrease as structural height increases or as ground condition level decreases. The estimated values of the yield frequency by the proposed method are almost the same as the analyzed values of the yield frequency. The drastic changing point of the yield frequency which is an important factor in seismic capacity or train running capacity can be detected easily by applying the proposed method to a series of structures in a railroad section.

Figure 13 shows an example estimation of the maximum displacement response by the above proposed method under L2spe.1 and L2spe.2. In this estimation, the single degree of freedom system, whose skeleton curve and hysteretic model have perfect elasto-plasticity and Clough model type respectively, was used. On one hand, the maximum displacement response by the proposed method was calculated on the condition that 0.6, the average value of $0.9 \times$ the maximum seismic coefficient and the estimated value are adopted as the yield seismic coefficient and the yield frequency respectively. On the other hand, the maximum displacement response by precise analysis was calculated on the condition that $0.9 \times$ the maximum seismic coefficient and the analyzed value are
adopted as the yield seismic coefficient and the yield frequency respectively. The figure illustrates that the values of the maximum displacement response by the proposed method and those by the precise analysis agree with each other and the proposed method make it possible to obtain an appropriate estimation of the maximum displacement response.

6. Conclusions

The purpose of this paper was to propose a method for estimating the yield frequency with micro-tremor measurements. For the existing RC railway viaducts, this paper allows the following conclusion.

(1) A method which combines ERA (Eigensystem Realization Algorithm) and some empirical hypotheses of vibration modes has been proposed for identifying the first natural vibration modes of structures.

(2) Numerical analyses have revealed that observed soil stiffness was between 4 and 10 times as large as that of the Seismic Standard.

(3) The ratio of yield frequency to natural frequency is almost a mostly fixed value and the average value of this ratio is 0.40 for wall-type piers, 0.39 for girder-type rigid frame viaducts and 0.55 for cantilever-type rigid frame viaducts.

(4) A simple procedure has been proposed for estimating the yield frequency with micro-tremor measurements; the measurement method, the identification method and the estimation method. Validation and application examples of the proposed method were shown.

References


