Modeling an Optimal Track Maintenance Schedule in Consideration of Train Derailment Risk

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Railway track irregularities need to be kept at a satisfactory level through appropriate maintenance work by using MTTs (Multiple Tie Tamper). We have developed an optimal track maintenance scheduling model taking train derailment accident risk into consideration. Firstly, we analyze derailment accident data in order to build a model for estimating the number of casualties when a train derailment accident occurs. Next, by applying the result, we develop an optimal track maintenance scheduling model in order to minimize the track irregularities and mitigate the train derailment accident risk. Finally, we apply our model to decision making for yearly tamping schedule, then try to show the optimal tamping schedule obtained by the model.

Keywords: train derailment accident, railway track irregularity, risk based maintenance

1. Introduction

It is imperative for us to keep the track irregularities at an optimal level through optimal maintenance strategies with due consideration of both tamping cost and the risk of occurrence of a train derailment accident. Thus, in this paper, we try to investigate an optimal railway track maintenance strategy by building mathematical models which take train derailment accident risk into consideration.

Firstly, we developed a model for estimating train derailment accident risk by applying a statistical modeling approach to the data of past derailment accidents in order to build a model which shows the relationship between quality of track irregularities and derailment accident risk as shown in Fig. 1. We can use the estimation model to identify the risk by multiplying the probability of a derailment accident occurring by the cost of the accident. When estimating the probability, we consider track irregularities, car profile, train operation condition, and track line condition. In particular, it is very important to estimate the number of casualties brought about by a derailment accident in order to build the model. When estimating the number, we also consider several events which accompany the accident. From the results of the above analyses, we developed a derailment risk estimation model in consideration of the magnitude of the accident, which includes the number of casualties, the loss of railway equipment, and the cost regarding suspension time. After the above estimation, we can obtain the total cost of the accident by considering various unit costs of the accident.

Next, by using the risk estimation model, we built on optimal track maintenance scheduling model for the MTT tamping operation for the whole year in consideration of the risk. The model enables us to decide which unit section should be selected and when we should carry out tamping with the MTT taking various conditions into consideration.

Finally, by applying these models to the actual railway data, we confirmed that our models were effective and useful enough to optimize maintenance strategies.

2. Derailment accident risk estimation model

In this chapter, we build a derailment accident risk estimation model to analyze the relationship between the magnitude of accidents and their characteristics [1-3].

2.1 Estimation model for the probability of a derailment accident occurring

For evaluating the probability of a derailment accident occurrence, we use the probability that the estimated derailment coefficient (EDC) exceeds an upper limit of the derailment coefficient (LDC). The derailment coefficient means lateral force $Y$ over vertical force (wheel load) $Q$ which presses on rails from cars. If either the EDC gets larger or the LDC gets lower, the probability of a derailment accident occurrence gets higher. To obtain the coefficients of EDC and LDC, we use the computing model used for evaluating safety in terms of derailment accidents on sharp curves at low speeds in Japan [4]. Regarding

Fig. 1 Risk estimation model for train derailment accident

Derailment accident risk ($P_D \cdot T$)
the computation model, we need to set both alignment irregularity and twist irregularity data as input data. In the event that these irregularities are high, the EDC increases.

2.1.1 Degrading rate of track irregularity

We show an actual distribution of degrading data of horizontal alignment and twist irregularities obtained from a curve section on a commercial line in Fig. 2. By applying the fitting test to several probability distributions, both of actual distribution of degrading data well fit log-normal distribution. Therefore, we build a degrading rate model for the irregularities as a log-normal distribution model. It should also be mentioned that as the correlation among the irregularities is low, we do not consider the correlation when estimating the probability of a derailment accident occurring.

![Fig. 2 Distribution of alignment and twist irregularities growth](image)

**Table 1 Computation condition**

| Car weight | 32 tf |
| Train velocity | 30 km/h |
| Curve radius | 400 m |
| Super elevation | 105 mm |
| Rate of change of super elevation | 1/400 |
| Alignment irr. | 5,7,10 mm |
| 2 m twist irr. | 3,5,7 mm |
| BC twist irr. | 5 mm |
| LDC | 1.6 |

2.1.2 Probability of a derailment accident occurring

Under the condition as shown in Table 1, we compute the probability of a derailment accident occurring by using a probability simulation technique. To compute the probability, we set three levels for each irregularity’s initial value. We then generate 10000 random values according to log-normal distribution for the degrading rate of each irregularity. After generating the values, we add them to the initial value. Therefore, we can compute EDC by using the obtained irregularity’s values. We show the results for the cumulative probability of EDC in Fig. 3. We also show LDC in the figure. Although the probability of EDC exceeding LDC is generally low, we can see a tendency for the probability to get larger as the irregularities increase.

![Fig. 3 Result of probabilistic simulation](image)

By using the above-mentioned method, we can obtain the probability in accordance with the magnitude of track irregularities.

2.2 Estimation model for the number of casualties brought about by a derailment accident

To build a model for estimating the number of casualties brought about by derailment accident, we analyze the derailment accident data obtained from the reports by JTSB (Japan Transportation Safety Board), which were published between October 2001 and May 2011. The reports contain 116 derailment accidents data.

2.2.1 The tendency of the number of casualties

We show the number of casualties (passengers and train crew) of each derailment accident in Fig. 4 in descending order. The accident which brought about the maximum number of casualties is the one in which a commercial train derailed and crashed into a large building at a high speed. Therefore, we can regard car behavior after derailment, velocity, the number of passengers and the car type (the structure inside car body) as important factors for increasing the number of casualties. Therefore, we classify the data by the events: (i) cars overturn or falling down after derailment, (ii) crashing into a heavy object, (iii) long seats (not cross seats).

The number of casualties is zero in most of the accidents. However, we can see a tendency for the number to be large in the case of accidents where the cars overturn/fall down or crash into a heavy object. On the other hand, it seems that the seat type does not have a great influence on the number. Those accidents which brought about a large number of casualties were greatly influenced by a car/cars either overturning/falling down or crashing into a heavy object. If we exclude the accidents including the above-mentioned events we then see that 2/3 of the accidents involving casualties occurred with cars with long seats.

From the results, we analyze the influence of each event (i)-(iii) on the values for both the casualty ratio (CR: the number of casualties / the number of passenger and train crew) and the death ratio (DR: the number of the deceased / the number of casualties). We show the result for the mean values and t-test values for the data classified by the events in Table 2 and 3.
The value of the CR is 92% for accidents involving cars overturning/falling down while it is 6.7% without such events. The value of the DR for accidents involving cars overturning/falling down is also greater than the value for accidents that do not involve such incidents. Moreover, from the result of the t-test, we find that the effect of this "cars overturning/falling down" event is significant factor for both the CR and DR.

b) Crashing into a heavy object

Classifying the casualty data into the cases with or without the event (i) the "cars overturn/falling down", we compute their mean values for both the CR and DR. The values for the CR are around 90–93% for accidents with the event (i) regardless of whether the event (ii), crashing into heavy objects occurs or not. However, for the accidents without the event (i), "cars overturn/falling down", the values for the CR were 24% in the accidents with the event (ii), while the values were 1.6% without it. The result of the t-test shows that the effect of the "crashing into heavy objects" event is significant different for the CR in the accidents without the event (i). According to the t-test, the DR is not significant enough for both accidents with or without the event (i).

c) Seat type

Classifying the casualty data into the cases with or without the event (i) the "cars overturn/falling down" and "crashing into a heavy object", we compute their mean values for both the CR and DR.

<table>
<thead>
<tr>
<th>Condition</th>
<th>CR</th>
<th>DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>With car overturn/falling</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without car overturn/falling</td>
<td></td>
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<td>Without</td>
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<td></td>
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<tr>
<td>With car overturn/falling</td>
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<tr>
<td>With crashing into a heavy object</td>
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<tr>
<td>Without</td>
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<td></td>
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<tr>
<td>Without</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long seat</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross seat</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With crashing into a heavy object</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without</td>
<td></td>
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</tr>
</tbody>
</table>

Table 2: Mean values of CR and DR for train derailment accident

<table>
<thead>
<tr>
<th>Car</th>
<th>Overtur/falling</th>
<th>No overturn/falling</th>
<th>Number of accidents</th>
<th>n=6</th>
<th>n=103</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>91.64%</td>
<td>6.65%</td>
<td>n=6</td>
<td>6</td>
<td>103</td>
</tr>
<tr>
<td>With crashing into a heavy object</td>
<td>92.90%</td>
<td>24.28%</td>
<td>n=6</td>
<td>3</td>
<td>23</td>
</tr>
<tr>
<td>Long seat</td>
<td>100%</td>
<td>21.04%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross seat</td>
<td>89.35%</td>
<td>25.70%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without crashing into a heavy object</td>
<td>90.38%</td>
<td>1.58%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long seat</td>
<td>-</td>
<td>2.72%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross seat</td>
<td>90.38%</td>
<td>0.65%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long seat</td>
<td>100%</td>
<td>5.70%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross seat</td>
<td>89.97%</td>
<td>7.33%</td>
<td></td>
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</tbody>
</table>

Table 3: Results of t-test

<table>
<thead>
<tr>
<th>Condition</th>
<th>CR</th>
<th>DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>With car overturn/falling</td>
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<tr>
<td>With car overturn/falling</td>
<td></td>
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<tr>
<td>With crashing into a heavy object</td>
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<td>Without</td>
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<tr>
<td>Without</td>
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<tr>
<td>Long seat</td>
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<tr>
<td>Cross seat</td>
<td></td>
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<tr>
<td>With crashing into a heavy object</td>
<td></td>
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<tr>
<td>Without</td>
<td></td>
<td></td>
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<tr>
<td>Without</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4 Distribution of casualties for train derailment accident
without the events (i) and (ii), and applying the above results, we try to compute mean values for the event (iii) long seats for both the CR and the DR. As for the CR, we know that the number of accidents including either the event (i) or the event (ii) is rather small. In addition, for the accidents with neither the event (i) nor the event (ii), we know that the values for the CR are 2.7% for the case of long seat and 0.65% for the cross seat case. From the result of the t-test on these cases, we can see that the difference between the two seat types’ effects is significant for the value of the CR only in the case of accidents with neither the event (i) nor (ii). As for the DR, we find from the results of the t-test that the seat type is not significant factor for any cases.

From the above results, we can conclude that the effects of the event (i), the event (ii) without the event (i), and the event (iii) with neither the event (i) nor (ii) have a great influence on the value of the CR. The event (i) also has a great influence on the value of the DR.

d) Train velocity

In the prior research reported in UK [5], train velocity has an influence on the number of casualties of fatal train accidents including derailment accident. Therefore, we show the relationship between train velocity and the number of casualties resulting from derailment accidents having occurred in Japan in Fig. 5.

We can see a tendency for the number of casualties in most train derailment accidents to be under 10. However, we can recognize a group of the data which tends to increase in proportion to train velocity. All accidents including “car overturn/falling down” are part of this group. Most of the accidents including “crashing into a heavy object” are also part of the group. Therefore, we can conclude that the probability of the number of casualties increasing may rise in proportion to the velocity at which accidents with such events take place.

e) The number of passengers

We can regard the number of passengers and train crew on derailed cars as a significant factor for increasing the number of casualties in derailment accidents. Therefore, we demonstrate a relationship between the number of passengers on cars and the number of casualties in Fig. 6 for each of the cases of cars overturn/falling down and not overturn/falling down.

As for the cars that do overturn/fall down, the number of casualties has a high correlation with the number of passengers and both of the slope on the graph is almost one regardless of the cases with or without the event (ii). Therefore, there is a high probability that passengers in cars that overturn/fall down will be injured or killed.

As for the cars that do not overturn/fall down, the correlation between the number of passengers on the cars and the number of casualty is lower. However, there is a high probability that passengers in cars that crash into a heavy object may be killed or injured.

f) Some other factors

As another factor, we can consider an event in which a derailed car runs or stops off the track, interfering with the opposite railway track, then another train running on the opposite track may collide with the derailed cars. It would be reasonable to assume that the casualties resulting from such an accident could be terrible. Such accidents were not included in the JTSB reports, because such accidents rarely occur in Japan. However, we should consider the probability of such events occurring when estimating the risk of train derailment accidents.

2.2.2 Estimation model for the number of casualties

From the results of the above-mentioned analyses, we have developed a model for estimating the number of casualties brought about by a train derailment accident as shown in Fig. 7. As for the model, we multiply the CR by the number of passengers on each car composing the train, sum up the number of casualties of each car, and obtain the total number of casualties of the accident. Then, we multiply the DR by the number of casualties by classifying the car as derailed or not and obtain the numbers of injured and deceased persons.

By considering the above-mentioned results of statisti-
cal analyses, we develop the model for computing the CR by the following expression.

\[ \text{CR} = aV + bL + c \]  

(1)

- \( V \): Train velocity (km/h), \( L \): Dummy variable for long seat, \( a, b, c \): Constant (Table 4)

The DR can be obtained by the result shown in Table 2.

We estimate the number of casualties and show the relation between the estimated values and actual ones in Fig. 8. We can see that the estimated number is close to the actual number.

### Table 4 Coefficient for the formula (1)

<table>
<thead>
<tr>
<th>Car overturn/falling</th>
<th>Crashing into a heavy object</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.00115</td>
<td>0.8269</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No, Yes</td>
<td>0.0048</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No, No</td>
<td>0.00024</td>
<td>0.01474</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Track maintenance scheduling model by considering the train derailment accident risk

In our past researches, we developed the optimal track maintenance scheduling (OTMS) model [6-9]. In the OTMS model, we can consider both surface irregularity and horizontal alignment irregularity for implementing the optimal track irregularity maintenance. In this research, we propose here a new scheduling model for preparing maintenance schedules ensuring greater safety by considering the derailment risk.
3.1 Considering the risk

3.1.1 OTMS model

Major input data for the OTMS model include the structure of the railway track network, the candidates for the lots for MTT’s operation, and the historical data on track (surface, horizontal alignment) irregularities for each lot. Each lot is 100m in length, and its degradation and restoration processes are predicted in the transition process model [9]. One unit consists of N (certain numbers) consecutive lots. An optimal operation schedule for an MTT is obtained for each unit and for each term (ex. 10 days) of every month. We show the structure of the model in Fig. 11.

The model, 0-1type integer programming model aims at finding optimal track tamping schedule, which maximizes the improvement of track irregularities under several related conditions when we execute tamping work by following the model solution. By applying this model, we can decide optimal operation of an MTT including the decisions regarding which depot and which unit should be chosen as well as its operation period.

For developing the model, we define the sets of months, terms, depots, and units which are denoted by \( M=\{1, 2, 3, \ldots, 12\} \), \( D=\{1, 2, 3, \ldots, D_{\text{max}}\} \), \( U=\{1, 2, 3, \ldots, U_{\text{max}}\} \), respectively. The OTMS model is given as follows.

(1) Decision variables

- \( z_{mkd} \) binary variable, \( m \in M, k \in K, d \in D \)
  - =1 MTT is located at depot \( d \) in month \( m \) and term \( k \).
  - =0 MTT is not located at depot \( d \) in month \( m \) and term \( k \).
- \( w_{mkj} \) binary variable, \( m \in M, k \in K, j \in U \)
  - =1 maintenance operation is executed at unit \( j \) in month \( m \) and term \( k \).
  - =0 maintenance operation is not executed at unit \( j \) in month \( m \) and term \( k \).

(2) Constraints

i) MTT location constraint

At most, the MTT can be located at one of the depots in each term of each month.

\[
\sum_j z_{mkd} \leq 1 \quad m \in M, k \in K \quad (2)
\]

ii) Upper bounding constraint for the number of tamped units

The number of units to be tamped has an upper bound in each term of each month.

\[
\sum_j w_{mkj} \leq A_{mk} \quad m \in M, k \in K \quad (3)
\]

where \( A_{mk} \) is the maximum number of units tamped in month \( m \) and term \( k \).

iii) Upper bounding constraint for the frequency of tamping

Each unit requires tamping operation at most once during the whole period of one year.

\[
\sum_k w_{mkj} \leq 1 \quad j \in U \quad (4)
\]

iv) Logical constraint for MTT location and operation

The tamping operation of MTT can be executed only when the MTT is located at the depot such that each unit is “covered” by the MTT located at the depot in each term.

\[
\sum_m z_{mkj} - A_{mk} \cdot \sum_d z_{mkd} \leq 0 \quad m \in M, k \in K \quad (5)
\]

where \( J_k \) is a set of units “covered” by depot \( d \) and \( D_d \) is a set of depots which “cover” UNIT \( j \).

v) MTT movement constraint

The MTT cannot move beyond the far distance from the presently located depot in the next term.

\[
C \cdot z_{mkj} + \sum_{d \in D_d} z_{mkd} \leq C \cdot m \in M, k \in K
\]

\[
C \cdot z_{mkj} + \sum_{d \in D_d} z_{mkd} \leq C \cdot m \in M, k \in K, j \in U \quad (6)
\]

\[
C \cdot z_{mkj} + \sum_{d \in D_d} z_{mkd} \leq C \cdot m \in M, k \in K, d \in D
\]

\[
C \cdot z_{mkj} + \sum_{d \in D_d} z_{mkd} \leq C \cdot m \in M, k \in K, d \in D
\]

where \( D_d \) is such a set of depots to which the MTT cannot move in the next term from the depot \( d \) (presently located) and \( C \) is a maximum value of \( \sum_{d \in D_d} z_{mkd} \) or \( \sum_{d \in D_d} z_{mkd} \).

vi) Unit specific operation constraint

If we identify the lots for which the track irregularities exceed the specified upper bound during the scheduling period, the units containing these lots need to be dealt with by tamping operation until the end of the scheduling period. Denoting such sets of units by \( J \), and corresponding pairs of month and term by \( (m,k) \in J \subseteq M \times K \) for \( j \in J \), we obtain the following unit specific operation constraint.

\[
\sum_{m,k,j} w_{mkj} = 1 \quad j \in J \subseteq U \quad (8)
\]

(3) Objective function

We assume that the objective function of this scheduling model is to minimize the mean value of standard deviation of track irregularities across the whole scheduling period.

\[
S = C_1 - C_2 \sum_{m} \sum_{k,j} S_{mkj} w_{mkj} \quad (9)
\]

where \( C_1 \) and \( C_2 \) are positive parameters, and \( S_{mkj} \) indicates the amount of improvement of track irregularities obtained from assigning an MTT to the unit \( j \) in term \( k \) of month \( m \). Thus, the objective function of the model is equivalent to maximizing the following expression.

\[
\text{Maximize} \quad \sum_{m,k,j} S_{mkj} w_{mkj} \quad (10)
\]
3.1.2 Considering the risk in the OTMS model

a) Considering the risk as constraint

We set a condition that we have to execute maintenance for the lot whose risk exceeds the limit if the maintenance will not be executed during the year for making tamping schedule. The condition should be set as the upper bound of the risk by considering the characteristics of the railway section. Therefore, the model has to select the lot which has large risk.

b) Consideration for the objective function

We can add the risk to the objective function of the OTMS model. In this case, we should divide the value for track irregularity by its target value and also divide the risk by its target value because the dimension of track irregularity and that of the risk are different.

3.2 Example of test computing

To confirm the validation of the model, we execute test calculation by applying the model to actual railway data (length:132km, the MTT can work for 48 days in a year). For the purposes of this calculation, we assume the above method b) which takes the risk into consideration. Then we compare the result with the result for the model that only takes into consideration surface irregularity.

We can compute the result that the risk is equal to zero for most of the lots. And a small number of the lots can be seen as having plus risk. We show the value and a comparison of the results between two models in Table 8. Lots A~F are examples which have a large level of risk. In case of considering only surface irregularity, maintenance work for several lots is not scheduled for completion in spite of having plus risk. However, by considering both surface irregularity and the risk, maintenance work of all the shown lots is scheduled to be done. Therefore, we can obtain an optimal maintenance schedule ensuring greater safety by considering both track irregularity and the risk of derailment accident.

4. Conclusions

We have developed the models for estimating train derailment accident risk and obtaining an optimal maintenance schedule. Furthermore, we have confirmed the effectiveness of our model through numerical calculations.

From the results of the investigation, we can conclude that we can estimate the magnitude of derailment accidents and the risk regarding such accidents. We can also decide an optimal maintenance schedule. The model therefore enables us to perform more effective railway track maintenance activities.

References

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