Non-linear Analysis Method of Ground Using Equivalent Single-degree-of-freedom Model

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This paper proposes a simple method to evaluate ground-surface waveforms using both the natural period of the ground and waveforms in the engineering bedrock. In this research, the "Equivalent Single Degree of Freedom method" is proposed and is based on the results of static push-over analyses of many grounds with various properties. A comparison was made of waveforms of multi-layered ground, obtained from the proposed method, using dynamic analysis. It was then confirmed that these waves are almost identical, and that the proposed method is applicable to the evaluation of the surface motion. Although information is still lacking about soil properties in regions where there are existing railway lines, proposed method enables economical dense calculation of trackside surface motion. These sets of waveforms will be utilized to identify locations where devastating damage is expected in the case of a strong earthquake.

Keywords: static push-over analysis, equivalent single degree of freedom method, natural period

1. Introduction

Social infrastructure facilities include not only those constructed according to earthquake resistance standards enacted following the Southern Hyogo Prefecture Earthquake, but also a large number of older structures. The earthquake resistance and the earthquake related risks of each structure differ greatly from one structure to another. An efficient method to reduce earthquake related risk of infrastructures located across an entire city each of which with a different earthquake resistance would be to effectively locate weak points in this fabric, and gradually roll out the required countermeasures. Achieving this would require the comprehensive evaluation of the earthquake resistance of a very large number of structures, and the effective evaluation of earthquake motion of each of these points, followed by precise individual calculation of earthquake related risk for each structure.

There are cases when multi-dimensional or multiple-degree-of-freedom response analysis methods are used in order to evaluate the behavior of an above-ground structure and also the degree of damage to the structure in the event of an earthquake. However, it is not realistic to analyze such a large group of structures at once. For this reason, it is necessary to carry out modeling and perform a response evaluation using a simpler method. For example, it has been confirmed that if an analysis is carried out on a relatively simple structure such as a viaduct, and the natural period, yield strength, and so on are known, it is possible to perform an accurate evaluation of these parameters by dynamic analysis employing in addition a model made using a simple single degree of freedom [1]. If detailed data concerning the structure exists, it will be possible to calculate the natural period or the yield strength with higher accuracy by implementing static non-linear analysis (push-

over analysis [2]). In addition, a method for easily calculating the natural period and the yield strength from only a small number of parameters, such as the height of the viaduct and the ground conditions, has also been proposed. Likewise, simple modeling and response evaluation are being studied for banked structures as well [3]. Consequently, by using these methods, even if the evaluation target consists of a large number of varied structures, it is possible to evaluate their behavior and the potential degree of damage by simple and minimal calculations, based on dynamic analysis, using surface earthquake motion and essential structural properties.

On the other hand, when evaluating the earthquake motion which will be exerted on each structure, it is important to take proper account of surface subsoil behavior, increasing the number of opportunities for implementing dynamic analysis based on the results of ground surveys. The most commonly used method for analyzing the ground response, apart from cases where the ground has undergone particular localized changes, is sequential non-linear analysis where the ground is discretized into a large number of single dimension thin elements [4] or equivalent linearization analysis [5]. To apply this method it is necessary to obtain the shear wave velocity V_s , the unit weight γ , non-linear characteristics (the $G - \gamma$ relationship, the $h - \gamma$ relationship, the adhesion c , and the internal friction angle ϕ), and other parameters for the location where earthquake motion is being evaluated. In addition, such a dynamic analysis requires a multi-degree-of-freedom model. For this reason, when considering the evaluation of earthquake motion applied to an existing facility, very often adequate ground data is unavailable. Also, if the evaluation target consists of a group of many structures, it is assumed that this will require a vast number of calculations. Many researchers have been obliged therefore to evaluate ground

behavior using simple methods, most of which only allow evaluation of a single earthquake motion index or amplification factor of the response spectrum while other simple methods have been introduced on the assumption that the amplification factor is the same regardless of the input [6, 7]. In sum, this situation called for the development a simple method to evaluate the time history waveform of the ground surface taking into consideration the degree of non-linearity.

In an attempt to meet this need, the present study proposes a simple method to evaluate the surface waveform considering the complicated strata composition of the ground and the localization of non-linearity according to the input level. This paper proposes a new method for analyzing the static non-linearity of the overall ground system, and studies a method to construct an equivalent single-degree-of-freedom model. More specifically, the mass distribution of the ground is roughly uniform, and the response characteristics change abruptly due to the localized non-linearization of the strata, which is clearly a contrast to the viaduct. That is to say, a static non-linear analysis method that results in a load equivalent to the same seismic intensity at each contact point, which is generally used for structures such as bridges, is not appropriate for estimating ground behavior. Accordingly, the new static non-linear analysis method proposes to consider changes in the mode shape accompanying an increase in ground deformation. Furthermore, by using a dynamic analysis method which employs an equivalent single-degree-of-freedom model, it is possible to confirm that the earthquake motion of the ground surface is accurately evaluated.

The proposed static non-linear analysis can then be employed to evaluate the behavior at multiple ground lo-

cations having different period characteristics and strata compositions. In addition, the proposed method is adapted to estimate the necessary parameters for the equivalent single-degree-of-freedom models using just the natural period of the ground location in question, which is relatively easily found, and also to confirm the appropriateness of the response prediction obtained using the proposed method. This approach will permit the evaluation of the earthquake motion of the ground surface with just a simple investigation and response analysis.

The method to be constructed here has the merits of allowing simple estimation of the earthquake motion of the ground surface after taking into consideration the non-linear behavior of the ground only from the natural period of the ground and the earthquake motion waveform of the bedrock position, and also permit implementation of a response analysis using a more accurate equivalent single-degree-of-freedom model by taking into consideration detailed ground data for certain points, when it is available.

2. Proposal of a method for performing static non-linear analyses of the ground

This section first proposes a method for evaluating the reduction in stiffness (lengthening of period) of the overall ground system and change in history damping, which accompany an increase in the displacement of the ground surface. Here, in the case of a general bridge or viaduct, girders are heavy, so when carrying out a static non-linear analysis, it is possible to roughly evaluate the relationship between the appropriate load and the displacement by gradually increasing the inertial effect on the assumption

Fig. 1 Procedure for performing a static non-linear analysis of the ground

that the acceleration acting on each node is equal. On the other hand, if the ground is defined as the target, it is conceivable that the weight distribution is roughly uniform, but that the distribution and the response acceleration and response displacement vary in a complicated way according to the degree of non-linearity resulting from the changes in the strata composition and input level of earthquake motion. Consequently, it is not appropriate to carry out static non-linear analysis using the same procedure as that used for bridges or viaducts. For this reason, the evaluation of changes in stiffness and damping of the overall ground system, which take into consideration changes in the mode shape along with deformation progression, is carried out by means of static non-linear analysis that conforms to the procedure indicated in Fig. 1. The actual procedure is shown below.

- ① It is assumed that the ground that has k degrees of freedom, and the values of the physical properties of each stratum (stratum thickness, weight, initial shear stiffness, damping, and non-linear characteristics) are fixed. For the initial step, the ground is assumed to be in an elastic region, and shear stiffness $G_i^{(1)}$ is assumed to be equal to $G_0(i)$. Here, $G_i^{(1)}$ is the initial (1st step) shear stiffness of the i^{th} stratum, and $G_0(i)$ is the initial shear stiffness of the i^{th} stratum.
- ②③ Eigenvalue analysis is performed using the stratum thickness, weight and shear stiffness $G_i^{(N)}$ of each stratum, and the primary natural frequency ${}_1\omega^{(N)}$ and corresponding eigenvector $\{{}_1u^{(N)}\}$ are calculated. Here, the eigenvector $\{{}_1u^{(N)}\}$ is normalized so that the value at the ground surface position is 1.0. Also, (N) of the superscript indicates that the result is for the N^{th} step.
- ④ The displacement distribution is assumed so that the increment of deformation at the ground surface is $\Delta\delta$. Here, the top to bottom displacement distribution is assigned according to the primary mode shape obtained from ③ above, as shown in (1).

$$\{\delta^{(N)}\} = \{\delta^{(N-1)}\} + \Delta\delta \cdot \{{}_1u^{(N)}\} \quad (1)$$

- ⑤ The tangential stiffness $G_i^{(N+1)}$ at each stratum and also the damping $h_i^{(N+1)}$ are evaluated.
- ⑥ The procedures ② to ⑤ are repeated until the predetermined displacement is obtained at the ground surface position.

Also, the relationships $G/G_0 - \delta$ and $h - \delta$ for the overall ground system can be calculated by using the static non-linear analysis shown below (Fig. 1).

- $G/G_0 - \delta$ relationship: The reduction of the stiffness of the overall ground system is evaluated at each step from the change in the characteristic angle frequency ${}_1\omega^{(N)}$ obtained from the above, which accompanies the increase in the ground surface displacement δ , as shown in (2).

$$(G/G_0)^{(N)} = ({}_1\omega^{(N)}/{}_1\omega^{(1)})^2 \quad (2)$$

- $h - \delta$ relationship: The average damping of the overall ground for each displacement condition is calculated from the weighted average of the damping $h_i^{(N)}$ of each stratum corresponding to the strain distribution $\{{}_1\gamma^{(N)}\}$ in the primary mode, according to ⑤ of the above, for each ground surface displacement δ , as shown in (3).

$$\begin{aligned} h_{eq}^{(N)} &= \{h_i^{(N)}\}^T \{{}_1\gamma^{(N)}\} \\ &= \{h_1^{(N)}, \dots, h_{k-1}^{(N)}, h_k^{(N)}\} \begin{Bmatrix} ({}_1u_1^{(N)} - {}_1u_2^{(N)})/D_1 \\ \vdots \\ ({}_1u_{k-1}^{(N)} - {}_1u_k^{(N)})/D_{k-1} \\ {}_1u_k^{(N)}/D_k \end{Bmatrix} \end{aligned} \quad (3)$$

Here, D_i is the thickness of the i^{th} stratum.

In addition, if response analysis is performed taking into consideration only the primary mode of the ground, it is necessary to take into consideration the participation function (PF) in order to convert the results into the response at the ground surface position. For this reason, when performing the static non-linear analysis of the ground, the primary mode participation function for the displacement δ at each step is also calculated.

3. Equivalent Single-degree-of-freedom Model in which Only the Natural Period of the Ground is used as an Index

3.1 Proposal of parameter standard values for which the natural period of the ground is used as an index

This section describes a study which aimed to evaluate seismic waveforms with greater ease across a wide range. A method is proposed for constructing an equivalent single-degree-of-freedom model for each ground location, using only the natural period T_g of the ground that can be evaluated relatively easily from microtremor measurements or microtopography classification. In the study, 60 ground locations possessing a large variety of period characteristics and strata compositions were used, and a static non-linear analysis for each ground location was performed. Through the analysis, the $G/G_0 - \delta$, the $h - \delta$, and $PF - \delta$ relationships were evaluated, from which an equivalent single-degree-of-freedom model of the ground was constructed from the natural period T_g alone. Note that these ground locations are also used to evaluate the ground surface design earthquake motion of railway structures, and possess a wide range of natural periods between 0.1 and 1.7 seconds. The non-linear characteristics of each ground location are expressed in a GHE-S model, and the parameters assigned to each stratum of each ground location are changed according to the soil quality classification such as sandy soil or cohesive soil.

Static non-linear analysis was carried out at all of these 60 ground locations. The conditions of the analysis were the same as those of the case described in the previous section, where analyses were performed in two ground locations. Figure 2 shows the arranged data representing the $G/G_0 - \delta$ and $h - \delta$ relationship. From this data, it can be seen that there is an overall tendency for areas where stiffness falls and damping increases for there to be higher displacement, although the degree of displacement differs greatly depending on the ground conditions. Consequently, it is difficult to create a simple model from these results alone.

Fig. 2 Results of static non-linear analysis of multiple ground locations with different characteristics

Consequently, it has been pointed out that if the results of element tests on the ground material are normalized and arranged using a strain value (standard strain γ_r) such that $G/G_0 = 0.5$, the results can be expressed using the same parameters irrespective of the ground material, and there is a possibility that this method can be applied to the overall ground system. Accordingly, the normalized relationships $G/G_0 - \delta$, and $h - \delta$ based on a displacement (this is defined as a standard displacement δ_r) such that $G/G_0 = 0.5$ are shown in Fig. 3 (a) and (b). Likewise, Fig. 3 (c) shows results after normalization of the participation function (PF), also using a standard displacement δ_r . First, from Fig. 3 (a) and (b) it can be seen that by normalizing the results of the static non-linear analysis of multiple ground locations by means of standard displacement δ_r , it is possible to express the change in stiffness and damping using roughly the same relationships, irrespective of the natural period, the stratum thickness, and the stratum composition of the ground. Also, from Fig. 3 (c), it can be seen the value of the participation function PF changes somewhat according to each ground location, but that the overall trend is roughly the same regardless of the ground condition. From the above results, it can be said that if the standard displacement δ_r is known, it is possible to predict to a certain extent the non-linear characteristics of the overall ground system,

regardless of the period and stratum composition of the ground.

Accordingly, the standard non-linear characteristics that can be set in common regardless of the ground conditions were evaluated. First, the $G/G_0 - \delta$ and $h - \delta$ relationships, were expressed by means of a GHE-S model, as in the previous section, and parameters that satisfied the average values of all of the results (blue lines in the figures) were determined by trial and error. The final parameters are indicated in Table 1, and based on these, the $G/G_0 - \delta$ and $h - \delta$ relationships, expressed using the proposed method are indicated by red lines in Fig. 3 (a) and (b). It can be seen that these eight parameters have made it possible to reproduce the average value of each result with good accuracy. Next, the trend of the participation function PF in Fig. 3 (c) was evaluated by means of the following equation. Note that each coefficient in (4) has been determined by the non-linear least-squares method.

$$PF = 1.4 - 0.07 \cdot \left(\frac{\delta}{\delta_r} \right)^{0.38} \quad (4)$$

Results obtained using (4) are also indicated in Fig. 3 (c) and illustrate the tendency for the participation function to decrease slightly along with the increase in the displacement as well as the average value for each set of the results. Fig. 3 (d) then shows the results arranged according to the relationship between the standard displacement δ_r and the natural period T_g of the ground. It can be seen that the standard displacement δ_r increases along with the increase in the natural period T_g . Here it is considered that the standard displacement δ_r varies not only with the natural period, but also with various other conditions including the stratum thickness and stratum composition, so it is desirable to carry out detailed evaluations at each ground location. However, the aim of the current study is to construct a model to evaluate the ground behavior easily from only a limited number of parameters, so the

Fig. 3 Results of the static non-linear analysis after normalization using standard displacement δ_r

Table 1 Results after setting standard parameters

$C_1(0)$	$C_2(0)$	$C_1(\infty)$	$C_2(\infty)$	$C_1(1)$	$C_2(1)$	κ	$h_{\max}$
1.00	1.00	0.15	2.5	0.87	1.15	1.30	0.21

evaluation was made only by using the natural period T_g as a parameter. More specifically, the model was expressed using a regressive formula (5), while reference was made to the trend of the change in the standard displacement δ_r with respect to the natural period T_g of Fig. 3 (d). Note that like (4), each coefficient of (5) is determined by a non-linear least-squares method.

$$\log \delta_r = 1.45 - \left(\frac{1}{T_g} \right)^{0.3} \quad (5)$$

As a result, the relationship between the natural period T_g and the standard displacement δ_r to be expressed is also indicated by the red line in Fig. 3 (d), thus expressing the general tendency for the standard displacement to increase along with the increase in the period. By integrating these results, it is possible to construct an equivalent single-degree-of-freedom model using the natural period T_g alone as a parameter.

3.2 Procedure for analyzing the ground response using an equivalent single-degree-of-freedom model in which the natural period of the ground alone, is used as an index

The above results were integrated, and a proposal for non-linear dynamic analysis was made, using only the natural period T_g as a parameter. Concretely, an equivalent single-degree-of-freedom model was constructed according to the steps described below, and the ground surface earthquake motion was calculated.

- Step 1 Set the initial stiffness K_0 using from the natural period T_g of the ground.
- Step 2 Set the GHE-S parameters and the natural period T_g from Table 1, and also set the standard displacement δ_r (cm) from (5).

Fig. 4 Velocity construction of the ground used in the calculation

- Step 3 Construct an equivalent single-degree-of-freedom model from Steps 1 and 2, and conduct a non-linear dynamic analysis.
- Step 4 Calculate the participation function PF from the maximum displacement $\delta_{\max}$ obtained from step 3 of the above and (4).
- Step 5 Multiply the response waveforms of the relative acceleration, the relative velocity and the relative displacement by the participation function PF , and convert the results into the response at the ground surface position.

In order to verify the accuracy of this method, an equivalent single-degree-of-freedom model was constructed, using the natural period T_g of the ground alone as a parameter for the two ground locations shown in Fig. 4, and a ground response analysis was carried out. The conditions here, other than those under which each parameter is set according to the above procedure (setting of input earthquake motion, damping, etc.), were all made identical to those in Section 2. The results of the comparison of the ground surface earthquake motion obtained using this procedure, with the response found by means of a detailed

Fig. 5 Results of the prediction of the ground surface waveform by using only the natural period T_g (Ground A ($T_g=0.36$ sec))

Fig. 6 Results of the prediction of the ground surface waveform by using only the natural period T_g (Ground B ($T_g=0.86$ sec))

Fig. 7 Results of the evaluation the ground surface displacement using the proposed method

ground response analysis are shown in Fig. 5 (ground A) and Fig. 6 (ground B). Both sets of results agree well with each other, which verifies the effectiveness of this method. The same study was carried out at all the 60 ground locations covered in this study, and the ground surface displacement obtained was compared with the displacement found using the detailed model. Both sets of results, which are shown in Fig. 7, agree with each other. However, because (5) is used when estimating the standard displacement δ_s , an estimating error of a magnitude that is roughly the same as that shown in Fig. 3 (d) exists, and in the case of the ground that has a long natural period, the result differs slightly from the detailed model. Furthermore, compared with the results of the response analysis using an equivalent single-degree-of-freedom model constructed based on the results of the static non-linear analysis, the predicted accuracy of the response derived from the results of this method was inadequate. For this reason, this method is effective when carrying out a simple prediction of the ground surface in a region where the results of the ground investigation are incomplete, or when screening locations which require priority attention during an earthquake.

4. Conclusion

This paper proposes a method for performing non-linear dynamic analysis using an equivalent single-degree-of-freedom model that can be used even for ground that has a complex stratum composition. The results obtained are shown below.

- (1) A method was proposed for performing static non-linear analysis of the ground, which makes it possible to successively take into consideration the accumulation of localized strain accompanying non-linear behavior of the ground. As a result, it was possible to identify a tendency where stiffness of the overall ground system decreased while damping increased, after taking into consideration changes in the vibration characteristics accompanying the response displacement.
- (2) Based on the results of the static non-linear analysis of the ground, a method involving the construction of an equivalent single-degree-of-freedom model of the ground, was proposed. This method was applied to sample ground types, and it was confirmed that the ground surface earthquake motion, which was roughly the same as that of the detailed dynamic analysis, could be evaluated.
- (3) As a result of the application of the static non-linear analysis method to the ground, it became clear that by normalizing ground displacement with a standard displacement δ_s , it was possible to uniformly express the tendency for stiffness to decrease and for damping to increase, irrespective of the softness or hardness of the ground, or the stratum composition. In addition, the standard parameters which express the $G/G_0 - \delta$ and $h - \delta$ relationships were fixed, and a proposal was made to easily estimate the standard displacement δ_s from the natural period alone. As a result, it was possible to construct an equivalent single-degree-of-freedom model from the natural period T_g of the ground alone. The results obtained using this model were compared with the results obtained using the detailed model, and it was confirmed that the proposed method was effective.

If adequate detailed data is available, applying the method constructed here makes it possible to easily evalu-

ate good ground surface earthquake motion in the case of arbitrary earthquake motion, provided that a static non-linear analysis is carried out in advance in order ascertain each parameter. Also, even if a ground survey has not been carried out, it is possible to construct an analysis model only from the natural period of the ground which is estimated from microtremor observations and microtopography, which enables us to evaluate the earthquake motion of the ground surface using the same method. Consequently, by using this method, it is possible to rationally extract locations which require priority attention in an earthquake, when the target of the measurement is an existing group of structures for which ground data is inadequate, and to predict ground surface earthquake motion immediately after a large scale earthquake.

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