Improvement of Accuracy of Method for Measuring Axial Force of Rail Based on Natural Frequency

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In order to evaluate the axial force of a continuous welded rail quantitatively in a simple manner, a method was developed to measure axial force based on the natural frequency. However, the accuracy of this method is insufficient because of certain variations in track condition. This study extracted factors which influence measurement accuracy and proposed an error correction method using track finite-element analysis for the purpose of improving the accuracy of this measuring method. Furthermore, measurements were taken of the natural frequency and axial force of a rail on a real track in order to validate the proposed method.

Keywords: continuous welded rail, measurement method of the axial force, natural frequency, FEM, field test, rail excitation test

1. Introduction

Continuous welded rails (CWR) run the risk of buckling from compressive stresses in summer and of breaking from tensile stresses in winter. Therefore, the axial stress of the CWR should be monitored and controlled appropriately. Although a number of solutions to deal with axial force have been suggested in the past, they have not come into use yet because they had too many drawbacks making them impractical.

(1) Method based on measuring rail strain

The thermal stress of the rail is directly measured by using a strain sensor such as a strain gauge[1, 2]. This method can precisely measure changes in the rail axial force, but it is impossible to obtain the absolute value of the axial force.

(2) Method using the magnetoelasticity effect

This method including Barkhausen Noise technology[3] or MAPS-SFT[4] exploits the relationship between the stress and the magnetic properties of ferromagnetic materials. Unfortunately, magnetic properties are sensitive to total rail stresses (thermal stress plus residual stress at the time of production). Therefore, this method needs to eliminate the effect of residual stresses by using magnetic measurements taken while in an unloaded state for measuring the absolute value of the axial force.

(3) Measurement of the stress with X-rays [5]

This is a technique measuring the stress state of the rail by focusing on the deformation of the rail steel crystal using X-rays. However, this approach has the same problem as technique (2), because measured stress includes not only thermal stress but residual stress as well.

(4) Rail lifting method

This method is based on the measurement of the reaction force while the rail is lifted. This method makes it possible to obtain the absolute value of the axial force, but the measurement can only be taken if the rail is under tensile stress. In addition, the VERSE method[6], for example, is labor intensive since rail fasteners must be removed over a distance of 30 meters before the rail can be lifted.

(5) Ultrasonic method

Certain methods using the acoustoelastic effect have been proposed for measuring the axial force of the rail, from the propagation velocity of elastic waves[7, 8]. However, these measurements have low sensitivity to axial stress and high temperature dependency. To overcome these disadvantages, some methods based on ultrasonic guided waves[9] or Rayleigh wave polarization[10] were suggested, but this research is still at an early stage of development. In the method using second-harmonic generation caused by elastic wave propagation in rails, neutral temperatures can be found by the wave pattern of the ultrasonic nonlinear parameters measured around the time when the axial force becomes zero[11]. Therefore, this method has several limitations related to time or environment.

(6) Rail vibration method

In order to measure the axial force of a CWR, a simple method was proposed based on changing vibration mode characteristics of the rail with the axial force[12-14]. This method has many advantages; for example, the absolute value of the axial force is measurable, it is neither labor intensive nor does it require large-scale measuring equipment, and measuring time is short. However, the accuracy of this method is insufficient because not only the axial force but the variations of track conditions such as the state of the rail and the rail fastening affect the vibration mode of the rail.

This paper investigated the influence of variations in track condition and proposed a correction method to increase the accuracy of measurements by using a finite element track model. The axial force and natural frequency of the CWR were also measured under various temperature
conditions on an actual ballasted track to validate the correction method.

2. Axial force measuring method based on the natural frequency

The schematic system of the method for measuring axial force is shown in Fig. 1. In this method, impulse hammer excitation is performed horizontally at the rail head at the centre of the sleeper bay, and the response acceleration measured by an acceleration sensor attached to the rail head is provided to perform frequency-analysis in order to identify the natural frequency of the lateral pinned-pinned mode. Then, the axial force is estimated by using the relation between the axial force and the natural frequency derived from the numerical simulation. The pinned-pinned apparatus for measuring the natural frequency are an impulse hammer, an acceleration sensor, and a FFT analyzer. All of them are very portable. Since the natural frequency can be measured rapidly and precisely in a short time (around one minute), the precision of this method for measuring the axial force depends on the relation between the axial force and the natural frequency.

![Fig. 1 Axial force measuring method based on the natural frequency](image)

3. Track FEM model

3.1 Overview of the model

Figure 2 shows the structure of the track FEM model for the relation between the axial force and the natural frequency. Solid elements are used to simulate the detailed geometry of a JIS 60 kg type rail on one side, which is the target for the vibration analysis, and the opposite rail and JIS 3PR type sleepers are modeled as beam elements having an effective cross section of each component. To represent JIS 5N type rail fasteners, spring elements in which the spring coefficient changed with temperature was used to tie rail elements to the sleeper elements. The ballast layer was modeled with spring elements under the sleepers. The analytical parameters are summarized in Table 1. The sleeper bays were 600 mm. The track model had 200 sleepers, which is equivalent to a 120 meters in length, so that the influence of the reflection wave occurring at both ends of the rail model could become small. Axial force was provided by setting a constant temperature on the solid elements of the rail.

As a result of the analysis using this model, the natural frequency of the lateral pinned-pinned mode at the time the axial force was zero was calculated to be 504.0 Hz, while the sensitivity of the natural frequency to the axial force was 90.6 kN/Hz.

3.2 Spring coefficient measurement in part of the rail fastener

In order to evaluate the spring coefficient in part of the rail fastener to be tested for the natural frequency measurement, the acceleration response was measured of a specimen composed of a cut rail, a cut sleeper and a rail fastener during the hammer excitation, as shown in Fig. 3(a). Furthermore, the parameter in the 6 directions was identified for the spring element, i.e., spring coefficient $k$ and loss factor $\eta$, which reproduces the measured acceleration response by the modal analysis using the finite element model (Fig. 3(b)). The identified parameters $k$ and $\eta$ were assumed to be the measured values for the spring coefficient.

![Fig. 2 Track model to calculate the relation between the axial force and the natural frequency](image)
Table 1 Track model parameters

(a) Solid and beam elements

<table>
<thead>
<tr>
<th>Component</th>
<th>Element</th>
<th>Young's modulus [GPa]</th>
<th>Poisson's ratio</th>
<th>Mass density [kg/m³]</th>
<th>Coefficient of thermal expansion [1/℃]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>Solid</td>
<td>206</td>
<td>0.29</td>
<td>7820</td>
<td>1.14 × 10⁻³</td>
</tr>
<tr>
<td>Rail</td>
<td>Beam</td>
<td>206</td>
<td>0.29</td>
<td>7820</td>
<td>-</td>
</tr>
<tr>
<td>Sleeper</td>
<td>Beam</td>
<td>45</td>
<td>0.167</td>
<td>2350</td>
<td>-</td>
</tr>
</tbody>
</table>

(b) Spring element

<table>
<thead>
<tr>
<th>Component</th>
<th>Element</th>
<th>Symbol</th>
<th>Direction</th>
<th>Value (per element)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail fastener (Rail to sleeper)</td>
<td>Spring</td>
<td>$k_f$</td>
<td>TX, TY, TZ, RX</td>
<td>205.9 MN/m / 129.9 MN/m / 786.4 MN/m / 1.90 MNm/rad / 1.44 MNm/rad / 1.30 MNm/rad</td>
</tr>
<tr>
<td>Loss factor</td>
<td>$\eta_f$</td>
<td>TX, TY, TZ, RX</td>
<td>0.11 / 0.1 / 0.19 / 0.11 / 0.09 / 0.04</td>
<td></td>
</tr>
<tr>
<td>Sleeper (end) to ballast</td>
<td>Spring</td>
<td>$k_e$</td>
<td>TX, TY, TZ, RX</td>
<td>6.19 MN/m / 6.96 MN/m / 4.05 MN/m / 0.076 MNm/rad</td>
</tr>
<tr>
<td>Loss factor</td>
<td>$\eta_e$</td>
<td>TX, TY, TZ, RX</td>
<td>4 / 4 / 4 / 4</td>
<td></td>
</tr>
<tr>
<td>Sleeper (center) to ballast</td>
<td>Spring</td>
<td>$k_c$</td>
<td>TX, TY, TZ, RX</td>
<td>4.09 MN/m / 6.36 MN/m / 3.45 MN/m / 0.076 MNm/rad</td>
</tr>
<tr>
<td>Loss factor</td>
<td>$\eta_c$</td>
<td>TX, TY, TZ, RX</td>
<td>4 / 4 / 4 / 4</td>
<td></td>
</tr>
</tbody>
</table>

※ 1 TX, TY, TZ: Translation direction of the X,Y,Z axis
   RX, RY, RZ: Rotation direction around the X,Y,Z, axis
※ 2 Value of the temperature being 25 ℃ (Varreid with its temperature)

4. Evaluation and correction of the influence of the variations in track conditions on the estimation accuracy of the axial force

The shifts in the natural frequency brought about by variations in track condition cause the changes in the relationship between the axial force and natural frequency, leading to estimation errors. This chapter presents three track conditions expected to greatly change the natural frequency and quantitatively evaluates the influence of these track conditions on the estimation accuracy of the axial force. This section also discusses the correction method for these errors.
4.1 Evaluation of the influence on the accuracy of the estimated axial force

Assuming that the accuracy of the estimated axial force required for utilizing this method is 90.6 kN (equivalent to 5 °C of rail temperature), the allowable shifts in natural frequency due to variations in track condition will be ± 1 Hz considering the frequency sensitivity in the previous section. The influence of the variations in actual track condition is evaluated by a margin of error of ± 1 Hz as a standard value.

(1) Variation of the sleeper bays

The natural frequency of the model was calculated with 200 sleeper bays arranged according to 7 patterns of regular random numbers, the average of \( \nu \) which was 600 mm and the standard deviations of \( \sigma \) which were 10 mm, 20 mm, 30 mm, 50 mm. Figure 4 shows the shift in natural frequency of the model with varied sleeper bays compared to the model with sleeper bays at constant intervals which had a natural frequency of 504.0 Hz. The shifts in natural frequency were less than 0.4 Hz when \( \sigma \leq 10 \) mm (see Fig. 4(a)), but rose to over 1 Hz when \( \sigma \geq 20 \) mm (Fig. 4(b-d)). Given that a variation in sleeper bays of over \( \sigma \geq 20 \) mm could occur in the actual ballasted track, this influence is not negligible in measuring the axial force.

(2) Wear of the rail head

The cross-sectional geometry of the wear of the rail head was measured at six cross-sections, two of which were in two straight track sections and four of which were from two curved track sections (R320 m, and R400 m). Two of the four cross-sections were from the inside rail and the other two from the outside rail, as shown in Fig. 5. The geometries of the solid elements of the rail are depicted numerically. The results are shown in Fig. 6. The natural frequencies of the rails in the two straight track sections and that of the inside rails in the two curved track sections increased as the top surface of the rails became worn, whereas those of the outside rails in the two curved track sections decreased as the gauge corner became worn. Slight wear with a depth of about 2 mm on the straight track caused the natural frequency to change by more than 1 Hz.

(3) Individual difference and temperature dependency of the coefficient in part of the rail fastener

The spring coefficient of the rail was measured just above the fasteners at eight points, five of which were on a used rail pad (No.1 ~ No.5) and three of which were installed on a new one (No.6 ~ No.8), according to the method explained in section 3.2. The same pair of JIS-5N-type spring clips was used to tighten all eight specimens with a standard torque of 120 Nm. To evaluate the change in spring coefficient with the change in temperature, the specimen rail was heated to approximately 70 °C and then cooled at room temperature until the difference in temperature between the bottom of the rail and the top of the sleeper was less than 10 °C. Then excitation tests were performed each time the rail temperature dropped by approximately 5 °C until it reached normal temperature. Tests were conducted on specimens No.1 ~ No.7 in winter and tests on specimen No.8 in summer, to obtain test values for a wide temperature range. The measurement results of

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\begin{align*}
\text{Fig. 4} & \quad \text{Shift in natural frequency of the model with varied sleeper bays provided with regular random numbers (}\nu=600\ \text{mm}, \sigma) \\
\text{(a)} & \quad \sigma=10\ \text{mm} \\
\text{(b)} & \quad \sigma=20\ \text{mm} \\
\text{(c)} & \quad \sigma=30\ \text{mm} \\
& \quad \sigma=40\ \text{mm}
\end{align*}
\]
the spring coefficient $k_i$ in the RZ direction, which greatly affects the natural frequency, are plotted versus averaged temperature $T$ measured at the bottom of the rail and the top surface of the sleeper, as shown in Fig. 7. These results indicate that the more temperature falls, the more the spring coefficients in part of the rail fastener increases. To evaluate the influence of individual differences in rail pads, an approximation of $k_i$ by a quadratic function of $T$ identified the each spring coefficient for the specimens No.1 ~ No.8 at 25 ℃. Furthermore, the natural frequency of the pinned-pinned mode was calculated for each specimen by using these spring coefficients. Figure 8 shows the shift of the natural frequency from 504.0 Hz.

4.2 Extent of influence of variations in track condition

Here we investigate numerically the extent to which variations in track condition affect natural frequency. Four normal random numbers, pattern 01 and 02 of $\sigma$=30 mm shown in Fig. 4(c), and pattern 02 and 03 of $\sigma$=50 mm shown in Fig. 4(d), were applied to the sleeper bays only in the range of $L$ meters (varied between 0 to 120 meters) around the center of the track model. Figure 9(a) shows the results of changes in natural frequency corresponding to $L$. In cases where $L$ was less than 20 meters, the natural frequency rapidly changed with increasing $L$, but the natural frequency converged at an almost constant and its variation was below 0.2 Hz in cases where $L$ was more than 30 meters. As a result of similar calculations applied to the worn rail head, similar results were obtained, as shown in Fig. 6(b). In cases where $L$ was more than 30 meters, variation in the natural frequency of the rail were below 0.3 Hz. Judging from these results, the natural frequency depends on the track conditions such as the sleeper bays, the wear of the rail and other factors within a distance of 30 meters from the measurement point.

The variation in sleeper bays and the wear of the rail head existing in actual tracks cannot be ignored in measuring the axial force, therefore, the information of the sleeper bays and the wear of rail head (only that of the representative point if the wear of the rail head is constant) within 30 meters from the measurement point has to be reflected between actual track and the track model.

4.3 Correction method of the relation between the axial force and the natural frequency considering the track conditions

(1) Variation in sleeper bays

According to sleeper bays measured within 30 meters from the measurement point, the sleeper bays in the track model were modified.
(2) Wear of the rail head
The solid elements of the rail of the track model were deformed according to the cross sectional geometry of the actual rail head at the measurement point.

(3) Individual differences and temperature dependency of the spring coefficient in part of the rail fastener

The temperature dependency of the spring coefficient of the rail fastener affects the gradient of the axial force versus the natural frequency. In order to correct this, the values of spring coefficient according to measured temperature of the rail and the sleeper on the actual track (regression equation of Fig. 7) were set in the track model.

Correction for the individual differences in spring coefficient of the rail pad is difficult. It was found that the estimation error was as much as 100 kN (equal to the 1.1 Hz of the natural frequency).

5. Measuring tests in the field

5.1 Overview of the experiment

Measuring tests were performed on the CWR section on an actual track in winter (November 2013) and in summer (September 2014). Experiment sites were located in an unmovable straight section. The changes in natural frequency and the axial force of the rail as rail temperature rose from the early morning to the daytime were measured and compared. Figure 10 presents the measurement positions, the excitation method and the installation conditions of the measurement equipment. Impulse hammer excitation was performed horizontally at the rail head at the centre of the sleeper bay, and the response acceleration was measured by an acceleration sensor attached to the rail head to analyze frequency-response, obtaining the natural frequency. At the same time, the rail axial force was converted from the strain measured by strain gauges attached to both sides of the rail web. In this experiment, it was assumed that the neutral temperature of the rail was 25 °C (which is widely used in the track). Rail temperature was measured with a thermocouple attached to the rail web.

For the purpose of correcting the relation between the axial force and the natural frequency derived from the numerical model, the sleeper bays, the wear of the rail head and the temperature of the bottom of the rail were also measured.

5.2 Experimental results

100 sleeper bays were measured. The mean distance between them was 565 mm with a standard deviation of 28 mm in the winter, whereas the mean was 563 mm with a standard deviation of 26 mm in the summer. The maximum value of the wear of the rail head was 1.6 mm in the winter and 2.2 mm in the summer. Rail temperature rose from the early morning to the daytime with a variation of 5 °C to 24 °C in the winter, and of 16 °C to 44 °C in the summer.

Figure 11 shows the relationship between the natural frequencies and the rail axial force confirming that the proportionality between the rail axial force and the natural frequency.
frequency. The sensitivity of the axial force to the natural frequency measured at two different points in the same season was almost the same, but the sensitivity of the axial force found during the winter experiment was around 1.4 times bigger than that during the summer experiment. Regarding the difference in natural frequency between the two measurement points under the same axial force, it was 1.5 Hz in winter while there was little difference in the summer.

5.3 Comparing with numerical results

Figure 11 also shows the numerical values corrected with the method described in section 4.3 and the uncorrected values. In this case, the value of specimen No.6 was applied to the spring coefficient in the part of the rail fastener for the winter test and the value of specimen No.8 was applied for the summer test. The uncorrected numerical model provides only a single relation between the axial force and the natural frequency, but the numerical model with the correction in consideration of the track condition is able to reproduce the differences in natural frequencies observed in the experimental results between the two measurement points (1.5 Hz in winter, almost zero in summer). This means that the proposed method can correct the differences in estimated values of the axial force at each measurement point. However, the numerical results of the natural frequency when the axial force is zero differ from the experimental ones, even when the model is corrected. A difference of 7 Hz was observed in the summer test and a difference of 1 Hz was observed during the winter test. This is because the assumed neutral temperature (25 °C) may be different from the actual temperature.

In addition, the calculated sensitivity of the axial force to the natural frequency by using the correction agrees well with the experimental results. The corrected numerical model can provide much the same values as the summer experiment as shown in Fig. 11(b). On the other hand, with regards changes in natural frequency in relation to the axial force, the experimental values were approximately 1.2 times larger than the numerical values obtained in the winter test (Fig. 11(a)). These differences were caused by the inappropriate modeling of the thermal properties of the spring coefficient in the part of the rail fastener. Although Fig. 7 indicates that the change in the spring coefficient with the temperature in the part of the rail fastener in the RZ direction is almost constant between 10 °C to 40 °C, it is suggested by [15] that the change in the elastic modulus of the rail pad with the temperature increases as its temperature decreases. Assuming that the change in the spring coefficient in the part of the rail fastener with a temperature below 25 °C is 1.2 times as large as that at a temperature over 25 °C, numerically estimated sensitivity of the axial force to the natural frequency becomes approximately equal to the experimental value obtained in the winter test. Further tests intend to examine the temperature dependency of the spring coefficient including verification of the correlation between the spring coefficient, temperature of the rail and the sleeper, and measurement of the spring coefficient in a wide range of temperatures (0 °C~50 °C) including winter and summer inter alia.

6. Conclusions

An investigation was conducted into the influence of variations in track condition on the accuracy of estimated rail axial forces using a FEM track model. An error correction method was also proposed for the relation between the axial force and the natural frequency needed for the estimation of the axial force by considering the sleeper bays, the wear of the rail head, and the temperature dependency of the spring coefficient of the rail fastener. Finally, the proposed method was validated through field tests. The following conclusions can be drawn:

(1) Shifts in natural frequency due to variation in sleeper bays, wear of the rail head, individual difference and temperature dependency of the coefficient in part of the rail fastener cannot be ignored in measuring the axial force.

(2) In order to eliminate the effects of the sleeper bays and the wear of the rail head, the information of the sleeper bays and the wear of rail head within 30 meters from the measurement point of the actual track are required.

(3) The measurement results conducted in this study indicate that an estimation error of 100 kN at most
could occur due to the individual differences in spring coefficients of the rail pads.

(4) It was confirmed that the axial force of the rail is proportional to the natural frequency from the experimental results in the CWR section of the actual ballasted track.

(5) The proposed method can correct the difference in the estimation value of the axial force between the measurement points.

(6) Although the axial force of the actual track was estimated previously in a summer test, experimental values obtained during the winter test differed from the numerical values. These results suggest that further studies need to be conducted into the temperature dependency of the spring coefficient of the rail fastener, which causes evaluation errors.

References


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