Statistical Characterization of P-wave Growth for Earthquake Early Warning

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This article introduces a statistical relation between earthquake magnitude (M) and a characteristic measured from initial P-wave displacement records, which demonstrates that the final M can be inferred before the peak amplitude arrival (i.e., earthquake rupture completion) in the range of M < 7. Because it is unknown why and how the relation is established, we propose a model for explaining its physical background. We also discuss the application of the relation to earthquake early warning, which can help improve train running safety because M can be determined faster than with the conventional method while an earthquake rupture is underway.

Keywords: earthquake early warning, magnitude

1. Introduction

Rapid determination of earthquake magnitude (M) is critical for earthquake early warning (EEW) that is conducive to reducing the risks of running trains during an earthquake event [e.g., 1]. To estimate M for EEW while an earthquake rupture is still in progress, some approaches have already been proposed. For example, the algorithms of the current EEW system for Shinkansen and of the Japan Meteorological Agency EEW employ a ground motion prediction equation (GMPE) in which M is, at a given distance, proportional to the logarithm of displacement amplitude observed at seismic stations [e.g., 2, 3, 4] (hereafter, referred to as the GMPE approach). Acceleration amplitude will be used for the technique of the next Shinkansen EEW in addition to displacement [5]. With these methods, therefore, the final M cannot be determined before the arrival of the peak amplitude. Colombelli & Zollo [6] and Noda et al. [7], using displacement and acceleration records, demonstrated that it typically required about 1.5 s, 3–4 s, and 10 s after the P onset, respectively, for M 5, M 6, and M 7 until the peak amplitude arrives, which are consistent with typical earthquake rupture durations [e.g., 8]. In another approach, which was used in the previous Shinkansen EEW system [9], M was derived from the frequency content (i.e., predominant period) of initial P wave [e.g., 10, 11, 12]. However, this method may be problematic in use for EEW because it can produce less accurate estimates than those using the GMPE approach [4]. Moreover, this approach is controversial from a physical point of view, that is, Olson & Allen [13] concluded that it could infer the final M before the completion of earthquake rupture (i.e., the peak amplitude arrival), which Rydelek & Horiuchi [14] argued that was suspicious. We will discuss this later. Alternatively, Hoshiba & Aoki [15] proposed to estimate the seismic wave field using a data assimilation technique without M. This methodology should be reasonable for EEW because the ultimate aim of EEW is not to determine hypocentral information (including M) but to issue the ground motion distribution radiated from the source, although this method must wait for the peak amplitude arrival to obtain the ground motion distribution associated with the final M as well as the GMPE approach. In this article, to discuss those issues, we first review a recent study [16] that presented a new insight and groundwork for EEW and consider the implications of its results.

2. Statistical characterization of P-wave growth

This chapter introduces Noda & Ellsworth [16] who statistically characterized initial P-wave records to examine earthquake rupture growths.

2.1 Averaged absolute displacement

Noda & Ellsworth [16] analyzed the vertical component data from K-NET operated by the National Research Institute for Earth Science and Disaster Resilience (NIED).
The dataset contained 7,514 accelerograms from 150 earthquakes with $4.5 \leq M_w \leq 8.7$ and focal depth $\leq 60$ km ($M_w$: moment magnitude). Figure 1 shows the hypocenter distribution. Observations with a hypocentral distance ($R$) of less than 200 km were selected. Noda & Ellsworth adopted $M_w$ values determined by the NIED from the F-net moment tensor analysis (note that $M_w$ of the 2011 Tohoku earthquake was 8.7 in this dataset). The P onsets were all manually picked. The waveform data were doubly integrated to convert from acceleration to displacement and then filtered with the frequency of 0.075 – 3 Hz. See the original manuscript [16] which demonstrates that this band-pass filter did not affect the discussion shown below.

To look at the statistical characteristics of the initial P waves, the dataset was binned in terms of $M_w$ by 0.1 unit and $R$ by 25 km intervals. In each bin, the absolute values of the displacement data were averaged. This procedure relieves the influence of differences in radiation pattern, rupture directivity and crustal structure. Figure 2 presents all the averaged absolute displacements (AADs) obtained in that study by each hypocentral distance bin (the data are colored in terms of $M_w$ using the scale at the right-hand bottom of the figure).

Noda & Ellsworth [16] concluded that four characteris-
which will be discussed in Chapter 3.

amplitude in contrast with the AADs, suggesting that an event. The synthetic data, however, do not grow again in wave similarly begins and then departs earlier for smaller events. The similarity is important for EEW because it suggests that the peak amplitude arrival was not necessary for measuring the final earthquake magnitude. By the way of comparison, Fig. 3b shows synthetic displacement waveforms for $M_5$ (blue), $M_6$ (green), and $M_7$ (red) emitted from the theoretical circular fault model [17] (the stress drop = constant; the polar angle = 45°) which is commonly used in seismology. This also shows that the P wave similarly begins and then departs earlier for smaller events. The synthetic data, however, do not grow again in amplitude in contrast with the AADs, suggesting that an alternative model is needed to explain the observed data, which will be discussed in Chapter 3.

Figure 3 shows the relationship between $T_{dp}$ and $M_w$ by each hypocentral distance bin using the AADs and demonstrates that log $T_{dp}$ significantly correlates with $M_w$. For this result, the $DPD$ of 0.05 s displayed the best correlation between $T_{dp}$ and $M_w$ (see the original article [16] for the detail about the selection of $DPD$). Because this correlation was established up to about $M_w$ 7 (i.e., the data from the two largest events, the $M_w$ 7.9 2003 Tokachi and the $M_w$ 8.7

2.2 Scaling relation between $M_w$ and the departure time ($T_{dp}$)

An approach was proposed to measure the time of the departure from P-wave similar growth [16]. Figure 4 shows an example of the AAD (the bin of $M_w = 5.0$ and $R = 50 – 75$ km; the gray and red lines show the absolute displacement data in the bin and its average, respectively). As indicated in the previous section, the P-wave growth departed from the common trajectory and was followed by a period of declining amplitude. Now, a systematic decline in absolute displacement is called a “departure delay” (the green arrow in Fig. 4). To determine the departure time, a predefined threshold, $DPD$, was introduced and compared with the duration of the departure delay. When the departure delay exceeds $DPD$ for the first time after the P onset, the departure from the similar growth is considered to have occurred (the orange arrow in Fig. 4). The time from the P onset to the departure is referred to as $T_{dp}$ (the blue arrow in Fig. 4).

Figure 5 shows the relationship between $T_{dp}$ and $M_w$ by each hypocentral distance bin using the AADs and demonstrates that log $T_{dp}$ significantly correlates with $M_w$. For this result, the $DPD$ of 0.05 s displayed the best correlation between $T_{dp}$ and $M_w$ (see the original article [16] for the detail about the selection of $DPD$). Because this correlation was established up to about $M_w$ 7 (i.e., the data from the two largest events, the $M_w$ 7.9 2003 Tokachi and the $M_w$ 8.7.
2011 Tohoku earthquakes, were apparently outliers), the scaling relation between $T_{dp}$ and $M_w$ was determined on the basis of all data, except these two events, as:

$$M_w = 2.29 \times \log T_{dp} + 5.95,$$

(1)

where the results from all of the distance bins were used (Fig. 5i). Note that there was no clear evidence for the dependence of $T_{dp}$ on distance. Equation (1) demonstrated that $T_{dp}$ was approximately 0.4 s, 1.1 s, and 2.9 s, respectively for $M_5$, $M_6$, and $M_7$, that is, $T_{dp}$ occurred at about 30% of typical rupture duration (i.e., the peak amplitude arrival). Therefore, Noda & Ellsworth [16] concluded that a characteristic, which scaled with the final $M$, could be found in a statistical sense before the earthquake rupture was completed. The reason for saying “in a statistical sense” is that averaged data (AADs) were analyzed.

3. Discussion and conclusions

3.1 Physical background for the scaling relation

As shown in the previous chapter, Noda & Ellsworth [16] found an empirical relationship which suggested that the final $M$ could be inferred before the earthquake rupture completion. This leads to an essential question of seismology in a physical sense, that is, whether or not the final $M$ is deterministic at the time of initiation (i.e., whether or not rupture nucleation knows about its termination) [e.g., 18]. This is still a controversial question, although a number of
studies have tried to answer it [e.g., 13, 14]. Therefore, this paper shows the consistency between Noda & Ellsworth [16] and previous studies, and considers the physical background for why and how the scaling relation between $T_{\text{dp}}$ and $M_s$ can be established, which was not clearly explained by Noda & Ellsworth [16].

The specific data to support the premise that the final $M$ cannot be known until the rupture is completed (i.e., earthquakes are not deterministic) is often derived from the fact that P-wave amplitudes actually start in a similar fashion for a wide range of $M$ [e.g., 19, 20, 21, 22]. Because a similar growth in observed seismograms implies that smaller and larger earthquakes originate in similar ways, other factors, such as the fault geometry or the fault-surface complexity or the surrounding stress field, should control the final $M$ of the event rather than the origin itself. On the other hand, evidence which shows that this interpretation may not always be true is based on empirical characteristics relating to the final $M$ even before the end of typical source durations. For example, the maximum observed displacements or predominant frequencies measured using the time window of 2 to 4 s after the onset correlate well with the final $M$ up to $M_s \leq 7$ (or more) events in which the source duration must be about 10 s (or more) [e.g., 13, 23, 24]. We consider that the scaling relation between $T_{\text{dp}}$ and $M_s$ should make a compromise between those (i.e., those are not contradictory when looking at the scaling relation), because $T_{\text{dp}}$ is statistically shorter than the source durations even when a similar initial P-wave growth is also confirmed. That is to say, based on findings from the scaling relation, a correlation between the final $M$ and the maximum displacement or the frequency content is found after departure from indistinguishable growth even before the end of the rupture (i.e., the peak amplitude arrival).

Although the idea indicated above offers a new insight, it does not explain the physical mechanism of the scaling relation. To do this, we look at Uchide & Ide [25] who obtained the moment histories of 1.7 ≤ $M_s$ ≤ 6.0 earthquakes observed in Parkfield, California and found that there were two stages in rupture growth. First was the “growth stage” characterized as a rapid increase in moment release, while the other was the “decline stage” as a less rapid increase. Uchide & Ide concluded that the growth stage had a similar growth in moment history over the $M_c$ range they analyzed, and showed that the turning point from growth to decline should be proportional to $M_s$. We find that time can agree with $T_{\text{dp}}$ (e.g., that of the $M_s \leq 6.0$ event is about 1.0 s while $T_{\text{dp}}$ is about 1.1 s for $M_s \leq 6.0$), suggesting that earthquake ruptures before $T_{\text{dp}}$ are characterized by the growth stage while after $T_{\text{dp}}$ they were characterized as the decline stage (this can be also supported by the characteristic (4) in Section 2.1).

As shown in Chapter 2, earthquake rupture propagation can be simplified as a circular crack fault although the conventional kinematic model [17] cannot explain the characteristics of the observed data (Fig. 3). We consider that is because the model assumes that the fracture instantaneously stops all over the fault when the rupture front reaches the fault edge. As an alternative, we propose a model that makes it possible to physically explain the background of the $T_{\text{dp}}$ scaling with $M_s$ (that is equivalent with the mechanism for the two stages [25]) as follows (Fig. 6), examining Murphy & Nielsen [26] who dynamically simulated earthquake ruptures using a finite difference method (although they used an isolated single patch on the fault):

1. An earthquake rupture begins in the area of the “initial asperity” where stress has accumulated more than in the surrounding area. Initial asperities are distributed hierarchically according to size on the fault plane (Fig. 6). This is a similar idea to Ide & Aochi [27] who placed fracture patches on the fault in a fractal manner for their numerical simulation to represent the heterogeneities of the dynamic parameters.

2. When a rupture begins somewhere in the initial asperity, its moment is rapidly released during the rupture propagation within the area (Fig. 6a-ii and 6b-ii). If the size of the initial asperity increases, the growth in the release similarly increases as well (i.e., duration of similar growths becomes longer; Fig. 6a-iii, 6b-iii and 6b-iv). These are consistent with the features of the growth stage described by Uchide & Ide [25].

3. Even when the rupture front arrives at the edge of the initial asperity, the fracture is not terminated but percolated from the asperity area to the surrounding area (Fig. 6a-iv). This should correspond to the decline stage described by Uchide & Ide [25] (note that the rupture area should be larger than in the growth stage, suggesting that its moment rate emitted from the whole rupturing area should be greater).

4. Unless the size of the asperity size exceeds a certain level, the rupture percolating from the initial asperity naturally stops because its energy is not enough to generate the critical-slip distance ($D_c$) of the slip-weakening law [e.g., 8] in the surrounding area (Fig. 6a-v).

5. If it exceeds the threshold, the rupture propagation no longer completes by itself but spontaneously spreads out to the area surrounding the initial asperity (i.e., it cannot stop naturally) because the slip reaches $D_c$ even in the surrounding area (Fig. 6b-v). A barrier is required to arrest the rupture [e.g., 28].

The discussion (5) can explain why $T_{\text{dp}}$ did not correlate with $M_s > 7$ earthquakes (Fig. 5). That is, they turned out to be such huge events because the spontaneous propagation occurred during the rupture growths due to a series of factors: e.g., the initial asperities were large or had highly accumulated stress. For $M_s < 7$ earthquakes, spontaneous ruptures are, statistically, unlikely to happen, i.e., the size of the initial asperity should control the final $M$. Although it is possible even for $M_s < 7$ earthquakes that the spontaneous propagation follows the break of the initial asperity, such data might have been missed because the AADs were used in the analysis. We conclude that these can be the reasons why it is possible to see a significant correlation between $T_{\text{dp}}$ and the final size of earthquakes, while we cannot see it for larger events.
3.2 Application of the scaling relation to earthquake early warning

For applying the scaling relation to $M$ estimation in the EEW algorithms, the $T_{dp}$ measurement is available using Equation (1) for $M_w < 7$ earthquakes. The $T_{dp}$ values are easily measured even in real-time computation using the departure delay and DPD (Fig. 4). As mentioned above, however, $M$ estimates from displacement amplitude are more accurate than those from the contents of predominant periods which must have connection with $T_{dp}$ [4]. Considering this problem, Noda & Ellsworth [29] proposed a new method to upgrade the GMPE approach based on the scaling relation. In the method, the intercept of GMPE is determined using only the amplitude data that have passed $T_{dp}$. As a result, the intercept is time-dependent even though it is conventionally constant (see the paper [29] for more detail). Noda & Ellsworth [29] demonstrated that the method was effective in improving the speed of the GMPE approach without the loss of accuracy of $M$. We conclude that source information estimation is critical to halt running trains in the earliest opportunity, i.e., a tradeoff between the speed and accuracy needs to be considered for EEW.

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References


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