Hybrid Simulation Testing of a Pantograph-Catenary System Using a Dynamically Substructured System Framework and a MDOF Catenary Model

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This study proposes a hybrid simulation (HS) method for pantograph /catenary systems based on a dynamically substructured system (DSS) framework. In this method, the contact force between an actual pantograph and a hydraulic actuator is utilized to calculate the motion of the catenary in real-time, whilst the actuator is driven according to the calculated motion of the catenary. The advantage of the proposed method, when compared with commonly-used methods such as the inverse transfer function approach, is that DSS is better able to avoid instability that can be caused, for example, by pure delay characteristics in the actuator dynamics. The proposed method is also able to accurately represent dynamic interaction between the pantograph and the catenary. In this paper, the DSS methodology is introduced and then the proposed method is validated via simulation and experiment.

Keywords: pantograph-catenary systems, hybrid simulation, dynamically substructured systems

1. Introduction

In order to evaluate railway system current collection quality, dynamic interaction between the pantograph and catenary should be thoroughly understood and tested. Some methods such as numerical simulation, physical testing rigs and on-track testing are already available to evaluate the current collection quality. Although various conditions such as train velocity can be changed in the simulation of the current collection system, the simulation results are affected by pantograph and overhead contact line (OCL) modelling errors. A test facility has already been developed at the Railway Technical Research Institute (RTRI) to measure this dynamic interaction [1], consisting of a tracked running device (representing the vehicle) that supports a pantograph and a linear catenary of length 400 m. This equipment can evaluate current collection performance, but the maximum velocity of the running device is limited to 200 km/h, which is not always sufficient for high-speed railway investigations. Moreover, the ‘coasting section’ of the track is approximately 70 m long, which is not always of sufficient length for higher speed investigations. A distinct advantage of this facility is that the on-track testing enables evaluation of the dynamic interaction using a real OCL. However, this requires high labour costs, track availability and, due to the high-voltage environment, problematic signal and data acquisition.

A hybrid simulation (HS) method has also been developed by the RTRI, in order to evaluate the dynamic interaction based on a bench test of the pantograph. In the HS system, dynamic motion of the OCL is calculated based on a real-time simulation and the real pantograph is excited using the calculated displacement of the contact wire. In

the real-time simulation, the contact force between the servo-hydraulic actuator and the pantograph head is applied to the contact wire model, and the point of contact along the OCL is determined by the virtual distance travelled by the pantograph.

Recently, the authors have further developed the HS system by making use of the dynamically substructured system (DSS) approach of [2]. This new DSS-based HS system was shown to be more stable and accurate than the conventional inverse transfer function-based HS system of [3].

In the earlier HS work, the OCL was modelled as a single degree-of-freedom (SDOF) system and this is referred to as SDOF-HS in this present study. Since the wave propagation along the wires cannot be represented using the SDOF catenary model, the model does not possess sufficient complexity to represent the dynamic interaction.

In this present study, the OCL is now modelled as a multi degree-of-freedom (MDOF) system, in order to cater for the varying application point of the contact force with the contact wire model, due to the pantograph travel. Furthermore, a stable and accurate HS system is proposed by application of the DSS method to the MDOF catenary model. This particular HS system is referred to as MDOF-HS in this study and is the first known application of DSS to such a moving interaction force problem. This present work has resulted from a joint collaboration between RTRI and the Department of Mechanical Engineering, University of Bristol.

The purpose of this study is therefore as follows:

- (1) In order to deal with the moving interaction force problem, the OCL is modelled as a MDOF system. Furthermore, the degree-of-freedom of the model is

reduced, based on a modal analysis technique, in order to carry out the required real-time simulation of the dynamic interaction between the pantograph and the OCL model. The developed model is then applied to the HS system.

- (2) Comparative HS tests, based on (i) the benchmark inverse transfer function method and (ii) the MDOF-HS method, are carried out as pure numerical simulations. Then, the stability and accuracy resulting from the use of (ii) are compared with those of (i).
- (3) SDOF-HS and MDOF-HS testing are carried out under the assumption that the pantograph travels at 300 km/h. In order to show that the MDOF-HS better represents the dynamic interaction over a frequency range up to 20 Hz, (corresponding to the OCL hanger-passing frequency), each HS testing result is compared with the pure simulation of the pantograph-catenary system.

2. Hybrid simulation system

2.1 Overview of the hybrid simulation

A schematic diagram of the HS system is shown in Fig. 1. The HS consists of the real pantograph, the real-time simulator and the pantograph excitation equipment. A servo-hydraulic actuator is used for the excitation equipment, which includes a feedback controller to generate a displacement that corresponds with the input command signal to the controller.

A dynamic model of the OCL is implemented via the real-time simulator, thus the motion of the OCL model is calculated in real-time. The resulting contact force between the exciter and the pantograph head is then applied to the OCL model. The point of application of the contact force along the contact wire is based on the pantograph travel and the excitation equipment is, in turn, driven by the contact wire displacement at this specific point.

There are various methods for generating the command signal. In a conventional approach [2], the inverse of the transfer function between input signal and output displacement of the actuator is multiplied by the calculated displacement of the contact wire, in order to obtain the command signal. Since it is difficult to confirm the stability of the HS system using the inverse transfer function method, due to the typically ill-conditioned inverse transfer function, the command signal is generated using the DSS method.

Fig. 1 Hybrid simulation of the pantograph-catenary system

2.2 The OCL model

2.2.1 Overview of the OCL model

In [2], the OCL is modelled as a SDOF lumped mass system that has a time-varying stiffness element, in order to express the stiffness variation of the OCL due to the travelling pantograph. Here, stiffness of the OCL is defined as the force required to lift the contact wire with a stationary pantograph, divided by the vertical deflection at the point of contact.

The OCL model that is used in this study is an MDOF system based on discrete masses representing the catenary and contact wires, as shown in Fig. 2. Each adjacent mass is connected with spring and dashpot elements, to represent the stiffness of the tensioned wires and the damping within them. In order to model the droppers, the contact and catenary wires are connected by spring and dashpot elements at the dropper positions. Both ends of the wires are rigidly fixed. Specifications of the OCL model that is used in this study are shown in Table 1; additionally, the span length and dropper interval are set as 50 m and 5 m, respectively.

Since the effect of the dynamic motion of the catenary wire is relatively small, the discrete masses of the catenary wire are set with 2.5 m intervals. On the other hand, the motion of the contact wire has to be calculated with relatively high accuracy. Therefore, the mass interval of the contact wire model is smaller than that of the catenary wire.

Fig. 2 The OCL model based on the MDOF system

Table 1 Specifications of the OCL model

Catenary wire	Tensile force	19,600 N
	Mass per unit length	1.375 kg/m
Contact wire	Tensile force	19,600 N
	Mass per unit length	0.935 kg/m

2.2.2 Reduction of the OCL model using modal analysis

In general, the number of discrete masses in the contact wire model is too large to simulate its motion in real-time. Therefore, the degrees-of-freedom of the contact wire model are reduced via the modal analysis technique, in order to carry out the real-time simulation in this study.

The equation of motion of the contact wire model is expressed as follows in a physical coordinate system:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \quad (1)$$

where, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{x}$ is the displacement vector and $\mathbf{f}$ is the external force vector.

Now, the displacement of (1) is approximated using its first r eigenvectors, where $r < n$ and n is the degree-of-freedom of the model. The relationship between displacement $\mathbf{x}$ in the physical coordinates and displacement ξ_r in the modal coordinates is determined by the matrix Φ_r , whose columns are the first r eigenvectors of (1), thus:

$$\mathbf{x} \cong \Phi_r \xi_r \quad (2)$$

By substituting (2) into (1), and pre-multiplying both sides of the resulting equation by Φ_r^T , the following is obtained:

$$\mathbf{M}_r \ddot{\xi}_r + \mathbf{C}_r \dot{\xi}_r + \mathbf{K}_r \xi_r = \Phi_r^T \mathbf{f} \quad (3)$$

where, $\mathbf{M}_r$ is the modal mass matrix, $\mathbf{C}_r$ is the modal damping matrix and $\mathbf{K}_r$ is the modal stiffness matrix.

Equation (3) describes the motion of the reduced OCL model from n to r . Hence, in the MDOF-HS system, motion of the catenary wire is calculated using (1) (based on physical coordinates) and motion of the contact wire is calculated using (3), (based on modal coordinates).

2.3 Hybrid simulation methodology

2.3.1 Inverse transfer function method

In this method, the dynamics of the pantograph excitation equipment is compensated using the inverse of the transfer function of the excitation equipment. Therefore, the excitation equipment can generate an accurate displacement of the contact wire model, *only if* the associated model is a precise representation of the physical system and also invertible. The transfer function of the excitation equipment, $G_A(s)$, can be expressed by assuming a first-order model with pure delay:

$$G_A(s) = \frac{b}{s+a} e^{-\tau s} \quad (4)$$

where a is the inverse time constant, $b = a K_{DC}$, K_{DC} is the static gain and τ is the pure delay time. In order to compensate for the first-order term in (4), a compensator of the following form is used:

$$G_C(s) = \frac{s+a}{\varepsilon s+b} \quad (5)$$

where $\varepsilon \approx 0$. If ε is set to the ideal value of zero, $G_C(s)$ is not a proper transfer function and it cannot be exactly realised in the HS system, so in this study $\varepsilon = 10^{-3}$. The command signal is then obtained by multiplying $G_C(s)$ by the displacement of the contact wire at the point of contact with the pantograph.

2.3.2 The DSS method

DSS minimises the error between the calculated displacement of the contact wire model and the generated displacement by the excitation equipment. Since details of the methodology to apply the DSS to the SDOF catenary model is described in [2], only the methodology for the DSS/MDOF catenary model is given below.

A block diagram of DSS using MDOF-HS is shown in Fig. 3. In conventional DSS, it is assumed that the external force is always applied at the same location within the dynamic model. However, when the MDOF catenary model

Fig. 3 A block diagram of MDOF-HS using DSS

is used for DSS, the external force is applied to a varying location determined by the pantograph travel.

This study assumes that, at a given time, the pantograph instantaneously makes contact with the i -th mass of the contact wire. Then, by assuming that the pantograph is subsequently located between the i -th and $i+1$ th masses, the DSS methodology can be extended to a moving force problem such as pantograph-catenary systems.

The equation of motion of the catenary is expressed as:

$$\begin{bmatrix} \dot{\mathbf{x}}_{N1} \\ \dot{\mathbf{x}}_{N2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{N11} & \mathbf{I} \\ \mathbf{A}_{N21} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{N1} \\ \mathbf{x}_{N2} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_{N2} \end{bmatrix} \mathbf{f} \quad (6)$$

where terms in (6), are defined as:

$$\mathbf{x}_{N1} \equiv \mathbf{x} \quad (7)$$

$$\mathbf{x}_{N2} \equiv \mathbf{M}^{-1} \int_0^t (-\mathbf{K} \mathbf{x}_{N1} + \mathbf{f}) dt \quad (8)$$

$$\mathbf{A}_{N11} \equiv -\mathbf{M}^{-1} \mathbf{C} \quad (9)$$

$$\mathbf{A}_{N21} \equiv -\mathbf{M}^{-1} \mathbf{K} \quad (10)$$

$$\mathbf{B}_{N2} \equiv \mathbf{M}^{-1} \quad (11)$$

A state-space model of the pantograph excitation equipment (Σ_A) is expressed as:

$$\dot{x}_p = -a x_p + b u \quad (12)$$

Now, displacement of the contact wire model at the i -th mass, $x_{N1(i)}$, and displacement of the pantograph excitation equipment, x_p , are synchronised, by assuming the pantograph is instantaneously beneath the i -th mass of the contact wire model. The dynamics of the synchronisation error, $x_{e(i)}$, between $x_{N1(i)}$ and x_p is therefore determined from (6) and (12) as:

$$\begin{aligned} \dot{x}_{e(i)} &\equiv \dot{x}_{N1(i)} - \dot{x}_p \\ &= A_{N11(i,i)} x_{e(i)} + A_{N11(i,i-1)} x_{N1(i-1)} \\ &\quad + A_{N11(i,i+1)} x_{N1(i+1)} + x_{N2(i)} \\ &\quad + (A_{N11(i,i)} + a) x_p - b u \end{aligned} \quad (13)$$

where indices within the subscripted parentheses indicate that only the corresponding entries in the vectors and matrices are used in (13).

In order to minimise the synchronisation error, $x_{e(i)}$, between the displacement of the contact wire model at the i -th mass, $x_{N1(i)}$, and the displacement of the pantograph excitation equipment, x_p , (13) is now rearranged into a homogeneous equation in $x_{e(i)}$. The command signal to the excitation equipment, u , is defined as:

$$u = K_{e(i)}x_{e(i)} + K_{N1(i-1)}x_{N1(i-1)} + K_{N1(i+1)}x_{N1(i+1)} + K_{N2(i)}x_{N2(i)} + K_Px_P \quad (14)$$

where $K_{e(i)}$ is the feedback gain acting on $x_{e(i)}$. In addition, $K_{N1(i-1)}$, $K_{N1(i+1)}$, $K_{N2(i)}$, K_P are the feedback gains acting on $x_{N1(i-1)}$, $x_{N1(i+1)}$, $x_{N2(i)}$ and x_P , respectively.

By substituting (14) into (13), one obtains (15):

$$\begin{aligned} \dot{x}_{e(i)} = & \left(A_{N11(i,i)} - bK_{e(i)} \right) x_{e(i)} \\ & + \left(A_{N11(i,i-1)} - bK_{N1(i-1)} \right) x_{N1(i-1)} \\ & + \left(A_{N11(i,i+1)} - bK_{N1(i+1)} \right) x_{N1(i+1)} \\ & + \left(1 - bK_{N2(i)} \right) x_{N2(i)} + \left(A_{N11(i,i)} + a - bK_P \right) x_P \end{aligned} \quad (15)$$

In order to delete the 2nd-5th terms on the right-hand side of (15), the feedback gains $K_{N1(i-1)}$, $K_{N1(i+1)}$, $K_{N2(i)}$ and K_P are defined as:

$$K_{N1(i-1)} = \frac{A_{N11(i,i-1)}}{b} \quad (16)$$

$$K_{N1(i+1)} = \frac{A_{N11(i,i+1)}}{b} \quad (17)$$

$$K_{N2(i)} = \frac{1}{b} \quad (18)$$

$$K_P = \frac{A_{N11(i,i)} + a}{b} \quad (19)$$

By substituting (16) to (19) into (15), the required homogeneous dynamics of the synchronisation error, $x_{e(i)}$, are therefore obtained as:

$$\dot{x}_{e(i)} = \left(A_{N11(i,i)} - bK_{e(i)} \right) x_{e(i)} \quad (20)$$

The feedback gain, $K_{e(i)}$, is then determined from the desired closed-loop characteristics (e.g. eigenvalues) of (20).

The methodology described above is only applicable when the pantograph is instantaneously beneath a discrete mass of the contact wire model. If the pantograph exists between adjacent masses, the displacement of the contact wire model and the corresponding feedback gain can be obtained approximately, in order to apply the DSS method to the moving load problem. Figure 4 shows a situation where the pantograph exists between mass indices i and $i+1$ of the contact wire model. The variable q in Fig. 4 defines the distance between the pantograph and mass $i+1$ of the contact wire. Then the displacement x_j of the contact wire at the pantograph contact point is approximately obtained by interpolation between $x_{j(i)}$ and $x_{j(i+1)}$ as:

$$x_j = qx_{j(i)} + (1-q)x_{j(i+1)} \quad (21)$$

The feedback gain K_j ($j = N1(i-1), N1(i+1), P$) is also approxi-

Fig. 4 The case of the pantograph between adjacent masses

mately obtained by interpolation between $K_{j(i)}$ and $K_{j(i+1)}$ as:

$$K_j = qK_{j(i)} + (1-q)K_{j(i+1)} \quad (22)$$

3. Validation of the HS based on simulation studies

In this section, accuracy of the MDOF-HS and conventional HS are compared via simulation studies. Numerical simulation of the HS testing method is referred to as pseudo-HS in this paper. Accuracy is then assessed in relation to the response of a pure numerical simulation of the pantograph-catenary system. Since the results obtained from the pure simulation are considered as the reference data, the results are referred to as 'ref' in this section. The purpose of this simulation is to demonstrate the advantages of MDOF-HS over the inverse transfer function method, which is based on the MDOF catenary model. (The advantage of the SDOF-HS over the inverse transfer function method, which is based on the SDOF catenary model, has already been demonstrated in [2]).

Pseudo-HS is carried out using not only the catenary model, but also the pantograph and excitation equipment models. The dynamic model of the pantograph is shown in Fig. 5, with parameters of both this and the excitation equipment being summarized in Table 2.

The discrete masses of the contact wire model are selected with constant intervals of 0.5 m. In addition, the degrees-of-freedom of the MDOF contact wire model are reduced using natural modes up to 50 Hz. However, the degrees-of-freedom of the MDOF contact wire model in the pure simulation are not reduced. The gain $K_{e(i)}$ is designed so that the closed-loop settling time $t_s \approx 40$ ms, thus ensuring that the hydraulic actuator ($t_s \approx 20$ ms) is able to adequately respond within this time. Hence, $K_{e(i)} = 0.2$ in this present study. The travelling velocity of the pantograph is 300 km/h and numerical integration is carried out by the Euler method with a 0.2 ms interval.

For travel between 50 m and 150 m, the resulting contact force and the displacement of the pantograph head are shown in Fig. 6 and Fig. 7, respectively. Although it can be confirmed that the difference of the contact force between

Fig. 5 A dynamic model of the pantograph

Table 2 Parameters of the pantograph model and pantograph excitation equipment

Symbol	Value	Symbol	Value
m_1	3 kg	k	38,000 N/m
m_2	15 kg	P_0	114 N
c_1	100 Ns/m	a	266.4 s ⁻¹
c_2	100 Ns/m	b	1.332 m/(sV)

Fig. 6 Simulation results of contact force

Fig. 7 Simulation results of pantograph head displacement

Fig. 8 ISE comparison of pantograph head displacement error from the simulation of the inverse transfer function and MDOS-HS methods

the ref and HS results becomes small when using MDOF-HS, it is difficult to recognise the difference in terms of the displacement of the pantograph head in Fig. 7. In order to better display these errors, an integrated squared error (ISE) measure is used, being defined as [4]:

$$ISE = \left[\left(1/t_f \right) \int_0^{t_f} (y_1 - y_0)^2 dt \right]^{0.5} \quad (23)$$

where, y_0 and y_1 are displacement of the pantograph head that are obtained from ref and the pseudo-HS, respectively, and t_f is the length of time of the simulation. The calculated ISE is shown in Fig. 8. It is confirmed that the error of the displacement is reduced by ~ 0.5 mm using the MDOF-HS, when compared with the inverse transfer function method.

4. HS testing

The HS testing was carried out using the pantograph excitation equipment and the real pantograph at RTRI, as shown in Fig. 9. In order to demonstrate the advantage of the proposed MDOF-HS, the SDOF-HS and MDOF-HS

Fig. 9 HS testing using a real pantograph

Fig. 10 Comparison of contact force from pure simulation and HS testing in the time domain

Fig. 11 Comparison of the pantograph head displacement from pure simulation and HS testing in the time domain

methods were both implemented with a travelling velocity of 300 km/h. Since the number of spans of the catenary model could not be more than 4, due to restrictions within the real-time simulation, a 4-span catenary model was used in the MDOF-HS. The constant interval between adjacent discrete masses of contact wire was 0.833 m. The degrees-of-freedom of the contact wire model were reduced using natural modes up to 17.5 Hz, in order to realise a stable real-time simulation. A pure numerical simulation of the pantograph-catenary system was also carried out, to provide the reference data, ref. The pure simulation was executed using the Euler method with a 0.2 ms step length. In the pseudo-HS method, numerical integration was also executed using the Euler method, but now with a 1.0 ms step length.

A comparison of the generated contact forces from HS testing and the ref is shown in Fig. 10 and a comparison of the displacement of the pantograph head is also shown in Fig. 11.

Power spectrum density (PSD) curves of the contact force and displacement are shown in Fig. 12 and Fig.13, respectively. It is seen that the phenomena that are induced by the span-passing frequency (1.67 Hz) and the dropper-passing frequency (16.7 Hz) are exhibited with higher accuracy by

Fig. 12 Comparison between pure simulation and the HS testing in the frequency domain (contact force)

Fig. 13 Comparison between pure simulation and the HS testing in the frequency domain (displacement of the pantograph head)

using MDOF-HS, compared with the use of SDOF-HS.

5. Conclusions

A hybrid simulation (HS) system that considers the dynamic interaction between a pantograph and catenary was developed. In the HS system, the applied position of the contact force changes with time, due to pantograph travel. The main conclusions from this study are summarized as follows.

- (1) A novel hybrid simulation testing method for pantograph/catenary systems was developed, based upon the dynamically substructured system (DSS) framework of Stoten and Hyde [2].

- (2) The catenary was modelled as a multi degree-of-freedom (MDOF) system. In order to realise the real-time simulation and consideration of the dynamic interaction between the pantograph and the catenary, the degrees-of-freedom of the MDOF contact wire model were reduced via a modal analysis technique.
- (3) HS testing based on the inverse transfer function method and MDOF catenary model were respectively carried out via pure numerical simulations (called the pseudo-HS method). By comparing the results obtained, it was shown that the error between pure simulation and HS testing was reduced using the MDOF catenary model.
- (4) Single degree-of-freedom (SDOF) HS testing and MDOF HS testing were carried out using a real pantograph at the RTRI, with a travelling velocity of 300 km/h. By comparing the results of the pure numerical simulation of the pantograph-catenary system and each HS test, it was shown that dynamic phenomena (behaviour corresponding to the span-passing frequency and dropper-passing frequency) over a frequency range of up to 20 Hz, can be expressed with high accuracy using the MDOF-HS testing.

Our future work will focus on increasing the stability bounds of HS testing by optimising the feedback gain K_e within DSS.

References

- [1] Koyama, T., Usuda, T., Kawasaki, K., Nakamura, K. and Kawamura, T., "Methods for detecting pantograph defects using sensors installed on contact lines," *Quarterly Report of RTRI*, Vol. 57, pp.207-212, August 2016.
- [2] Stoten, D., P., and Hyde, R., A., "Adaptive control of dynamically substructured systems: the single-input, single-output case," *Proc. IMechE Part I: J. Systems and Control Engineering*, Vol. 220, pp.63-79, March 2006.
- [3] Yamashita, Y., Kobayashi, S., and Usuda, T., "Hybrid Simulation Method for Catenary/Pantograph System," *RTRI Report*, Vol. 31, No. 4, pp. 11-16, April 2017 (in Japanese).
- [4] Dorf, R. C. and Bishop, R. H., *Modern Control Systems (13th Edition)*, Pearson Education Ltd., England, UK, p. 346, 2017.

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