Growth Mechanism and Evolution Process of Rail Corrugation

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Rail corrugation is a phenomenon where roughness patterns of regular wavelengths are formed on the rail running surface by passing vehicles. In order to elucidate the mechanisms underlying corrugation, rail roughness growth mechanisms and evolution process were analyzed using theoretical and numerical methods. The resulting characteristics obtained in this manner were verified on the basis of rail roughness measured on domestic commercial lines. These results confirmed that corrugation growth factors consist of four anti-resonance phenomena, and that there are three stages in the corrugation evolution process: formation, growth and saturation. Finally, this paper proposes possible approaches for developing countermeasures.

Keywords: corrugation, track, vehicle, anti-resonance, mechanical impedance

1. Introduction

Rail corrugation, where roughness patterns with regular wavelengths are formed on the rail surface by passing vehicles (Fig. 1), causes vibration and noise. To date, although many investigations and studies have been conducted [1-4], the mechanisms underlying this phenomenon have not been fully elucidated.

This paper describes the growth mechanism of rail roughness and its wavelength determination mechanism using theoretical and numerical analyses from the viewpoint of dynamics. On the basis of rail roughness measured on domestic commercial lines, the resulting characteristics, obtained in this manner, were verified. Finally, this paper proposes possible approaches for developing countermeasures.

2. Theoretical analysis of growth mechanism

2.1 Theoretical analysis model

Figure 2 shows the theoretical analysis model [5]. The track system is modeled as an elastically supported beam, and the vehicle as a mass-spring system moving at speed v . The elastic support models of the track are shown in Fig. 3. ρ and EI are the line density and the bending rigidity of the rail respectively. A_R indicates the amplitude of the rail roughness, and Z_s is the mechanical impedance of the elastic support of the rail. f_a, f_b, f_A and f_B represent the contact force fluctuations between the rail and the wheelsets. The track is modeled on one rail and the vehicle is modeled as a 1/4 car body. The validity of expressing the actual track structure by the elastically supported beam model as shown in Fig.3 has been verified in another

Fig. 1 An example of rail corrugation

paper [6]. Table 1 shows the common values of the parameters used in chapters 2 and 3.

2.2 Mechanical impedance of track and vehicle

When the rail is excited by a force f at $x=0$, the wave equation of the rail is given by (1), where ω is the angular frequency of the excitation, i is an imaginary unit and $\delta(x)$ is Dirac's delta-function.

$$EI \frac{\partial^4 y}{\partial x^4} + \rho \frac{\partial^2 y}{\partial t^2} + i\omega Z_s y = f \delta(0) \quad (1)$$

If the roots of the characteristic equation above are $k_1 \sim k_4$ (k_1 and k_3 have a positive real part, and k_2 and k_4 have a negative real part), the rail displacement y_1 and y_2 are given by (2).

Fig. 2 Theoretical analysis model

Fig. 3 Elastic support models of track

Table 1 Common values of parameters

(a) Track				(b) Vehicle	
Parameter	Single	Double	Triple	Parameter	value
ρ (kg/m)	50	50	50	M_a, M_b (kg)	10^3
EI (Nm ²)	4×10^6	4×10^6	4×10^6	M_c (kg)	10^3
K_{S1} (N/m)	10^8	10^8	10^8	M_d (kg)	6×10^3
D_{S1} (Ns/m)	10^3	10^3	10^3	K_a, K_b (N/m)	10^6
M_{S1} (kg)		10^2	10^1	D_a, D_b (Ns/m)	0
K_{S2} (N/m)		10^7	10^8	K_c (N/m)	10^5
D_{S2} (Ns/m)		10^3	10^3	D_c (Ns/m)	0
M_{S2} (kg)			10^2	L (m)	2.1
K_{S3} (N/m)			10^7	v (m/s)	15.12
D_{S3} (Ns/m)			10^3		

※ The parameters of the elastic support of the track are the values per unit length.

$$\left. \begin{aligned} y_1 &= (A_1 e^{k_1 x} + A_2 e^{k_2 x}) e^{i\omega t} \quad (x < 0) \\ y_2 &= (A_3 e^{k_3 x} + A_4 e^{k_4 x}) e^{i\omega t} \quad (x \geq 0) \end{aligned} \right\} \quad (2)$$

$A_1 \sim A_4$ are coefficients obtained from the boundary condition at $x=0$. From this, the mechanical impedance (driving-point impedance) of the track Z_R can be obtained as in (3).

$$Z_R = \frac{f}{dy_2 / dt|_{x=0}} \quad (3)$$

Figure 4 shows an example of the mechanical impedance Z_R by using values in Table 1. These maxima are due to the anti-resonance phenomenon of the elastic support of the track.

When the rail is excited at two points $x=0$ and $x=L$ with a force f_A and f_B , the rail displacement $y_1 \sim y_3$ can be expressed by (4) by shifting (2) by the distance L and overlapping.

$$\left. \begin{aligned} y_1 &= \{A_{1A} e^{k_1 x} + A_{1B} e^{k_1(x-L)} + A_{3A} e^{k_3 x} + A_{3B} e^{k_3(x-L)}\} e^{i\omega t} \quad (x < 0) \\ y_2 &= \{A_{2A} e^{k_2 x} + A_{2B} e^{k_2(x-L)} + A_{4A} e^{k_4 x} + A_{4B} e^{k_4(x-L)}\} e^{i\omega t} \quad (0 \leq x \leq L) \\ y_3 &= \{A_{2A} e^{k_2 x} + A_{2B} e^{k_2(x-L)} + A_{4A} e^{k_4 x} + A_{4B} e^{k_4(x-L)}\} e^{i\omega t} \quad (L < x) \end{aligned} \right\} \quad (4)$$

If the exciting force f_B has the same amplitude as f_A and the phase is advanced by θ , and the coefficients $A_1 \sim A_4$ in (2) are applied to the corresponding $A_{1A} \sim A_{4B}$, then the mechanical impedances Z_A and Z_B at $x=0$ and $x=L$ can be obtained in the same way as (3).

Figure 5 shows a calculated example of Z_A for the double spring type of the elastic support. The multiple maxima (excluding the maximum in Fig. 4) are due to the interference of the wave propagating from the two excitation points.

The motion equations of the vehicle are expressed in (5), where $y_a \sim d$, $M_a \sim d$, $K_a \sim c$ and $D_a \sim c$ are displacement, mass, spring constant and damping coefficient of each part of the vehicle.

$$\left. \begin{aligned} M_a \frac{d^2 y_a}{dt^2} &= f_a - \tilde{K}_a (y_a - y_c), \quad M_b \frac{d^2 y_b}{dt^2} = f_b - \tilde{K}_b (y_b - y_c) \\ M_c \frac{d^2 y_c}{dt^2} &= \tilde{K}_a (y_a - y_c) + \tilde{K}_b (y_b - y_c) + \tilde{K}_c (y_d - y_c) \\ M_d \frac{d^2 y_d}{dt^2} &= -\tilde{K}_c (y_d - y_c) \\ \tilde{K}_a &= K_a + i\omega D_a, \quad \tilde{K}_b = K_b + i\omega D_b, \quad \tilde{K}_c = K_c + i\omega D_c \end{aligned} \right\} \quad (5)$$

If the displacement y_b of the wheelset is ahead of y_a by the phase θ , then the mechanical impedances Z_a and Z_b can be calculated in the same way as (3).

Figure 6 shows a calculated example of Z_a . The maximum is due to the anti-resonance phenomenon of the vibration mode in which the wheelset is the node.

Fig. 4 Mechanical impedance of track (one-point excitation)

Fig. 5 Mechanical impedance of track (two-point excitation; double spring type)

Fig. 6 Mechanical impedance of vehicle

2.3 Amplification coefficient of roughness amplitude

Combining the motion equations of the track and the vehicle shown in the previous section with (6), the contact force fluctuations between the rail and the wheelsets f_A , f_B are expressed as in (7) with coefficients H_A and H_B .

$$\left. \begin{aligned} y_a &= A_R e^{i\omega t} + y_A, \quad y_b = A_R e^{i\omega t} e^{i\theta} + y_B \\ f_a &= -f_A, \quad f_b = -f_B \end{aligned} \right\} \quad (6)$$

$$f_A = H_A A_R e^{i\omega t}, \quad f_B = H_B A_R e^{i\omega t} \quad (7)$$

To judge the amplification of the roughness amplitude, the average contact force f_{ave} is calculated in (8) by delaying f_B by the phase θ and adjusting it to the position where f_A acts. Note that H is a transfer function of the contact force fluctuation to the unit roughness amplitude.

$$f_{ave} = \frac{f_A + f_B e^{-i\theta}}{2} = \frac{1}{2} (H_A + H_B e^{-i\theta}) A_R e^{i\omega t} = H \cdot A_R e^{i\omega t} \quad (8)$$

Assuming that the rail wears in proportion to the contact force based on Archard's wear equation, and the amount of the wear per unit contact force is C_W , the roughness amplitude after running the vehicle becomes κ times that before running. κ is called "amplification coefficient of roughness amplitude" and is expressed by (9).

$$\kappa = \left| \frac{A_R e^{i\omega t} - C_W \cdot (-f_{ave})}{A_R e^{i\omega t}} \right| = |1 + C_W H| \quad (9)$$

If $\kappa > 1$, the roughness amplitude of that wavelength increases as the vehicles run, and if $\kappa < 1$, it decreases. That is, in order to grow as the corrugation, it is necessary that, in addition to $\kappa > 1$, κ has a maximum value. Figure 7 shows an example of the amplification coefficient of roughness amplitude in the case of the double spring type of the track and $C_W = 10^{-15}$ m/N.

2.4 Four anti-resonance phenomena as growth factors

Multiple maxima of $\kappa > 1$ appears in Fig. 7, and the following four anti-resonance phenomena can be identified as growth factors of the corrugation.

(1) Interference of rail wave motion with wheelbase of vehicle

The maximum spatial frequency ① in Fig. 7 corresponds to about 560 Hz, which is the converted value using the vehicle speed. Since this frequency corresponds to the frequency of maximum in Fig. 5 ($\theta = \pi$), it is due to the interference of rail waves between the wheelbases of the vehicle. Also, as shown enlarged in the right side of the figure, local maxima appear at regular intervals, and these intervals correspond to the reciprocal of the wheelbase distance.

(2) Anti-resonance phenomenon in elastic support in track system

The maximum spatial frequency ② in Fig. 7 corresponds to about 165 Hz. Since this frequency corresponds to the frequency of maximum in Fig. 4, it is due to the anti-resonance of the track system.

(3) Anti-resonance phenomenon in total system of track and vehicle

The maximum spatial frequency ③ in Fig. 7 corresponds to about 20 Hz. At this frequency, the track and the vehicle are equal in the absolute values of the mechanical impedances, and the series mechanical impedance of the track and the vehicle reach a maximum [5]. This maximum is considered to be an anti-resonance phenomenon where a large force is required to achieve forcible displacement between the rail and the wheelsets.

(4) Anti-resonance phenomenon in the vehicle system

The maximum spatial frequency of ④ in Fig. 7 corresponds to about 7 Hz. This frequency corresponds to the anti-resonance frequency of the vehicle shown in Fig. 6.

Fig. 7 Amplification coefficient of roughness amplitude (double spring type)

3. Numerical analysis of the evolution process

3.1 Dynamic simulation model

Figure 8 shows a dynamic simulation model of the track and the vehicle [7]. The rail is divided into mass points and the non-reflective elements are installed at

both ends of the rail to suppress the wave reflection. Following repeated passages of a vehicle, the rail wears in proportion to the wheel load according to Archard's wear equation. In order to reduce the calculation time, C_W is set to 10^{-11} m/N, which is 10^4 times the condition shown in Fig. 7.

3.2 Roughness formation of a specific wavelength

Figure 9 shows the transition of the rail roughness when a minute and irregular wheel load fluctuation (white noise of maximum amplitude -0.1 to 0.1 N) is applied to the state without initial roughness. The roughness of a specific wavelength having the maximum of the amplification coefficient of roughness amplitude shown in Fig. 7 is selectively formed.

3.3 Suppression of roughness growth on short wavelength

In the above simulation, it is assumed that the wheel is in contact with the rail at one point, but in this section the pressure distribution on the contact surface is set as an ellipse. Figure 10 shows a comparison of the power spectral densities of the rail roughness after 30 runs where the contact lengths (the length of the contact ellipse in the running direction) are 0.01 m and 0.02 m compared with the case with only one-point of contact (Fig. 9). It was found that the longer the contact length, the more the growth of roughness on the high frequency (short wavelength) side is suppressed.

Fig. 8 Dynamic simulation model

(a) Rail roughness waveform

(b) Power spectrum density

Fig. 9 Transition of rail roughness at initial formation stage

Fig. 10 Comparison of rail roughness PSD by contact length

3.4 Saturation of roughness amplitude

In this section, the wheels are allowed to lose contact with the rail when the wheel load becomes negative. The initial roughness is set as a sine wave with the wavelength of 0.092 m and the amplitude of 10^{-5} m before the running. The contact condition between the rail and the wheel is one point.

Figure 11 shows the transition of the rail roughness and the contact loss ratio. After the wheels lose contact with the rail, growth of the roughness is suppressed and there is a tendency for saturation. In addition, as shown in Fig. 11(a), the roughness waveform collapses from the sine wave, and higher-order component of the roughness appears.

Fig. 11 Transition of rail roughness at saturation stage

4. Verification based on field data

4.1 Measurement method of rail roughness

The rail surface roughness on commercial lines was measured using a rail surface roughness measurement trolley [8] shown in Fig. 12. On the trolley, laser gap sensors are arranged at unequal intervals to constitute an asymmetrical chord. It is possible to measure the rail surface roughness continuously and accurately independently of travel speed of the trolley.

Fig. 12 Rail surface roughness measurement trolley

4.2 Case of interference of rail wave motion between wheelbase of vehicle

Figure 13 shows a measured example of rail corrugation [5]. STFT means a short time Fourier transform, and is modified according to the diminishing multiplication rule of 10^{-3} . Around the spatial frequency 20~25 (1/m), relatively remarkable roughness is seen at regular spatial frequency intervals.

Figure 14 shows the amplification coefficient of roughness amplitude under the condition of this line. This coefficient has a maximum around the spatial frequency of 20~25 (1/m), and the interval of the maximum spatial frequency corresponds to the reciprocal of the wheelbase of the representative vehicle running in this section. Therefore, the cause of this corrugation can be identified as the interference of the rail wave motion with the wheelbase of the vehicle.

(a) Photo of rail corrugation (b) STFT of rail roughness

Fig. 13 Measured result of rail roughness

Fig. 14 Amplification coefficient of roughness amplitude in the field condition of Fig. 13

4.3 Case of anti-resonance phenomenon in elastic support in track system

Figure 15 shows another example of rail corrugation [5]. Corrugation occurs when the train speed is in the 20~60 km/h range, and spatial frequency is 10~30 (1/m): the spatial frequency tends to be inversely proportional to train speed.

Figure 16 shows the mechanical impedance of the elastic support of the track under the condition of this section. It can be seen that the anti-resonance frequency of the track 140 Hz

(a) Photo of rail corrugation (b) Relationship between train

Fig. 15 Measured result of rail roughness

Fig. 16 Mechanical impedance of the elastic support in the field condition of Fig. 15

matches the relationship between the train speed and the spatial frequency of the corrugation. Therefore, the cause of this corrugation can be identified as the anti-resonance phenomenon in the elastic support of the track system.

4.4 Case of anti-resonance phenomenon in total system of track and vehicle

Figure 17 shows another example of rail corrugation in the case of a single spring rack. The spatial frequency of the corrugation is 5~8 (1/m), which tends to be inversely proportional to train speed. Since higher-order components of the roughness are seen in this section, this corrugation is considered to be in the saturation stage.

Figure 18 shows the amplification coefficient of roughness amplitude under the conditions of this section. This coefficient has a maximum in the spatial frequency of 5~10 (1/m), and it matches the situation at the field. Therefore, the cause of this corrugation can be deemed to be anti-resonance phenomenon in the total system of the track and the vehicle.

(a) Photo of rail corrugation (b) Relationship between train speed and STFT of rail roughness

Fig. 17 Measured result of rail roughness

Fig. 18 Amplification coefficient of roughness amplitude in the field condition of Fig. 17

4.5 Classification and characteristics of corrugation by growth factors

Table 2 shows the classification and characteristics of corrugation based on verifications in 15 cases on 8 domestic commercial lines including the above cases. Wavelengths are generally short in case of the interference of the rail wave motion with the wheelbase of the vehicle, and long in case of anti-resonance phenomena in the total system of the track and the vehicle. However, growth conditions due to anti-resonance phenomena in the vehicle are limited [5], so no cases were identified on these lines.

Table 2 Classification and characteristics of corrugation (15 cases of 8 lines)

Growth factor	Spatial frequency (1/m)	Wave length (mm)	Train speed (km/h)	Frequency (Hz)
①Interference of rail wave motion between wheelbase of vehicle	12-33	30-83	55-119	230-1000
②Anti-resonance phenomenon in elastic support in track system	3.9-26	38-260	22-120	110-330
③Anti-resonance phenomenon in total system of track and vehicle	1.0-12	83-1000	37-115	19-130

5. Evolution process and approaches for developing rail corrugation countermeasures

Figure 19 is a schematic representation of the rail corrugation evolution process [7].

- (1) Formation stage: This is the stage where a minute irregular roughness is formed due to fluctuation factors such as initial rail irregularities or wheel load fluctuations.
- (2) Growth stage: This is the stage where the roughness of specific wavelength selectively grows and appears as the corrugation.
- (3) Saturation stage: When the roughness amplitude increases and the wheel loses contact with the rail, the growth is suppressed, and the roughness amplitude saturates.

Fig. 19 Image of evolution process of rail corrugation

Fig. 20 Ways to approach countermeasures

Figure 20 shows possible approaches for developing countermeasures. It is effective to suppress fluctuation factors, suppress progress of wear, and restrain anti-resonance phenomena during the formation or growth stage, and to remove roughness by rail grinding or replacement at the saturation stage.

6. Conclusion

In this paper, rail corrugation growth mechanisms and the wavelength determination mechanism were analyzed from the viewpoint of dynamics, and these characteristics were verified in the light of field data. Based on these results, insight was gained into the evolution process of rail corrugation, and approaches are proposed for developing countermeasures. The results are as follows:

- (1) The corrugation wavelength of the corrugation corresponds to the wavelength at which the amplification coefficient of roughness amplitude reaches a maximum exceeding 1.0.
- (2) The growth factors of the corrugation are ①interference of the rail wave motion with the wheelbase of the vehicle, ②anti-resonance phenomena in the elastic support in the track system and ③anti-resonance phenomena in the total system of the track and the vehicle.
- (3) Verification were made to ensure that the wavelengths of the corrugation measured on the commercial lines agreed well with the theoretical analyses.

- (4) There are three stages in the evolution process of corrugation: formation, growth and saturation.
- (5) Possible approaches to develop countermeasures include the suppression of fluctuation factors, suppression of wear progress, restraint of anti-resonance phenomena and removal of rail roughness.

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