Growth Mechanism and Evolution Process of Rail Corrugation

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Rail corrugation is a phenomenon where roughness patterns with regular wavelengths are formed on the rail running surface by passing vehicles. In order to elucidate the mechanisms underlying corrugation, rail roughness growth mechanisms and evolution process were analyzed using theoretical and numerical methods. The resulting characteristics obtained in this manner were verified on the basis of rail roughness measured on domestic commercial lines. These results confirmed that corrugation growth factors consist of four anti-resonance phenomena, and that there are three stages in the corrugation evolution process: formation, growth and saturation. Finally, this paper proposes possible approaches for developing countermeasures.

Keywords: corrugation, track, vehicle, anti-resonance, mechanical impedance

1. Introduction

Rail corrugation, where roughness patterns with regular wavelengths are formed on the rail surface by passing vehicles (Fig. 1), causes vibration and noise. To date, although many investigations and studies have been conducted [1-4], the mechanisms underlying this phenomenon have not been fully elucidated.

This paper describes the growth mechanism of rail roughness and its wavelength determination mechanism using theoretical and numerical analyses from the viewpoint of dynamics. On the basis of rail roughness measured on domestic commercial lines, the resulting characteristics, obtained in this manner, were verified. Finally, this paper proposes possible approaches for developing countermeasures.

2. Theoretical analysis of growth mechanism

2.1 Theoretical analysis model

Figure 2 shows the theoretical analysis model [5]. The track system is modeled as an elastically supported beam, and the vehicle as a mass-spring system moving at speed \( v \). The elastic support models of the track are shown in Fig. 3. \( \rho \) and \( EI \) are the line density and the bending rigidity of the rail respectively. \( A_y \) indicates the amplitude of the rail roughness, and \( Z_s \) is the mechanical impedance of the elastic support of the rail. \( f_a, f_b, f_4 \) and \( f_0 \) represent the contact force fluctuations between the rail and the wheelsets. The track is modeled on one rail and the vehicle is modeled as a 1/4 car body. The validity of expressing the actual track structure by the elastically supported beam model as shown in Fig.3 has been verified in another paper [6]. Table 1 shows the common values of the parameters used in chapters 2 and 3.

2.2 Mechanical impedance of track and vehicle

When the rail is excited by a force \( f \) at \( x=0 \), the wave equation of the rail is given by (1), where \( \omega \) is the angular frequency of the excitation, \( i \) is an imaginary unit and \( \delta(x) \) is Dirac’s delta-function.

\[
EI \frac{\partial^4 y}{\partial x^4} + \rho \frac{\partial^2 y}{\partial t^2} + i\omega Z_s y = f \delta(0)
\] (1)

If the roots of the characteristic equation above are \( k_1 \sim k_4 \) (\( k_1 \) and \( k_3 \) have a positive real part, and \( k_2 \) and \( k_4 \) have a negative real part), the rail displacement \( y_1 \) and \( y_2 \) are given by (2).
Table 1 Common values of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single</th>
<th>Double</th>
<th>Triple</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ (kg/m)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>E (N/m²)</td>
<td>4 × 10³</td>
<td>4 × 10³</td>
<td>4 × 10³</td>
</tr>
<tr>
<td>η₀ (N/m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>D₀ (N/m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>M₀ (kg)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>K₀ (N/m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>M (kg)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>K (N/m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

= The parameters of the elastic support of the track are the values per unit length.

\[
y_1 = (A_1 e^{ikx} + A_2 e^{ik(x-L)}) e^{i\omega t} \quad (x < 0)
\]
\[
y_2 = (A_1 e^{ikx} + A_2 e^{ik(x-L)}) e^{i\omega t} \quad (x ≥ 0)
\]
\[A_1, A_2 \text{ are coefficients obtained from the boundary condition at } x=0.\]

From this, the mechanical impedance (driving-point impedance) of the track Z₀ can be obtained as in (3).

\[
Z_0 = \left. \frac{f}{d y} \right|_{y=0}
\]

Figure 4 shows an example of the mechanical impedance \(Z_t\) by using values in Table 1. These maxima are due to the anti-resonance phenomenon of the elastic support of the track.

When the rail is excited at two points \(x=0\) and \(x=L\) with a force \(f_a\) and \(f_b\), the rail displacement \(y_1 - y_2\) can be expressed by (4) by shifting (2) by the distance \(L\) and overlapping.

\[
y_1 = A_1 e^{ikx} + A_2 e^{ik(x-L)} + A_3 e^{ikx} + A_4 e^{ik(x-L)} e^{i\omega t} \quad (x < 0)
\]
\[
y_2 = (A_1 e^{ikx} + A_2 e^{ik(x-L)} + A_3 e^{ikx} + A_4 e^{ik(x-L)} e^{i\omega t}) e^{i\omega t} \quad (0 ≤ x ≤ L)
\]
\[
y_3 = (A_1 e^{ikx} + A_2 e^{ik(x-L)} + A_3 e^{ikx} + A_4 e^{ik(x-L)} e^{i\omega t}) e^{i\omega t} \quad (L < x)
\]

If the exciting force \(f_a\) has the same amplitude as \(f_b\) and the phase is advanced by \(\theta\), and the coefficients \(A_1 - A_4\) in (2) are applied to the corresponding \(A_{1a} - A_{4b}\), then the mechanical impedances \(Z_t\) and \(Z_0\) at \(x=0\) and \(x=L\) can be obtained in the same way as (3).

Figure 5 shows a calculated example of \(Z_i\) for the double spring type of the elastic support. The multiple maxima (excluding the maximum in Fig. 4) are due to the interference of the wave propagating from the two excitation points.

The motion equations of the vehicle are expressed in (5), where \(y_a, y_b, M_{w-a}, K_a\), and \(D_{w-a}\) are displacement, mass, spring constant and damping coefficient of each part of the vehicle.

\[
M \frac{d^2 y_a}{dt^2} = f_a - \bar{K}_s (y_a - y_s) \quad M \frac{d^2 y_b}{dt^2} = f_b - \bar{K}_s (y_s - y_b)
\]
\[
M \frac{d^2 y_s}{dt^2} = \bar{K}_s (y_a - y_s) + \bar{K}_s (y_s - y_b) + \bar{K}_s (y_s - y_s)
\]
\[
M \frac{d^2 y_c}{dt^2} = \bar{K}_s (y_j - y_s) \quad \bar{K}_s = K_s + i\omega D_s, \quad \bar{K}_c = K_c + i\omega D_c
\]

If the displacement \(y_s\) of the wheelset is ahead of \(y_c\) by the phase \(\theta\), then the mechanical impedances \(Z_t\) and \(Z_0\) can be calculated in the same way as (3).

Figure 6 shows a calculated example of \(Z_c\). The maximum is due to the anti-resonance phenomenon of the vibration mode in which the wheelset is the node.

2.3 Amplification coefficient of roughness amplitude

Combining the motion equations of the track and the vehicle shown in the previous section with (6), the contact force fluctuations between the rail and the wheelsets \(f_a, f_b\) are expressed as in (7) with coefficients \(H_1\) and \(H_2\).

\[
y_s = A_1 e^{i\omega t} + y_a, \quad y_b = A_2 e^{i\omega t} + y_a + \bar{K}_s (y_s - y_b)
\]
\[
f_a = -f_a, \quad f_b = -f_b
\]
\[
f_a = H_1 A_1 e^{i\omega t}, \quad f_b = H_2 A_2 e^{i\omega t}
\]

To judge the amplification of the roughness amplitude, the average contact force \(f_{am}\) is calculated in (8) by delaying \(f_a\) by the phase \(\theta\) and adjusting it to the position where \(f_a\) acts. Note that \(H\) is a transfer function of the contact force fluctuation to the unit roughness amplitude.

\[
f_{am} = \frac{f_a + f_b e^{i\theta}}{2} = \frac{1}{2} (H_1 + H_2 e^{i\theta}) A_1 e^{i\omega t} = H \cdot A_1 e^{i\omega t}
\]

Assuming that the rail wears in proportion to the contact force based on Archard’s wear equation, and the amount of the wear per unit contact force is \(C_s\), the roughness amplitude after running the vehicle becomes \(\kappa\) times that before running. \(\kappa\) is called “amplification coefficient of roughness amplitude” and is expressed by (9).
\[
\kappa = \frac{A_y e^{i\omega t} - C_y \cdot (-f_y e^{i\omega t})}{A_y e^{i\omega t}} = \| + C_y H \]

If \( \kappa > 1 \), the roughness amplitude of that wavelength increases as the vehicles run, and if \( \kappa < 1 \), it decreases. That is, in order to grow as the corrugation, it is necessary that, in addition to \( \kappa > 1 \), \( \kappa \) has a maximum value. Figure 7 shows an example of the amplification coefficient of roughness amplitude in the case of the double spring type of the track and \( C_y = 10^{-12} \text{ m/N} \).

2.4 Four anti-resonance phenomena as growth factors

Multiple maxima of \( \kappa > 1 \) appears in Fig. 7, and the following four anti-resonance phenomena can be identified as growth factors of the corrugation.

(1) Interference of rail wave motion with wheelbase of vehicle

The maximum spatial frequency \( \frac{2\pi}{\lambda} \) in Fig. 7 corresponds to about 560 Hz, which is the converted value using the vehicle speed. Since this frequency corresponds to the frequency of maximum in Fig. 5 (\( \theta = \pi \)), it is due to the interference of rail waves between the wheelbases of the vehicle. Also, as shown enlarged in the right side of the figure, local maxima appear at regular intervals, and these intervals correspond to the reciprocal of the wheelbase distance.

(2) Anti-resonance phenomenon in elastic support in track system

The maximum spatial frequency \( \frac{2\pi}{\lambda} \) in Fig. 7 corresponds to about 165 Hz. Since this frequency corresponds to the frequency of maximum in Fig. 4, it is due to the anti-resonance of the track system.

(3) Anti-resonance phenomenon in total system of track and vehicle

The maximum spatial frequency \( \frac{2\pi}{\lambda} \) in Fig. 7 corresponds to about 20 Hz. At this frequency, the track and the vehicle are equal in the absolute values of the mechanical impedances, and the series mechanical impedance of the track and the vehicle reach a maximum [5]. This maximum is considered to be an anti-resonance phenomenon where a large force is required to achieve forcible displacement between the rail and the wheelsets.

(4) Anti-resonance phenomenon in the vehicle system

The maximum spatial frequency \( \frac{2\pi}{\lambda} \) in Fig. 7 corresponds to about 7 Hz. This frequency corresponds to the anti-resonance frequency of the vehicle shown in Fig. 6.

3. Numerical analysis of the evolution process

3.1 Dynamic simulation model

Figure 8 shows a dynamic simulation model of the track and the vehicle [7]. The rail is divided into mass points and the non-reflective elements are installed at both ends of the rail to suppress the wave reflection. Following repeated passages of a vehicle, the rail wears in proportion to the wheel load according to Archard’s wear equation. In order to reduce the calculation time, \( C_y \) is set to \( 10^{-11} \text{ m/N} \), which is \( 10^3 \) times the condition shown in Fig. 7.

3.2 Roughness formation of a specific wavelength

Figure 9 shows the transition of the rail roughness when a minute and irregular wheel load fluctuation (white noise of maximum amplitude -0.1 to 0.1 N) is applied to the state without initial roughness. The roughness of a specific wavelength having the maximum of the amplification coefficient of roughness amplitude shown in Fig. 7 is selectively formed.

3.3 Suppression of roughness growth on short wavelength

In the above simulation, it is assumed that the wheel is in contact with the rail at one point, but in this section the pressure distribution on the contact surface is set as an ellipse. Figure 10 shows a comparison of the power spectral densities of the rail roughness after 30 runs where the contact lengths (the length of the contact ellipse in the running direction) are 0.01 m and 0.02 m compared with the case with only one-point of contact (Fig. 9). It was found that the longer the contact length, the more the growth of roughness on the high frequency (short wavelength) side is suppressed.
3.4 Saturation of roughness amplitude

In this section, the wheels are allowed to lose contact with the rail when the wheel load becomes negative. The initial roughness is set as a sine wave with the wavelength of 0.092 m and the amplitude of $10^{-5}$ m before running. The contact condition between the rail and the wheel is one point.

Figure 11 shows the transition of the rail roughness and the contact loss ratio. After the wheels lose contact with the rail, growth of the roughness is suppressed and there is a tendency for saturation. In addition, as shown in Fig. 11(a), the roughness waveform collapses from the sine wave, and higher-order component of the roughness appears.

4. Verification based on field data

4.1 Measurement method of rail roughness

The rail surface roughness on commercial lines was measured using a rail surface roughness measurement trolley [8] shown in Fig. 12. On the trolley, laser gap sensors are arranged at unequal intervals to constitute an asymmetrical chord. It is possible to measure the rail surface roughness continuously and accurately independently of travel speed of the trolley.

4.2 Case of interference of rail wave motion between wheelbase of vehicle

Figure 13 shows a measured example of rail corrugation [5]. STFT means a short time Fourier transform, and is modified according to the diminishing multiplication rule of $10^{-5}$. Around the spatial frequency $20 \sim 25$ (1/m), relatively remarkable roughness is seen at regular spatial frequency intervals.

Figure 14 shows the amplification coefficient of roughness amplitude under the condition of this line. This coefficient has a maximum around the spatial frequency of $20 \sim 25$ (1/m), and the interval of the maximum spatial frequency corresponds to the reciprocal of the wheelbase of the representative vehicle running in this section. Therefore, the cause of this corrugation can be identified as the interference of the rail wave motion with the wheelbase of the vehicle.

4.3 Case of anti-resonance phenomenon in elastic support in track system

Figure 15 shows another example of rail corrugation [5]. Corrugation occurs when the train speed is in the $20 \sim 60$ km/h range, and spatial frequency is $10 \sim 30$ (1/m); the spatial frequency tends to be inversely proportional to train speed.

Figure 16 shows the mechanical impedance of the elastic support of the track under the condition of this section. It can be seen that the anti-resonance frequency of the track 140 Hz.
matches the relationship between the train speed and the spatial frequency of the corrugation. Therefore, the cause of this corrugation can be identified as the anti-resonance phenomenon in the elastic support of the track system.

### 4.4 Case of anti-resonance phenomenon in total system of track and vehicle

Figure 17 shows another example of rail corrugation in the case of a single spring rack. The spatial frequency of the corrugation is $5 \sim 8 \text{ (1/m)}$, which tends to be inversely proportional to train speed. Since higher-order components of the roughness are seen in this section, this corrugation is considered to be in the saturation stage.

Figure 18 shows the amplification coefficient of roughness amplitude under the conditions of this section. This coefficient has a maximum in the spatial frequency of $5 \sim 10 \text{ (1/m)}$, and it matches the situation at the field. Therefore, the cause of this corrugation can be deemed to be anti-resonance phenomenon in the total system of the track and the vehicle.

#### 4.5 Classification and characteristics of corrugation by growth factors

Table 2 shows the classification and characteristics of corrugation based on verifications in 15 cases on 8 domestic commercial lines including the above cases. Wavelengths are generally short in case of the interference of the rail wave motion with the wheelbase of the vehicle, and long in case of anti-resonance phenomena in the total system of the track and the vehicle. However, growth conditions due to anti-resonance phenomena in the vehicle are limited [5], so no cases were identified on these lines.

<table>
<thead>
<tr>
<th>Growth factor</th>
<th>Spatial frequency (1/m)</th>
<th>Wave length (mm)</th>
<th>Train speed (km/h)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Interference of rail wave motion between wheelbase of vehicle</td>
<td>12.33</td>
<td>30-83</td>
<td>55-119</td>
<td>230-1000</td>
</tr>
<tr>
<td>2 Anti-resonance phenomenon in elastic support in track system</td>
<td>3.9-26</td>
<td>38-260</td>
<td>22-120</td>
<td>110-330</td>
</tr>
<tr>
<td>3 Anti-resonance phenomenon in total system of track and vehicle</td>
<td>1.0-12</td>
<td>83-1000</td>
<td>37-115</td>
<td>19-130</td>
</tr>
</tbody>
</table>

#### 5. Evolution process and approaches for developing rail corrugation countermeasures

Figure 19 is a schematic representation of the rail corrugation evolution process [7].

1. **Formation stage**: This is the stage where a minute irregular roughness is formed due to fluctuation factors such as initial rail irregularities or wheel load fluctuations.
2. **Growth stage**: This is the stage where the roughness of specific wavelength selectively grows and appears as the corrugation.
3. **Saturation stage**: When the roughness amplitude increases and the wheel loses contact with the rail, the growth is suppressed, and the roughness amplitude saturates.
6. Conclusion

In this paper, rail corrugation growth mechanisms and the wavelength determination mechanism were analyzed from the viewpoint of dynamics, and these characteristics were verified in the light of field data. Based on these results, insight was gained into the evolution process of rail corrugation, and approaches are proposed for developing countermeasures. The results are as follows:

(1) The corrugation wavelength of the corrugation corresponds to the wavelength at which the amplification coefficient of roughness amplitude reaches a maximum exceeding 1.0.

(2) The growth factors of the corrugation are: (i) interference of the rail wave motion with the wheelbase of the vehicle, (ii) anti-resonance phenomena in the elastic support in the track system and (iii) anti-resonance phenomena in the total system of the track and the vehicle.

(3) Verification were made to ensure that the wavelengths of the corrugation measured on the commercial lines agreed well with the theoretical analyses.

(4) There are three stages in the evolution process of corrugation: formation, growth and saturation.

(5) Possible approaches to develop countermeasures include the suppression of fluctuation factors, suppression of wear progress, restraint of anti-resonance phenomena and removal of rail roughness.

References


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