NUMERICAL ANALYSES ON CONSOLIDATION OF CLAYEY GROUND IMPROVED BY VERTICAL DRAIN SYSTEM BASED ON 3-D ELASTO-VISCOUS MODEL

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ABSTRACT

This paper addresses a proposal on the three-dimensional elasto-viscous consolidation theory and discusses on its applicability. In order to predict the three-dimensional consolidation behavior of clayey ground improved by vertical drain system, one-dimensional consolidation theory based on an elasto-viscous liquid model proposed by Yoshikuni et al. (1994), which can express secondary consolidation without considering any distinction between normally consolidated and overconsolidated regions, is extended to three-dimensional condition. This theory is applied to the consolidation of a hollow cylindrical body that models a clayey ground improved by a vertical drain system. Then, a rigorous solution of elastic consolidation theory, a FEM solution based on elasto-plastic theory and a FDM solution using three-dimensional elasto-viscous theory are compared with the test results obtained using the three-dimensional consolidation test apparatus developed by the authors (Baek and Moriwaki, 2004). Based on the results of analyses, it is evident that the three-dimensional consolidation theory proposed by authors could simulate the consolidation behavior of clayey ground improved by vertical drains, such as the secondary consolidation and the radial displacement during the three-dimensional consolidation process.

Key words: elasto-viscous model, radial displacement, secondary consolidation, stress relaxation, three-dimensional consolidation, vertical drain (IGC: D5/E2/E14)

INTRODUCTION

Vertical drain method is one of the most effective soil improvement techniques employed to accelerate the consolidation process in clayey ground. The design of vertical drain system is generally based on the theoretical solution developed by Barron (1948). Yoshikuni and Nakanodo (1974) proposed a rigorous solution considering the finite permeability of the drain and sand mat materials while Hansbo (1981) compared the results derived from his own approximate solution considering the reduced permeability in the smeared zone around the drain with the above solutions. These conventional solutions, however, cannot predict the residual settlement induced by secondary consolidation because these solutions are based on elastic theory. In order to express the secondary consolidation governing the residual settlement of clayey ground, Taylor (1948) considered a family of equi-strain rate lines having zero strain rate line (ultimate compression line) while Bjerrum (1967) proposed a similar family of equi-strain rate lines having infinite strain rate line (instant compression line). However, both of them failed to specify these compression lines experimentally and, consequently, these theories were not used for practical predictions. The

mainstream of the study of secondary consolidation has shifted, therefore, toward modeling of clays as an elasto-visco plastic body and some contributions have been made by Sekiguchi and Toriihara (1976), Adachi and Oka (1982) and Matsui and Abe (1982), to name a few. Although their models could well simulate the time-dependent behavior of the normally consolidated clay, they also have some defects, e.g., they cannot explain the rate dependency of the yield stress observed in the constant rate of strain consolidation tests and the secondary consolidation appearing even in overconsolidated state. Yoshikuni et al. (1994) proposed a one-dimensional consolidation theory based on an elasto-viscous liquid model. Imai et al. (2003) also proposed a similar consolidation theory based on the isotach law. These theories have successfully expressed the secondary consolidation behavior without considering any distinction between normally consolidated and overconsolidated states. It has been demonstrated that the proposed models could explain various aspects of clay behavior in a unified manner together with experimental verifications, such as the secondary consolidation, stress relaxation, quasi-overconsolidation, strain rate dependency of yielding stress, and creep in overconsolidated state. However, as these consolidation theories were derived under one-

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dimensional condition, it is necessary to extend these theories to three-dimensional condition in order to explain the three-dimensional consolidation behavior of clayey ground improved by vertical drain system.

From these viewpoints, the following were performed. Firstly, in order to predict the three-dimensional consolidation behavior, the one-dimensional consolidation theory based on the elasto-viscous liquid model proposed by Yoshikuni et al. (1994) was extended to three-dimensional condition, and an application method of the extended theory to the consolidation by vertical drain system was proposed. Secondly, a new method for measuring the radial displacement in a model ground specimen in the laboratory was developed (Baek et al., 2003). Finally, the rigorous solution of elastic consolidation theory and the analytical solution by the finite element method (FEM) based on elasto-plastic theory and the analytical solution by the finite difference method (FDM) based on three-dimensional elasto-viscous consolidation theory were compared with the test results obtained by using the three-dimensional consolidation test apparatus developed by the authors (Baek and Moriwaki, 2004). Based on the results, the applicability of the proposed theory was discussed.

THREE-DIMENSIONAL ELASTO-VISCOUS CONSOLIDATION THEORY

One-Dimensional Condition

A clay skeleton is regarded as a non-linear elasto-viscous liquid in the elasto-viscous consolidation theory proposed by Yoshikuni et al. (1994) and its constitutive equation is expressed as follows:

$$-\frac{de}{dt} = (1 + e_0)m_v \frac{d\sigma'}{dt} + \frac{\sigma'}{\eta} \quad (1)$$

where e is void ratio, σ' is effective stress, m_v is the coefficient of volume compressibility and η is the coefficient of viscosity. m_v and η are determined from the following equations.

$$m_v = 0.434 \frac{C_\gamma}{1 + e_0 \sigma'} \quad (2)$$

$$\eta = \eta_0 \cdot 10^{\frac{1}{C_\alpha}(e_0 - e) + \frac{C_\beta - C_\alpha}{C_\alpha} \log \left(\frac{\sigma'_0}{\sigma'} \right)} \quad (3)$$

$$C_\alpha = \frac{de}{d \log(de/dt)}, C_\beta = \frac{de}{d \log \sigma'} \Big|_{\text{on equi-creep rate line}}, C_\gamma = \frac{de}{d \log \sigma'} \Big|_{\text{on unloading \& reloading line}} \quad (4)$$

where C_α is the gradient of creep lines on the e -log t plane, C_β is the gradient of equi-creep rate lines on the e -log σ' plane and C_γ is the gradient of unloading-reloading lines on the e -log σ' plane. Although the values of C_α , C_β and C_γ have to be defined by using the irrecoverable changes in void ratio, i.e., the viscous component Δe^v , they are practically equal to the coefficient of secondary compression, C_α , the compression index, C_c , and the swelling index, C_s , respectively, which are defined by the total

changes in void ratio.

The one-dimensional consolidation equation based on the elasto-viscous model is represented as follows:

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w \cdot m_v} \frac{\partial^2 u}{\partial z^2} + \frac{1}{\gamma_w \cdot m_v} \frac{\partial k}{\partial z} \frac{\partial u}{\partial z} + \frac{dp}{dt} + \frac{\gamma' z + p - u}{(1 + e_0) \cdot m_v \cdot \eta} \quad (5)$$

where t is the elapsed time after the load application and u is the excess pore water pressure at a depth z .

The sum of the first and second terms on the right side of Eq. (5) is the drain term and takes a negative value as the consolidation progresses. The third term is the surcharge term, which is positive during loading and negative during unloading, while the final term is caused by the relaxation of effective stress and always takes a positive value. The change in pore water pressure is interpreted as the resultant phenomenon of these four terms.

Extension to Three-Dimensional Condition

The following conditions and assumptions have been made to derive the three-dimensional consolidation equation for a hollow cylindrical body with a vertical drain (Moriwaki et al., 1998).

[Conditions]

- ① Void in clay is completely saturated with water.
- ② Deformation is small.

[Assumptions]

- ① Soil particle and pore water are incompressible.
- ② The clay skeleton behaves as an elasto-viscous liquid, and the three-dimensional rheology equation is expressed by Eq. (6).

$$\dot{\sigma}'_{ij} = \left(K + \frac{4}{3} G \right) \dot{\epsilon}_{kk} \delta_{ij} - 2G (\dot{\epsilon}_{kk} \delta_{ij} - \dot{\epsilon}_{ij}) - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\dot{\sigma}'_{kk}}{3} \delta_{ij} - \frac{G}{\eta_G} \dot{\sigma}'_{ij} \quad (6)$$

where K is the bulk modulus, G is the shear modulus, η_K and η_G are the coefficients of bulk viscosity and shear viscosity, respectively, and are dependent on the stress and strain.

- ③ Flow of pore water is governed by Darcy's law.
- ④ The boundary surfaces of the hollow cylinder are smooth and the shear stresses on these planes are zero.
- ⑤ The top and bottom surfaces of the hollow cylinder are displaced parallel to their original surfaces.
- ⑥ There is no vertical flow in the hollow cylinder; only internal radial flow.

Considering the above conditions and assumptions, the three-dimensional consolidation equation of a hollow cylindrical body is obtained as follows (Moriwaki et al., 1998):

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w} \left(K + \frac{4}{3} G \right) \frac{\partial^2 u}{\partial x_i \partial x_i} + \frac{1}{\gamma_w} \left(K + \frac{4}{3} G \right) \frac{\partial k}{\partial x_i} \frac{\partial u}{\partial x_i} + \frac{d\phi}{dt} - \frac{G}{\eta_G} u + \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\sigma'_{kk}}{3} \quad (7)$$

where ϕ is called the consolidation potential and is uniformly distributed throughout a body. Therefore, ϕ is

Fig. 1. Coordinates of a hollow cylindrical body

expressed as a function of time only as follows:

$$\frac{d\varphi}{dt} = \left\{ \left(K + \frac{4}{3} G \right) \dot{\varepsilon}_{kk} - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\sigma_{kk}}{3} + \frac{K}{\eta_K} u + \dot{u} \right\} \quad (8)$$

The coefficient of volume compressibility m_v , the relaxation time of volume deformation T_K and the relaxation time of shear deformation T_G are defined as follows:

$$m_v = 1 / \left(K + \frac{4}{3} G \right), \quad T_K = \frac{\eta_K}{K}, \quad T_G = \frac{\eta_G}{G} \quad (9)$$

Therefore, Eq. (7) is re-written as

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w m_v} \frac{\partial^2 u}{\partial x_i \partial x_i} + \frac{1}{\gamma_w m_v} \frac{\partial k}{\partial x_i} \frac{\partial u}{\partial x_i} + \frac{d\varphi}{dt} - \frac{u}{T_G} + \left(\frac{T_G - T_K}{T_K T_G} \right) \frac{\sigma'_{kk}}{3} \quad (10)$$

Again, the combination of first and second terms on the right side of Eq. (10) is the drain term and takes a negative value as the consolidation progresses. The combination of third and fourth terms expresses the relaxation of effective stress during the three-dimensional deformation and always takes positive value. The final term is caused by the difference in the relaxation time between volume deformation and shear deformation. Pore water pressure increases when T_G is larger than T_K .

Boundary Conditions

The coordinates of the hollow cylindrical body are shown in Fig. 1. In this figure, h , r_w and r_e are the height, the radius of the central drain and the effective external radius of the specimen, respectively, while n is the dimensionless ratio of radii, $n = r_e / r_w$. In the following equations, the upper bar and dot represent the average value in radial direction and the time rate of components, respectively.

Considering the principle of effective stress, the stresses on the boundary surface of the hollow cylindrical body are:

Top and bottom surfaces:

$$\bar{\sigma}_z = \bar{p}_z = \bar{\sigma}'_z + \bar{u} \quad (11)$$

Internal periphery:

$$\sigma_{r|r=r_w} = p_{rw} = \sigma'_{r|r=r_w} + u_{|r=r_w} \quad (12)$$

External periphery:

$$\sigma_{r|r=r_e} = p_{re} = \sigma'_{r|r=r_e} + u_{|r=r_e} \quad (13)$$

where $\bar{p}_z$, p_{rw} and p_{re} are the average applied load on the top surface, the applied loads on the internal periphery and on the external periphery of a hollow cylindrical body, respectively. On the top surface, the load is applied as an average value because the load changes in radial direction. In addition, the consolidation potential is assumed to be uniformly distributed throughout the body in this study and is a function of time only. Therefore,

$$\frac{d\varphi}{dt} = \left\{ \left(K + \frac{4}{3} G \right) \frac{\partial \bar{\varepsilon}_{kk}}{\partial t} - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk}}{3} + \frac{K}{\eta_K} \bar{u} + \frac{\partial \bar{u}}{\partial t} \right\} \quad (14)$$

From the three-dimensional elasto-viscous rheology Eq. (6), the relations between stresses and strains on the boundary surfaces are:

$$\begin{aligned} \bar{\sigma}'_{z|z=0} &= \left(K + \frac{4}{3} G \right) \bar{\varepsilon}_{kk|z=0} - 2G \left(\bar{\varepsilon}_{r|z=0} + \bar{\varepsilon}_{\theta|z=0} \right) \\ &\quad - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk|z=0}}{3} - \frac{G}{\eta_G} \bar{\sigma}'_{z|z=0} \end{aligned} \quad (15)$$

$$\begin{aligned} \bar{\sigma}'_{r|r=r_w} &= \left(K + \frac{4}{3} G \right) \bar{\varepsilon}_{kk|r=r_w} - 2G \left(\bar{\varepsilon}_{r|r=r_w} + \bar{\varepsilon}_{\theta|r=r_w} \right) \\ &\quad - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk|r=r_w}}{3} - \frac{G}{\eta_G} \bar{\sigma}'_{r|r=r_w} \end{aligned} \quad (16)$$

$$\begin{aligned} \bar{\sigma}'_{r|r=r_e} &= \left(K + \frac{4}{3} G \right) \bar{\varepsilon}_{kk|r=r_e} - 2G \left(\bar{\varepsilon}_{r|r=r_e} + \bar{\varepsilon}_{\theta|r=r_e} \right) \\ &\quad - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk|r=r_e}}{3} - \frac{G}{\eta_G} \bar{\sigma}'_{r|r=r_e} \end{aligned} \quad (17)$$

The relations between strains and displacements are:

$$\varepsilon_z = \bar{\varepsilon}_z = \frac{w_{z|z=0}}{h} = \frac{w_{z0}}{h} \quad (18)$$

$$\varepsilon_{\theta|r=r_w} = - \frac{w_{r|r=r_w}}{r_w} = \frac{w_{rw}}{r_w} \quad (19)$$

$$\varepsilon_{\theta|r=r_e} = - \frac{w_{r|r=r_e}}{r_e} = \frac{w_{re}}{r_e} \quad (20)$$

$$\varepsilon_r + \varepsilon_{\theta} = - \frac{\partial w_r}{\partial r} - \frac{w_r}{r} = - \frac{1}{r} \frac{\partial}{\partial r} (r \cdot w_r) \quad (21)$$

$$\begin{aligned} \bar{\varepsilon}_r + \bar{\varepsilon}_{\theta} &= \frac{2\pi}{\pi(r_e^2 - r_w^2)} \int_{r_w}^{r_e} r(\varepsilon_r + \varepsilon_{\theta}) dr = \frac{2(n \cdot w_{r|r=r_e} - w_{r|r=r_w})}{r_w(n^2 - 1)} \\ &= \frac{2(n \cdot w_{re} - w_{rw})}{r_w(n^2 - 1)} \end{aligned} \quad (22)$$

$$\bar{\varepsilon}_{kk} = \bar{\varepsilon}_z + \bar{\varepsilon}_r + \bar{\varepsilon}_{\theta} = \frac{w_{z0}}{h} - \frac{2(n \cdot w_{re} - w_{rw})}{r_w(n^2 - 1)} \quad (23)$$

where w_{z0} , w_{rw} and w_{re} are the displacements on the top surface, on the internal periphery and on the external periphery, respectively. Substituting Eqs. (11) ~ (13) into Eqs. (14) ~ (23), the relationships of consolidation potential and boundary values are obtained as follows:

$$\frac{d\phi}{dt} = \bar{p}_z - 4G \left\{ \frac{2(n \cdot \dot{w}_{re} - \dot{w}_{rw})}{r_w(n^2 - 1)} \right\} + \frac{G}{\eta_G} \bar{p}_z \quad (24)$$

$$\frac{d\phi}{dt} = \bar{p}_{rw} - 2G \left\{ \frac{\dot{w}_{z0}}{h} - \frac{\dot{w}_{rw}}{r_w} \right\} + \frac{G}{\eta_G} \bar{p}_{rw} \quad (25)$$

$$\frac{d\phi}{dt} = \bar{p}_{re} - 2G \left\{ \frac{\dot{w}_{z0}}{h} - \frac{\dot{w}_{re}}{r_w} \right\} + \frac{G}{\eta_G} \bar{p}_{re} \quad (26)$$

$$\begin{aligned} \frac{d\phi}{dt} = & \left(K + \frac{4}{3} G \right) \left\{ \frac{\dot{w}_{z0}}{h} - \frac{2(n \cdot \dot{w}_{re} - \dot{w}_{rw})}{r_w(n^2 - 1)} \right\} \\ & - \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk}}{3} + \frac{K}{\eta_K} \bar{u} + \frac{\partial \bar{u}}{\partial t} \end{aligned} \quad (27)$$

In these equations, the unknowns are $\bar{p}_z$, $\bar{p}_{rw}$, $\bar{p}_{re}$, $\dot{w}_{z0}$, $\dot{w}_{rw}$, $\dot{w}_{re}$ and $d\phi/dt$. If among these, three values are specified as boundary values, the remaining unknowns can be obtained from Eqs. (24)~(27).

Application to Vertical Drain System

As typical boundary conditions of the consolidation by vertical drain system, the radial displacements at the internal and external peripheries are kept under restraint and a constant vertical load is applied under equal vertical strain condition. Considering a uniform state of initial stress in the radial direction, the consolidation potential and the boundary conditions in this case are expressed as:

$$\frac{d\phi}{dt} = \frac{G}{\eta_G} p \quad (28)$$

$$\begin{aligned} \bar{p}_r = & \frac{G}{\eta_G} (p - p_r) - \frac{2G}{K + \frac{4}{3} G} \left\{ \frac{G}{\eta_G} p + \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk}}{3} \right. \\ & \left. - \frac{K}{\eta_K} \bar{u} + \frac{\partial \bar{u}}{\partial t} \right\} \end{aligned} \quad (29)$$

$$\dot{\epsilon}_z = \frac{\dot{w}_{z0}}{h} = \frac{1}{K + \frac{4}{3} G} \left\{ \frac{G}{\eta_G} p + \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\bar{\sigma}_{kk}}{3} - \frac{K}{\eta_K} \bar{u} + \frac{\partial \bar{u}}{\partial t} \right\} \quad (30)$$

where p_r and w_{z0} are the boundary stress on the periphery and the vertical displacement on the top surface, respectively.

Next, the strains are obtained from Eqs. (14), (21) and (28) as follows:

$$\begin{aligned} \dot{\epsilon}_{kk} = \dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = & -\frac{1}{K + \frac{4}{3} G} \left\{ \left(\frac{K}{\eta_K} - \frac{G}{\eta_G} \right) \frac{\sigma'_{kk}}{3} \right. \\ & \left. + \frac{G}{\eta_G} (p - u) - \dot{u} \right\} \end{aligned} \quad (31)$$

$$\dot{\epsilon}_r + \dot{\epsilon}_\theta = \dot{\epsilon}_{kk} - \dot{\epsilon}_z = -\frac{1}{r} \left(\frac{\partial}{\partial r} \right) (r \dot{w}_r) \quad (32)$$

where ϵ_{kk} , ϵ_r and ϵ_θ are the volumetric, the radial and the circumferential strains, respectively.

Finally, the consolidation equation for vertical drain consolidation is obtained from Eqs. (7) and (28) as follows:

$$\begin{aligned} \frac{\partial u}{\partial t} = & \frac{k}{\gamma_w} \left(K + \frac{4}{3} G \right) \frac{\partial^2 u}{\partial x_i \partial x_i} + \frac{1}{\gamma_w} \left(K + \frac{4}{3} G \right) \frac{\partial k}{\partial x_i} \frac{\partial u}{\partial x_i} \\ & + \frac{p - u}{T_G} + \frac{T_G - T_K}{T_K T_G} \left(\frac{p + 2p_r}{3} - u \right) \end{aligned} \quad (33)$$

Mechanism of Consolidation by Three-Dimensional Elasto-Viscous Theory

The four terms on the right-hand side in Eq. (33) can be rewritten in the form:

$$\frac{\partial u}{\partial t} = \left(\frac{\partial u}{\partial t} \right)_D + \left(\frac{\partial u}{\partial t} \right)_R + \left(\frac{\partial u}{\partial t} \right)_{DR} \quad (34)$$

The first term on the right side of Eq. (34), which corresponds to the first and second terms on the right side of Eq. (33), is the drain term and takes a negative value as the consolidation progresses. The second term is caused by the relaxation of effective stress during the three-dimensional deformation and always takes positive value, while the final term is caused by the difference in the relaxation time between volume deformation and shear deformation. Moreover, the pore water pressure increases when T_G is larger than T_K . Thus, the first term governs the rate of dissipation of pore water pressure, while the second and third terms control the rate of generation of pore water pressure. Thus the magnitude of pore water pressure is determined as the resultant of these three terms.

Furthermore, when $T_K = T_G$, the elasto-viscous three-dimensional consolidation equation for a hollow cylindrical body is obtained from Eq. (33) as follows and is equivalent to the one-dimensional consolidation equation.

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w m_v} \frac{\partial^2 u}{\partial x_i \partial x_i} + \frac{1}{\gamma_w m_v} \frac{\partial k}{\partial x_i} \frac{\partial u}{\partial x_i} + \frac{p - u}{m_v \eta} \quad (35)$$

where

$$\eta = \eta_K + \frac{4}{3} \eta_G \quad (36)$$

Determination of Elasto-Viscous Parameters

Referring to the paper by Yoshikuni et al. (1995), the following assumptions are made to determine the elasto-viscous parameters.

[Assumptions]

- (1) Creep lines in the one-dimensional consolidation test are a family of straight lines on e -log t plane with a gradient C_α .
- (2) Creep lines in the isotropic consolidation test are a family of straight lines on e -log t plane with a gradient $C_{\alpha i}$.
- (3) Equi-creep rate lines in the one-dimensional consolidation test are a family of straight lines on e -log σ'_z plane with a gradient C_β .
- (4) Equi-creep rate lines in the isotropic consolidation test are a family of straight lines on e -log σ'_m plane with a gradient $C_{\beta i}$.

- (5) Unloading-reloading curves on e -log σ'_z plane in the one-dimensional consolidation test are a family of straight lines with a gradient C_{γ} .
- (6) Unloading-reloading curves on e -log σ'_m plane in the isotropic consolidation test are a family of straight lines with a gradient $C_{\gamma i}$.
- (7) K and G are function only of stress, and η_K and η_G are function of both void ratio and stress.

Using these assumptions, the elasto-viscous parameters are given by the following equations.

[Coefficients of Elasticity]

$$K = 2.3 (1 + e_0) \frac{\sigma'_m}{C_{\gamma i}} \quad (37)$$

$$G = \frac{3}{4} \left(\frac{1}{m_v} - K \right) \quad (38)$$

[Coefficients of Viscosity]

$$\eta_K = \eta_{K0} \cdot 10^{\frac{1}{C_{ai}}(e_0 - e) + \frac{C_{\beta i} - C_{ai}}{C_{ai}} \log \frac{\sigma'_{m0}}{\sigma'_m}} \quad (39)$$

$$\eta_G = \frac{4}{3} \left(\frac{G}{K} \right)^2 \frac{\eta \cdot \eta_K (m_v \cdot K)^2}{\eta_K - \eta (m_v \cdot K)^2} \quad (40)$$

where m_v and η are the parameters under one-dimensional condition.

Calculation Method

Equation (33) can be numerically solved using the finite difference method, in which the differentiation with respect to time t is approximated by the forward finite differences while the differentiation with respect to depth z is by the central finite differences, i.e.,

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t_i} \quad (41)$$

$$\frac{\partial u}{\partial z} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta z} \quad (= U_1) \quad (42)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta z^2} \quad (= U_2) \quad (43)$$

$$\frac{\partial k}{\partial z} = \frac{k_{i+1,j} - k_{i-1,j}}{2\Delta z} \quad (= K_1) \quad (44)$$

Since only $u_{i,j+1}$ is unknown in Eqs. (41) ~ (44), substitution of Eqs. (41) ~ (44) into Eq. (33) and rearranging results in

$$u_{i,j+1} = A_{ij} \cdot U_2 + \frac{1}{r_i} \cdot (A_{ij} \cdot U_1) + \frac{A_{ij}}{k_{ij}} \cdot K_i \cdot U_1 + B_{ij} + C_{ij} + u_{i,j} \quad (45)$$

where

$$A_{ij} = \Delta t_j \frac{k_{i,j}}{\gamma_w} \left(K_{i,j} + \frac{4}{3} G_{i,j} \right) \quad (46)$$

$$B_{ij} = \Delta t_j \frac{p - u_{i,j}}{T_{Gi,j}} \quad (47)$$

$$C_{ij} = \Delta t_j \left(\frac{1}{T_{Ki,j}} - \frac{1}{T_{Gi,j}} \right) \cdot \sigma'_{mi,j} \quad (48)$$

Therefore, the pore water pressure $u_{i,j+1}$ at $t + \Delta t$ for the given boundary conditions is calculated from Eq. (45), and the increments of the vertical strain $\Delta \varepsilon_z$, and of displacement Δw_{z0} , from t to $t + \Delta t$ are calculated by substituting the pore water pressure obtained by Eq. (45) into Eq. (30). As the increment of the volumetric strain $\Delta \varepsilon_{kk}$ is calculated from Eq. (31), the summed increment of the radial and circumferential strains $\Delta \varepsilon_r + \Delta \varepsilon_\theta$, is calculated from Eq. (32). Moreover, as the increment of the radial displacement Δw_r is related to $\Delta \varepsilon_r + \Delta \varepsilon_\theta$ as shown in Eq. (32), Δw_r at any radial position in the specimen is calculated by the differential equation in the radial direction as in the following equation:

$$\Delta w_{r|i,j+1} = \frac{r_{i-1}}{r_{i-1} + \Delta r} (\Delta w_{r|i-1,j+1} - (\Delta \varepsilon_r + \Delta \varepsilon_\theta)_{i,j+1} \cdot \Delta r) \quad (49)$$

where $\Delta w_{r|i=0,j+1}$ is always zero and $r_{i=0} = r_w$ because of the boundary conditions. Therefore, the increments of the individual values of radial and circumferential strains are calculated by using $\Delta w_{r|i,j+1}$ and $(\Delta \varepsilon_r + \Delta \varepsilon_\theta)_{i,j+1}$ as follows:

$$\Delta \varepsilon_{\theta|i,j+1} = - \frac{\Delta w_{r|i,j+1}}{r_i} \quad (50)$$

$$\Delta \varepsilon_{r|i,j+1} = (\Delta \varepsilon_r + \Delta \varepsilon_\theta)_{i,j+1} - \frac{\Delta w_{r|i,j+1}}{r_i} \quad (51)$$

Using the calculated values of the strains and the pore water pressure, the increments of the effective stresses are calculated from the three-dimensional elasto-viscous rheology equation of Eq. (6). When the incremental values of the strains, stresses and displacements from t to $t + \Delta t$ are calculated by the above-mentioned procedure, the total values of each parameter at time $t + \Delta t$ are obtained by adding the incremental values to the total values at time t .

THREE-DIMENSIONAL CONSOLIDATION TEST AND NUMERICAL ANALYSIS

Three-Dimensional Consolidation Test

The clay used in this test was an alluvial clay reconstituted from the slurry of Maizuru clay, which was taken from the seabed of Maizuru Bay at Maizuru city of Kyoto Prefecture. The clay sample was pre-consolidated at 49 kPa under the one-dimensional condition in laboratory. The physical and mechanical properties of this clay are shown in Table 1.

Figure 2 shows the magnet and magnetometers used for measuring the radial displacement in the specimen. As shown in this figure, the radial displacement in the specimen during the three-dimensional consolidation process can be obtained by measuring the intensity of the magnetic field induced by a small magnet buried in the specimen through the magnetometers set in the consolidation ring and in the vertical drain installed at the center of the specimen. In this test apparatus, the vertical drain made of stainless steel, which does not have a magnetic field, is not deformed during the test process. Refer to the paper by Baek et al. (2003) for details.

Table 1. Physical and mechanical properties

Physical properties				Mechanical properties				
Liquid limit w_L (%)	Plastic limit w_p (%)	Plasticity index I_p	Specific gravity G_s	Compression index C_c	Swelling index C_s	Coefficient of consolidation c_v (cm ² /min)	Coefficient of secondary consolidation	
83.3	31.2	52.1	2.737	0.665	0.084	0.0388	C_{α}^*	$C_{\alpha\epsilon}^{**}$
							0.018	0.006

$$*C_{\alpha} = \frac{de}{d \log t} \quad **C_{\alpha\epsilon} = \frac{d\epsilon_v}{d \log t}$$

(a) Schematic diagram **(b) Plane section** **(c) Cross section****Fig. 2. Magnet and magnetometers for measuring displacement in the specimen****Table 2. Cases of three-dimensional consolidation tests**

	Magnet position r (mm) a/b/c	Initial pressure (kPa)	Consolidation pressure (kPa)	Increment pressure (kPa)
Case I-1	12.5/17.5/22.5	78.5	157	78.5
Case I-2	12.5/17.5/22.5	157	314	157
Case I-3	12.5/17.5/22.5	314	628	314

In this study, three-dimensional consolidation tests were carried out under the conditions shown in Table 2. The clay specimens were initially consolidated under one-dimensional condition (vertical flow only) and then were consolidated under three-dimensional condition (internal radial flow only). The applied consolidation pressures are listed in Table 2 and the load increment/decrement ratio $\Delta p/p$ was selected as ± 1.0 . For each loading condition, a magnet (2 mm $\times$ 4 mm $\times$ 3 mm) was buried in the specimen 3 mm from the top surface to avoid friction between the loading plate and the magnet. Three patterns of magnet position ($r = 12.5, 17.5$ and 22.5 mm, where r is a radial distance from the center of the specimen to the buried magnet), and three patterns of the final consolidation pressure ($p = 157, 314$ and 628 kPa) were considered.

Modified Method for Measuring Radial Displacement

The authors (Baek and Moriwaki, 2004) have proposed a method for measuring the radial displacement in a specimen with vertical drain by measuring the intensity of the magnetic field induced by a small magnet buried in the specimen using magnetometers. The measuring method using the calibration curves, which are relationships between the position of the magnet and the measured intensity of the magnetic field in the calibration test, is modified by using the theoretical solution of the magnetic field induced by a magnet. The theoretical solution is calculated by considering the influence of changes in temperature, electric voltage by human activities, etc. as described in detail in the Appendix of this paper.

Figure 3 shows the radial displacement-time curves at the drained and undrained sides in Case I-2 obtained from the calibration curve method and the modified method based on the theoretical equation. As shown in this figure, the radial displacement obtained using the modified method is smaller than that by the calibration

Fig. 3. Radial displacement-time curves (Case I-2)

curve method. In reality, the environment in the laboratory is influenced by temperature, human activities, etc. Therefore, the radial displacements should be taken as those obtained by the modified method, which combines the theoretical solutions with the calibration curves and with the cyclic changes in the magnetic field considered.

Numerical Analyses based on Elastic, Elasto-Plastic and Elasto-Viscous Theories

In this study, numerical analyses for the three-dimensional consolidation with internal radial flow through the vertical drain were carried out using three types of constitutive models, namely elastic, elasto-plastic and elasto-viscous models. The elastic solutions were calculated by the rigorous solution for the three-dimensional consolidation theory proposed by Yoshikuni (1974) while for the elasto-plastic analysis, the finite element method based on the Modified Cam-clay model was used. For the elasto-viscous analysis, numerical solutions for the elasto-viscous consolidation theory proposed by the authors were obtained by the finite difference method. The parameters used in the elastic, elasto-plastic and elasto-viscous analyses are presented in Tables 3, 4 and 5, respectively. These values were determined from the results of one-dimensional consolidation test, isotropic consolidation test and consolidated-undrained triaxial compression test. The coefficients of permeability in the elasto-plastic and elasto-viscous analyses were regarded to vary with the void ratio, assuming a linear relationship between e and $\log k$ with the gradient denoted by λ_k . The poison ratio ν was assumed to be $1/3$.

Comparison of Test Results and Numerical Analyses

Figure 4 shows the settlement-time curves obtained from the three-dimensional consolidation tests and numerical analyses using the elastic, elasto-plastic and elasto-viscous models. As shown in these figures, the elastic and elasto-plastic analyses can simulate the primary consolidation process but cannot replicate the secondary consolidation process. The reason for this is because viscosity, which can explain secondary consolidation, is not considered in the elastic and elasto-plastic theories. On the other hand, the results of the elasto-

Table 3. Parameters used in elastic analysis

	Case 1	Case 2	Case 3
m_v (m^2/kN)	9.87E-4	4.62E-4	2.10E-4
c_h (cm^2/min)	3.07E-2	3.88E-2	4.86E-2
h (mm)	17.96	16.50	15.11
Δp (kPa)	78.5	157.0	314.0

Table 4. Parameters used in elasto-plastic analysis

	Case 1	Case 2	Case 3
λ	0.2787	0.2388	0.2105
κ	0.0139	0.0119	0.0105
M	1.333	1.333	1.333
ν	0.333	0.333	0.333
σ_{vc} (kPa)	78.5	157.0	314.0
σ_{v0} (kPa)	78.5	157.0	314.0
K_0	0.613	0.613	0.613
e_0	1.545	1.315	1.171
λ_k	0.898	0.500	0.337
k_0 (cm/min)	5.23E-5	5.15E-5	4.48E-5
h (mm)	17.96	16.50	15.11
Δp (kPa)	78.5	157.0	314.0

Table 5. Parameters used in elasto-viscous analysis

	Case 1	Case 2	Case 3
C_α	0.018	0.018	0.020
C_β	0.641	0.549	0.484
C_γ	0.032	0.027	0.024
$C_{\alpha i}$	0.024	0.024	0.024
$C_{\beta i}$	0.641	0.549	0.484
$C_{\gamma i}$	0.034	0.034	0.034
σ'_{z0} (kPa)	78.5	157.0	314.0
σ'_{m0} (kPa)	52.3	104.7	209.3
e_0	1.545	1.315	1.171
λ_k	0.898	0.500	0.337
k_0 (cm/min)	5.23E-5	5.15E-5	4.48E-5
η_0 (kPa·min)	1.0E7	1.0E7	1.0E7
η_{k0} (kPa·min)	2.0E8	2.0E8	2.0E8
h (mm)	17.96	16.50	15.11
Δp (kPa)	78.5	157.0	314.0

viscous analysis for the three-dimensional consolidation by vertical drain agree well with the whole consolidation process of the test results, including the secondary consolidation process.

(a) Elastic

(a) Elastic ($\Delta p=314\text{kPa}$)

(b) Elasto-Plastic

(b) Elasto-Plastic ($\Delta p=314\text{kPa}$)

(c) Elasto-Viscous

(c) Elasto-Viscous ($\Delta p=314\text{kPa}$)

Fig. 4. Settlement-time curves

The dissipation of excess pore water pressures at $r=8$, 17, 26 mm in the tests and those obtained by numerical analyses are shown in Fig. 5. In these figures, the excess pore water pressure is normalized by the incremental consolidation pressure Δp . As shown in these figures, the elasto-plastic analysis shows a behavior wherein the normalized pore water pressure ($u/\Delta p$) exceeds 1.0 in the early stage of consolidation near the undrained side, as also shown in the test results. However, the normalized pore water pressures calculated by the elastic and elasto-viscous theories do not exceed 1.0 because negative dilatancy of normally consolidated clay is not considered

in both analyses.

Figure 6 shows the comparison of the radial displacements obtained from the tests and from the numerical analyses using the elastic theory at the drained ($r=12.5$ mm) and undrained ($r=22.5$ mm) sides. From these figures, it is found that the radial displacements obtained by the elastic analysis occur toward the drained side in the early stage of the three-dimensional consolidation process and then completely return to the original position at the later stage. However, the radial displacements in the test results do not completely return to the original position at the end of consolidation. This is because clay is not an elastic material. Note that the radial displace-

(a) $r=12.5\text{mm}$ (b) $r=22.5\text{mm}$

Fig. 6. Radial displacement-time curves based on elastic theory

(a) $r=12.5\text{mm}$ (b) $r=22.5\text{mm}$

Fig. 7. Radial displacement-time curves based on elasto-plastic analysis

ments obtained from elastic analysis have larger peak value in both drained and undrained sides at lower consolidation pressure. The radial displacements in the test results have the same tendency as that shown by numerical analysis results on the drained side but show opposite tendency on the undrained side. The reason for the occurrence of such different tendency as compared to the test results needs further investigation. Peak radial displacements occur at earlier time on the drained side than on the undrained side in both experiments and numerical analyses.

Figure 7 shows the comparison of the radial displacements obtained from the tests and from the numerical analyses using the elasto-plastic model. From these figures, the radial displacements obtained in the elasto-plastic analysis have a tendency to have larger peak value in both drained and undrained sides at lower consolidation pressure, similar to the results of the elastic analysis. It is also found that the elasto-plastic analysis could replicate the behavior of radial displacements, i.e., they do not return to the original position at the end of consolidation as observed in the test results. The amounts of radial displacements of the elasto-plastic analysis are larger on the drained side than on the undrained side and this tendency also agree with that observed in the test results. Although the final values of the radial displacements

based on the elasto-plastic analysis almost agree with those of the test results, the peak values of the elasto-plastic analysis are smaller than those of the test results.

Figure 8 shows the comparison of the radial displacements between the test results and the numerical analyses using the elasto-viscous model. As shown in these figures, the radial displacements obtained from elasto-viscous analysis show larger peak value on both drained and undrained sides at lower consolidation pressure, similar to those obtained in elastic and elasto-plastic analyses. The peak radial displacements on the drained side are larger than those on the undrained side and these results agree with the tendency indicated by the test results. It is also found that the radial displacements obtained by the elasto-viscous analysis did not return to the original position at the end of consolidation and these agree with the test results. Both peak and final values of the radial displacements from the elasto-viscous analysis almost agree with those of the test results. In addition, radial displacements calculated by the elasto-viscous analysis develop toward the drained side with time when the excess pore water pressures are completely dissipated throughout the specimen, as shown in Fig. 8 (a).

(a) $r=12.5\text{mm}$ (b) $r=22.5\text{mm}$

Fig. 8. Radial displacement-time curves based on elasto-viscous analysis

Interpretation of Consolidation Behavior of the Vertical Drain by Elasto-Viscous Theory

In order to predict the three-dimensional consolidation behavior of clayey ground improved by vertical drains, one-dimensional consolidation theory based on an elasto-viscous liquid model proposed by Yoshikuni et al. (1994) was extended to the three-dimensional condition under axisymmetric condition. Then, the finite difference method solution of the proposed elasto-viscous consolidation theory was compared with the test results.

From Figs. 4, 5 and 8, the elasto-viscous analysis agreed well with the test results of the secondary consolidation in the three-dimensional consolidation process with vertical drain. However, it could not express the phenomenon observed near the undrained side wherein the pore water pressure exceeded the applied consolidation pressure in the early stage of the three-dimensional consolidation process because negative dilatancy was not considered in the elasto-viscous model. Furthermore, the elasto-viscous analysis can simulate the internal behavior of clayey ground improved by vertical drains in three-dimensional consolidation in various ways. Firstly, the volumetric strain in the elements near the drained side is largely generated by the dissipation of the excess pore water pressure, but the volumetric strain near the undrained side is scarcely generated because of the slow

(a) Settlement-time curves

(b) Radial displacement-time curves

Fig. 9. Effect of the ratio T_G/T_K

dissipation of the excess pore water pressure in the early stage of the consolidation process. Therefore, radial displacement is generated toward the drained side to maintain the equal vertical strain condition. Secondly, in the later stage of the consolidation process, shear strain is predominant in the elements near the drained side because their volumetric strain is almost terminated, but the volumetric strain in the element near the undrained side is generated by the dissipation of pore water pressure. Due to this difference in the component of strains in the elements between the drained side and the undrained side and to maintain equal vertical strain condition, it would appear that the radial displacement is generated toward the undrained side from the drained side in the later stage of consolidation. Finally, when the excess pore water pressure is completely dissipated throughout the specimen, it is believed that the radial displacement in the drained side occurs toward the inner side again while that in the undrained side occurs toward the outer side because volumetric strain is generated throughout the specimen by the secondary compression of clay.

Effects of Relaxation Time on Three-Dimensional Consolidation Behavior

In order to clarify the influence of the relaxation time of the volume deformation and shear deformation, Fig. 9

is presented which shows the settlement and radial displacement-time curves with the change in T_G/T_K obtained from the elasto-viscous analysis for Case I-2. As mentioned above, the drain term in Eq. (31) governs the rate of dissipation of pore water pressure, while the relaxation of effective stress and the influence of the ratio of T_G/T_K terms control the rate of generation of pore water pressure. In Case I-2, however, the term corresponding to the relaxation of effective stress strongly affects the dissipation of pore water pressure because T_K has larger value. Thus, the fourth term on the right-hand side in Eq. (31) hardly affects the dissipation of pore water pressure. Therefore, the settlement in the case of small T_G/T_K is delayed at the later stage of consolidation and is largely generated because the dissipation of pore water pressure is delayed by the relaxation of effective stress. The radial displacement in the drained side in the case of small T_G/T_K is largely generated toward the undrained side in the later stage of consolidation. This is considered to be due to the delay in the dissipation of pore water pressure caused by the generation of positive pore water pressure induced by the relaxation of effective stress. In addition, it is also found that the time at peak radial displacement is almost the same.

CONCLUSIONS

The one-dimensional consolidation theory based on the elasto-viscous model was extended to the three-dimensional condition in order to simulate the consolidation behavior of clayey ground improved by vertical drain method. The applicability of the proposed elasto-viscous theory was discussed by comparing the analytical results obtained through elastic, elasto-plastic and elasto-viscous theories with the test results with emphasis on radial displacements in the specimen, which were obtained by the modified measuring method that combined the theoretical solutions of the magnetic field with the calibration curves. In addition, the influence of the relaxation time of volume and shear deformations in the elasto-viscous analysis was discussed. The main conclusions are summarized as follows:

- (1) From the numerical analyses, it was found that not only vertical displacement but also radial displacement developed in the specimen during the three-dimensional consolidation process. Radial displacement developed toward the drained side in the early stage of the three-dimensional consolidation process and then toward the undrained side in the later stage. It was also found that the radial displacement in the specimen was larger in the drained side than in the undrained side. The time at peak radial displacement occurred earlier in the drained side than in the undrained side.
- (2) Elastic analysis could not express the phenomena wherein the pore water pressure near the undrained side exceeded the applied consolidation pressure in the early stage of the three-dimensional

consolidation process and the secondary consolidation which occurred in the late stage of consolidation. The radial displacements obtained through the elastic analysis completely returned to the original position at the end of the consolidation process, but those of the test results showed otherwise.

- (3) Elasto-plastic analysis using the Modified Cam-clay model could simulate the phenomena wherein the pore water pressure near the undrained side exceeded the applied consolidation pressure in the early stage of the three-dimensional consolidation process and the radial displacements at the end of consolidation did not return to the original position. However, the elasto-plastic analysis could not reproduce the secondary consolidation and also underestimated the peak value of the radial displacement in the specimen during the three-dimensional consolidation process.
- (4) Elasto-viscous model extended to the three-dimensional condition based on an one-dimensional elasto-viscous liquid model proposed by Yoshikuni et al. (1994) could not replicate the phenomenon where the pore water pressure near the undrained side exceeded the applied consolidation pressure in the early stage of the three-dimensional consolidation process. However, it could simulate the secondary consolidation and the tendency for the radial displacements not to return to the original position at the end of consolidation. Moreover, the peak and final values of the radial displacements obtained by the elasto-viscous model are almost in agreement with those of the test results.
- (5) The settlement in the case of the small T_G/T_K was delayed in the later stage of consolidation and was largely generated because the dissipation of the pore water pressure was delayed by the generation of the pore water pressure due to the relaxation of effective stress. Radial displacement at the drained side in the case of the small T_G/T_K was largely generated toward the undrained side in the later stage of consolidation. The time at peak radial displacement was almost not influenced by the difference in the ratio T_G/T_K .

From this study, it was confirmed that the proposed three-dimensional consolidation theory based on the elasto-viscous model could express the behavior of three-dimensional consolidation by vertical drain during the whole consolidation process, including the secondary consolidation process. Therefore, the residual settlement of clayey ground improved by vertical drain method and the pre-loading method, which cannot be predicted by the consolidation theories widely used at present, can be predicted by this theory without considering the distinction between normally consolidated and the overconsolidated regions.

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APPENDIX

Method for Measuring Radial Displacement in a Specimen by Theoretical Equation of Magnetic Flux Density

Theoretical Solution of Magnetic Flux Density

In this study, a review of method for measuring the radial displacement in the specimen with the vertical drain was carried out by combining the theoretical solution of the magnetic field with the calibration curves, which are the relationships between the magnetic flux density measured by the magnetometer and the vertical position of the magnet buried at the specified radial positions (Baek et al., 2004). Considering the magnetic field induced by a magnet having an intensity M of a magnetic pole and the distance l of poles, this magnet is called a magnetic dipole when the distance l of poles is sufficiently smaller than the distance ρ from the center of the magnet to the target point P . The magnetic field H

Fig. A1. Magnetic field induced by magnetic dipole

generated by magnetic dipole is shown in Fig. A1, and components of ρ and ϕ directions of H are given by the following equations:

$$H_\rho = \frac{M}{4\pi\mu_0} \frac{2 \cos \phi_1}{\rho^3} \quad (A1)$$

$$H_\phi = \frac{M}{4\pi\mu_0} \frac{\sin \phi_1}{\rho^3} \quad (A2)$$

where μ_0 is called the vacous magnetic permeability and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

The magnetometer used in this study detects the magnetic flux density perpendicular to a detection surface of the magnetometer, and the detected magnitude of the magnetic field is given by:

$$H_n = H_\rho \cdot \sin \phi_2 + H_\phi \cdot \cos \phi_2 \quad (A3)$$

The magnitude H_t of the magnetic flux density when the detection surface is parallel to the direction of the magnetic pole, and H_v when the detection surface is perpendicular to the direction of the magnetic pole are given respectively by the following equation:

$$H_t = H_\rho \cdot \sin \phi_1 + H_\phi \cdot \cos \phi_1 = \frac{3M}{4\pi\mu_0} \frac{\sin \phi_1 \cos \phi_1}{\rho^3} \quad (A4)$$

$$\begin{aligned} H_v &= H_\rho \cdot \sin \left(\frac{\pi}{2} + \phi_1 \right) + H_\phi \cdot \cos \left(\frac{\pi}{2} + \phi_1 \right) \\ &= \frac{M}{4\pi\mu_0} \frac{1}{\rho^3} (2 \cos^2 \phi_1 - \sin^2 \phi_1) \end{aligned} \quad (A5)$$

In this study, to measure the radial displacement in the specimen with vertical drain, the magnet was buried in the specimen so that the direction of a magnetic pole would coincide with the vertical direction z , as shown in Fig. A2. The magnetic flux densities H_7 and H_8 measured by magnetometers Ch7 and Ch8 are given respectively by the following equation:

$$H_7 = \frac{3M}{4\pi\mu_0} \frac{(z_7 - z)[(r_7 - r \cos \theta_7)^2 + (r \sin \theta_7)^2]^{1/2}}{[(r_7 - r \cos \theta_7)^2 + (r \sin \theta_7)^2 + (z_7 - z)^2]^{5/2}} \quad (A6)$$

$$H_8 = \frac{M}{4\pi\mu_0} \frac{2(z_8 - z)^2 - (r_8 \cos \theta_8 + r)^2 - (r_8 \sin \theta_8)^2}{[(r_8 \cos \theta_8 + r)^2 + (r_8 \sin \theta_8)^2 + (z_8 - z)^2]^{5/2}} \quad (A7)$$

Since two magnetometers and one magnet have been arranged on the same vertical section in this study, $\theta_7 = \theta_8 = 0$. Thus, Eqs. (A6) and (A7) are rewritten as:

(a) Plane section

(b) Cross section

Fig. A2. Position of magnet and magnetometers

$$H_7 = \frac{3M}{4\pi\mu_0} \frac{(r_7 - r)(z_7 - z)}{[(r_7 - r)^2 + (z_7 - z)^2]^{5/2}} \quad (\text{A8})$$

$$H_8 = \frac{M}{4\pi\mu_0} \frac{2(z_8 - z)^2 - (r_8 + r)^2}{[(r_8 + r)^2 + (z_8 - z)^2]^{5/2}} \quad (\text{A9})$$

Figure A3 shows the theoretical solution of the magnetic field at Ch7. In this figure, the magnitude of the magnetic flux density is represented by $(4\pi\mu_0/M)H_7$. The magnetic flux density obtained from the theoretical solution is similar to the calibration curves, which are the relationships between the magnetic flux density measured by the magnetometer and the vertical position of the magnet buried at specified radial positions (Baek et al., 2004).

Figure A4 shows the relationship between the theoretical value of the magnetic flux density and the measured values in the calibration tests at Ch7 when the magnet is located at the same position in the specimen. It is recognized from this figure that a linear relationship exists between the theoretical values $(4\pi\mu_0/M)H_7$ calculated by Eq. (A8) and the measured values V_7 by the magnetometer at Ch7.

Fig. A3. Theoretical value of magnetic flux density

Fig. A4. Relationship between theoretical value of the magnetic flux density and measured value of magnetometer

Measuring Method by Combining Theoretical Solution of Magnetic Field with Calibration Curves

From the results mentioned above, the relationship between the value of the magnetometer and the position of the magnet is given by the following equations. In these equations, a and b are determined from Fig. A4 considering the degradation of the magnet with time.

$$V_7 = a_7 \cdot \frac{(r_7 - r)(z_7 - z)}{[(r_7 - r)^2 + (z_7 - z)^2]^{5/2}} + b_7 \quad (\text{A10})$$

$$V_8 = a_8 \cdot \frac{2(z_8 - z)^2 - (r_8 + r)^2}{[(r_8 + r)^2 + (z_8 - z)^2]^{5/2}} + b_8 \quad (\text{A11})$$

These equations indicate that the radial position of the magnet in the specimen could be calculated when the value of the magnetometer and the vertical position of the magnet are known. Therefore, as the value of the magnetometer could be measured during the whole consolidation process and the vertical position of the magnet could be determined from the settlement of the specimen because of the uniform distribution of the vertical strain in the specimen during the three-dimensional consolidation process, the radial displacement in the specimen δ_r could be determined as the radial movement of the magnet Δr using Eqs. (A10) and (A11).

In addition, the magnet is considered to rotate slightly with the compression of the specimen even in one-dimensional consolidation process (vertical flow only) caused by a small heterogeneity in the specimen, etc. Figure A5 shows the relationship between the increment of the

Fig. A5. Normalized movement by settlement of magnet

radial movement normalized by the increment of vertical movement $\Delta r/\Delta z$ and the settlement of magnet z . As shown in Fig. A5, $\Delta r/\Delta z$ varies cyclically, and the average value changes almost linearly with the increase in z .

Figure A6 shows the periodic movement of buried magnet in one-dimensional consolidation process. It is found from this figure that $\Delta r/\Delta z$ varies periodically with a period of about one day. This variation with time is considered to be caused by the cyclic change in measuring environment which is influenced by the change in temperature, human activities, etc. with a period of one day. This variation is fitted by a sine function with a period of one day as shown in Fig. A6.

Therefore, the variation of $\Delta r/\Delta z$ under one-dimensional consolidation condition is expressed by the following equation considering the change in the vertical position and the periodic variation with a period of one day.

Fig. A6. Periodic variation of buried magnet

$$\left(\frac{\Delta r}{\Delta z} \right)_{1D} = \alpha \cdot \Delta z + \beta + \gamma \cdot \sin \left(\frac{2\pi}{1440} (t + t_0) \right) \quad (A12)$$

where α and β are determined by the minimum square fitting method in Fig. A5, and γ and t_0 are determined from Fig. A6, respectively.

Finally, since it can be regarded that the variation of $\Delta r/\Delta z$ in one-dimensional consolidation process also occurs in three-dimensional consolidation process, the radial displacement in the specimen in three-dimensional consolidation process δ_r is calculated by the following equation using the difference between $(\Delta r/\Delta z)_{1D}$ obtained by Eq. (A12) for one-dimensional consolidation condition and $(\Delta r/\Delta z)_{3D}$ measured in three-dimensional consolidation process.

$$\delta_r = \left\{ \left(\frac{\Delta r}{\Delta z} \right)_{3D} - \left(\frac{\Delta r}{\Delta z} \right)_{1D} \right\} \cdot \Delta z \quad (A13)$$