A NEW NONLINEAR HYSTERETIC RULE FOR WINKLER TYPE SOIL-PILE INTERACTION SPRINGS THAT CONSIDERS LOADING PATTERN DEPENDENCY

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ABSTRACT

We propose a new hysteretic rule for p - y curves to be used in the dynamic analysis of deep foundations. We first examine the results of past lateral cyclic pile load experiments to clarify the characteristics that are to be modeled in the load transfer hystereses in p - y curves. We then undertake an analytical study of soil element behavior when subject to cyclic passive (compressive) and active (extensile) deformation. We found that the soil resistance intensity to piles varies with different cyclic loading patterns, as the stress-dilatancy behavior in soil varies with cyclic loading patterns. We developed a new hysteretic rule that satisfies the observed dominant characteristics. Although the proposed hysteretic rule has its background in the peak-oriented rule, it is further extended to be a function of the loading pattern. Numerical tests using the proposed model showed that the model is capable of reproducing observed differences in the behavior of piles subjected to fully-reversed cyclic loading and one-sided cyclic loading, even though the typical peak-oriented rules are unable to predict these outcomes.

Key words: dynamic interaction, earthquake, horizontal load, pile, (p - y curve) (IGC: E12/E14/H1)

INTRODUCTION

Seismic design methods and structural design codes have been developed on the basis of lessons learned from past damage associated with large earthquakes. A recent trend in seismic design to protect against the effects of large earthquakes in earthquake-prone regions such as Japan is to control structural damage so that the structure can quickly return to service, considering not only the strength of the structure but also the ductility when subjected to large earthquakes. A practical method of assessing the ductile response of foundations to large earthquakes is therefore required.

In the code of practice for the design of highway bridge foundations in Japan, nonlinear ductile behavior of the foundation is examined by the pushover analysis using the Beam on Nonlinear Winkler Foundation concept. The Winkler hypothesis herein is taken to mean that the function of the soil resistance p at a particular depth is independent of the soil properties and pile displacement at other locations (Poulos and Davis, 1980). The so-called p - y curves express the lateral soil-pile interaction at each depth, with p representing the soil resistance stress to a pile, and y representing the corresponding displacement of the pile relative to the far-field. The pile shaft (or deep foundation body) is modeled as a beam,

considering that the pile shaft may become plastic.

A future alternative to the current seismic design of pile foundations is likely to involve extending the current design using pushover analysis in the Beam on Nonlinear Winkler Foundation concept toward the nonlinear dynamic analyses of foundations in the time domain. In such a case, a nonlinear hysteretic p - y curve is necessary.

p - y curves have previously been developed empirically (see review by Reese, 1993). Many monotonic p - y curves have been proposed based on the results of lateral load experiments of piles (e.g., Reese et al., 1974; Matlock, 1970; Kubo, 1965; Fukui et al., 1997). Many studies have also sought to incorporate the dynamic effects of stiffness and damping into p - y curves, based on elastic wave theory and finite element calculations (e.g., Novak, 1974; Nogami and Novak, 1980; Gazetas and Dobry, 1984). Other studies have attempted to develop the nonlinear hysteretic p - y curve (e.g., Nogami et al., 1992; Boulanger et al., 1999; Curras et al., 2001; Kondou et al., 1998; Maki, 2002). In terms of soil nonlinearity, most previous studies have simply adopted conventional hysteretic rules such as Masing's rule, and have verified them by comparing the numerical pile responses with experiments that involve piles subjected to harmonic loading or fully-reversed cyclic loading. However, it is considered that there is insufficient data to confirm that such a conven-

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tional hysteretic rule represented by Masing's rule mostly provides relevant results for nonlinear soil-pile interactions when piles are subjected to random cyclic loading. In reality, not only are earthquake motions random, but structural aspects can also cause various loading patterns. For example, in highway bridges and overpasses, the transition from reversed-cyclic loading to one-sided cyclic loading is anticipated in the situation where $P-\Delta$ effects appear, resulting from damage to the bottom of a pier or the top of piles; the $P-\Delta$ effect results from secondary overturning moment loads to the pier and foundation associated with the movement of the point of gravity vertical loads applied to the superstructure. Although some past studies have compared the results of experiments involving piles subjected to earthquake motions with numerical results calculated via a hysteretic $p-y$ curve (Boulanger et al., 1999; Curras et al., 2001), additional systematic research is also required to determine hysteretic mechanisms for various loading patterns.

The aim of the current paper tackles to investigate the hysteresis of $p-y$ curves, based on both an experimental study of pile behavior and an analytical study of soil element behavior. We propose a new hysteretic rule that can be used in numerical simulations within a framework of total stress analysis. First we review past experiments that involved piles subjected to different patterns of lateral loading. We focus on the hysteretic $p-y$ curves obtained by Shirato et al. (2006) from the pile load tests conducted by Kimura et al. (1998) and Fukui et al. (1998). We then conduct an analytical investigation using a simple model at the soil element level. An important finding of this study is that the hysteresis of $p-y$ varies with different loading patterns; we then model this phenomenon. A new hysteretic rule for $p-y$ curves is developed that accounts for the modeled characteristics described above. Finally, we compare experimental results with the numerical accuracy of the behavior of single piles embedded in sand and subjected to either lateral reversed cyclic loading or lateral one-sided cyclic loading; this is considered in terms of overall pile behavior and load transfer hysteresis between soil and pile.

LOADING PATTERN DEPENDENCY OF $p-y$

Loading Pattern Dependent Soil Resistance Observed in Previous Experiments of Cyclic Lateral Pile Loading

From previous cyclic lateral loading experiments of piles, we first review an experiment of single piles subjected to either fully-reversed or one-sided lateral cyclic loading. This experiment was originally conducted by Kimura et al. (1998) and Fukui et al. (1998); experimental data analysis was refined by Shirato (2004) and Shirato et al. (2006). Figure 1 shows the experiment setup. A saturated soil deposit was made from Kashima sand ($D_{50}=0.67$ mm and $U_c=2.66$) in a deep test pit at the Foundation Engineering Laboratory in the Public Works Research Institute in Tsukuba, Japan. The deep test pit has a length of 4 m, a width of 3 m, and a depth of 11 m.

Fig. 1. Illustration of the test setup (units: mm)

The sand deposit was very loose because of the construction procedure, with an average relative density of 17.3% and average dry unit mass of 1.51 t/m³. The internal friction angle obtained by drained triaxial compression tests was 39°, with a relative density of 30% and a confining stress of 29.4 kN/m². A small strain shear modulus G_0 was obtained from cyclic triaxial compression laboratory tests:

$$G_0 = 199.63 (\sigma'_c/98)^{0.65} \times 10^3 \text{ (kN/m}^2\text{)}, \quad (1)$$

where σ'_c is confining pressure expressed in kN/m².

The experiment involved steel pipe piles with a diameter of 318.5 mm and an embedded depth of 8 m. Piles with two different wall thicknesses were used: 10.3 mm and 5.6 mm. A pinned device was attached to the bottom of each pile to clarify the boundary condition. Strain gauges were arranged on the sides of the piles to estimate $p-y$ curves. Lateral cyclic loads were applied to the piles at an average rate of loading of 60 mm/min. The point of application of the lateral cyclic loading was 0.7 m above the initial sand deposit surface (ground level).

The experimental cases examined herein are listed in Table 1. Case S1 used a pile with a larger wall thickness than the other tested piles and was subjected to fully-reversed cyclic loading. Cases S2 and S3 used piles with the same details and were subjected to fully-reversed cyclic loading and one-sided cyclic loading, respectively. As these experiments dealt with two extreme cyclic load patterns in random loading, we expected more detailed data than that generated in past tests; these data could then be used to model the hysteretic rule of $p-y$. The material element test results on the pile specimens are also

tabulated in Table 1. In all cases, the amplitude of the specified displacement was gradually increased from δ to $n \times \delta$ ($n = 2, 3, 4, \dots$), in which $\delta = 15$ mm. Three cycles were repeated at every displacement amplitude. During each experiment, part of the soil surface around the pile subsided markedly and water appeared inside the subsided area.

Figure 2 shows observed p - y curves at a depth of GL -0.96 m for all cases. The results shown in Fig. 2 are sourced from earlier stages in the experiments because of the increased reliability in data processing for p - y relationships. In the fully-reversed cyclic loading case,

- the p - y path always approaches the previous peak displacement point on the envelope curve, as predicted by Masing's rules and other conventional peak-oriented hysteretic rules; the envelope curves are denoted by dashed lines in Fig. 2.
- unloading paths are stiffer than the reloading paths, and the unloading gradient appears to be maintained even when the peak displacement level increases.

The term 'envelope curve' is defined as a curve that traces the peak points of the first loops. The term 'loading' refers to the case when the increment of $|p|$ is positive, while the term 'unloading' refers to the case when the increment of $|p|$ is negative. In the one-sided cyclic

loading case (Case S3),

- the mobilized soil resistances are clearly smaller than those for the fully-reversed cyclic loading cases (Cases S1 and S2) at the same displacement levels; the conventional hysteretic rules are not able to reproduce this tendency.

The load-displacement curves at the loading point also show the same trends varying with cyclic loading patterns.

Similar loading-pattern dependent differences in lateral soil resistance are evident in the results of the experiments conducted by Agaiby et al. (1992) and Mayne et al. (1992). The drained and undrained lateral behavior of drilled shafts were investigated in various densities of sand deposits and various consolidation conditions of clay deposits; the model drilled shafts were subjected to monotonic and various patterns of one-sided cyclic loading. The lateral resistance of the drilled shafts in one-sided cyclic loading tended to be smaller than that in monotonic loading. Therefore, it is considered a general tendency that the lateral soil resistance varies with the cyclic loading pattern, independent of soil classification.

Source of Loading Pattern Dependency and Modeling of Predominant Characteristics in Loading Pattern Dependency

To examine the soil resistance mobilized in the near-field as simply as possible, we used a model that simplifies the soil-pile interaction system in a way as shown in Fig. 3. A very thin slice and small depth rigid body sandwiched between two very thin slice and small depth soil blocks in a plane strain condition was subjected to cyclic loading under a vertical compression stress, assuming a pile shaft, surrounding soil, and an overburden stress in the ground. The major and minor principle stress axes are x_1 and x_2 , respectively; the x_1 -direction corresponds to the horizontal direction in the field. Each soil block is modeled using one element. The boundary conditions are given such that strains in the x_1 and x_2 directions can be freely mobilized. Accordingly, only

Fig. 2. Comparison of p - y hysteresis between the three experimental cases (GL -0.96 m)

Fig. 3. Simple model used to examine the influence of soil nonlinearity on soil-pile interaction (from Shirato et al., 2006)

Table 1. Description of experimental cases

Case	Cyclic loading pattern	Pile wall thickness	Yield stress	Young's modulus
S1	Fully-reversed	10.3 mm	562 N/mm ²	227 kN/mm ²
S2	Fully-reversed	5.6 mm	589 N/mm ²	238 kN/mm ²
S3	One-sided

Fig. 4. Plane strain cyclic compression-extension test results for Toyoura sand from Masuda et al. (1999)

horizontal displacement is fixed at both ends. The load-displacement responses of the rigid body shown in Fig. 3 are calculated for many displacement histories of the rigid body.

Soil in the near-field around the piles can be assumed to be primarily subjected to cyclic passive (compression) and active (extension) deformation because of the difference in the lateral displacement between soil and pile, not only in cyclic load experiments but only in the field during earthquakes; hysteretic p - y curves are assumed to be associated with the nonlinear behavior of soil subjected to cyclic compression-extension deformation. We therefore use a constitutive model based on the cyclic compression-extension plane strain element test results for sand derived by Masuda et al. (1999). Their tests were conducted under saturated and drained conditions. Figure 4 shows typical test results obtained by Masuda et al. (1999), in which σ_{11} and σ_{22} are principal stresses, ϵ_{11} and ϵ_{22} are corresponding strains, γ is shear strain, ϵ_{vol} is volumetric strain, and ϕ_{mob} is the mobilized friction angle. As indicated by Masuda et al., the stress-strain relationship of the sand blocks can be described by a combination of a $\sin \phi_{mob}$ - γ relationship and a stress-dilatancy relationship. In the present paper, the $\sin \phi_{mob}$ - γ relationship is given with a hyperbolic function backbone curve and Masing's hysteretic rule, in which;

$$\sin \phi_{mob} = (R - 1) / (R + 1), \quad \gamma = \epsilon_{11} - \epsilon_{22}, \quad (2)$$

where R is the principal stress ratio defined by $R = \sigma_{11} /$

Fig. 5. Analytical results of the effects of stress-dilatancy behavior of sand on p - y relationships

σ_{22} , ϕ_{mob} is the mobilized internal friction angle, γ is shear strain, and ϵ_{ij} is the (i, j) component of strains. The backbone curve (or the monotonic loading curve) is symmetric about the origin:

$$\sin \phi_{mob} = \frac{\gamma/\gamma_r}{1 + \gamma/\gamma_r} \times \sin \phi_{max}, \quad (3)$$

in which ϕ_{max} is the maximum internal friction angle and γ_r is the reference strain. We assume that $\phi_{max} = 30^\circ$ and $\gamma_r = 0.0002$. The stress-dilatancy relationship is Rowe's stress-dilatancy law (Rowe, 1962), which is expressed with a combination of:

$$R_1 = K_1 D_1, \quad R_1 = \frac{\sigma_{11}}{\sigma_{22}}, \quad D_1 = \frac{d\epsilon_{22}^p}{d\epsilon_{11}^p}, \quad (\text{when } \Delta\sigma_{11} > 0), \quad (4)$$

and

$$R_2 = K_2 D_2, \quad R_2 = \frac{\sigma_{22}}{\sigma_{11}}, \quad D_2 = \frac{d\epsilon_{11}^p}{d\epsilon_{22}^p}, \quad (\text{when } \Delta\sigma_{11} < 0) \quad (5)$$

in which the values of the coefficients of dilatancy, K_1 and K_2 , are assumed to be 3.5. Note that Shirato et al. (2006) document the results of more detailed investigations with several combinations of sand parameter values and several loading patterns.

Figure 5 shows some of the results from the present analysis. The left-hand diagram shows results for fully-reversed cyclic loading, while the right-hand diagram shows results for one-sided cyclic loading. The dashed lines represent the corresponding monotonic loading curves. In particular, the p - y curve for the case of one-sided cyclic loading trends is considerably lower than the previous peak states.

Figure 6 shows the calculated $\sin \phi_{mob}$ - γ relationships in the sand blocks, in which a plane strain sand block on the positive side with respect to $y=0$ is referred to as a positive-side sand block, and that existing on the negative side with respect to $y=0$ is referred to as a negative-side sand block. The center points of loops of $\sin \phi_{mob}$ - γ shift to the negative side of γ because the negative dilatancy (i.e. volume contraction, $\Delta\epsilon_{vol} > 0$) is mobilized at every displacement reversal. The magnitude of the shift varies with loading pattern: the mobilized value of $\sin \phi$ in the positive-side sand block in the one-sided cyclic loading

case, i.e. the passive-side sand block, is smaller than the mobilized value in the sand blocks subjected to fully-reversed cyclic loading. Accordingly, the mobilized soil resistance stress, p , in the one-sided cyclic loading case decreased to a value clearly smaller than that in the fully-reversed cyclic loading case.

Soil stress-dilatancy characteristics played an important role in the observed dependence of loading pattern on soil resistance. The ε_{vol} - γ relationship shown in Fig. 4 indicates the following points. For fully-reversed cyclic deformation in both compression and extension, a large part of the incremental negative dilatant strain ($\Delta\varepsilon_{vol} > 0$) that is generated during a compressive deformation phase (passive phase) is countered during the following extensile deformation phase (active phase). For one-sided cyclic deformation only in compression, the residual negative dilatant strain accumulates and the negative dilatancy effect becomes stronger as it is not countered during the extensile phase. A larger axial deformation is therefore required to mobilize large shear strain and shear strength in the one-sided cyclic loading case. Ultimately, the mobilized soil resistance to a pile is assumed to be dependent on the degree of eccentricity of the cyclic loading direction.

Based on the calculated results with various displacement histories of y in the model, we recognize the following trends, as illustrated in Fig. 7(a). In a p - y cycle, the p - y curve returns to a point close to the preceding

reversal point when the plus and minus intensities of p at the preceding and most recent reversal points are almost equal (trend 1 in Fig. 7(a)); as the difference in $|p|$ of the preceding and most recent reversal points increases, the reversed p - y curve trends are lower (trends 2 and 3); and if the most recent reversal occurs in the vicinity of the preceding reversal point, the p - y curve returns to the vicinity of the preceding reversal point (trend 4). These phenomena can also be reinterpreted in the manner illustrated in Fig. 7(b). The backbone curve of p - y virtually drifts from the original position; the degree of drift is related to the degree of eccentricity in the loading direction of the previous cyclic loading history. While the interpretation in Fig. 7(a) is based on a viewpoint of soil resistance stress, that in Fig. 7(b) is based on a viewpoint of displacement.

As described in the above review of previously published pile load experimental results, variation in lateral soil resistance with cyclic loading pattern is observed even in cohesive soil. Although stress-dilatancy behavior is still considered to play a key role in loading pattern dependency in cohesive soil, we encourage future investigations that seek to confirm this hypothesis via relevant element test results. Note that the above analysis does not consider the states in which the influence of the increase in excess pore pressure on the overall pile behavior cannot be neglected.

PROPOSED HYSTERETIC RULE FOR p - y CURVES FOR SINGLE PILES

Here we propose a new hysteretic rule for p - y as a function of the loading pattern. This proposed model can be widely used within a framework of total stress computation, in which the generation of excessive pore pressure in the soil can be ignored.

To describe the various conditions of the nonlinear relationship of p - y , we describe the hysteretic rule using combinations of piecewise straight branches. We disregard inertial effects on the soil wedge in the interaction zone, because considering temporal variations in the inertial effect makes the present problem complex and may also mask the essential elements of the present problem.

Fig. 6. Calculated $\sin \phi_{mob}$ - γ relationships for the sand blocks, considering stress-dilatancy behavior

Fig. 7. Characteristics of degradation in soil resistance stress p with loading pattern

Fig. 8. Backbone curve of the p - y relationship

Modeling of Backbone Curve

This paper simplifies the backbone curve as an elasto-perfectly plastic bilinear curve, as shown in Fig. 8, in which p_U is the ultimate soil resistance stress and y_y is the displacement level that the soil resistance stress achieves at the ultimate value on the backbone curve. We expect that the hysteretic rule, which will be introduced later in the text, can be applied to any shape functions of the backbone curve; the choice of the shape of the backbone p - y curve is optional.

The ultimate soil resistance stress can be defined as a function of the passive resistance to the pile. Therefore, a simple expression of the ultimate soil resistance stress is

$$p_U = \alpha_p p_P \quad (6)$$

where p_P is the ultimate passive earth pressure and α_p is a modifier for the three-dimensional effects of soil resistance on a pile. The value of α_p can be estimated from computations of the behavior of single piles subjected to monotonic loading or fully-reversed cyclic loading with a gradually increasing amplitude in an inverse manner. If we derive the ultimate passive earth pressure in seismic situations from the Japanese Specifications for Highway Bridges, we arrive at the following passive earth pressure coefficient K_{EP} :

$$K_{EP} = \frac{\cos^2 \phi}{\cos \delta_E \left(1 - \sqrt{\frac{\sin(\phi - \delta_E) \sin \phi}{\cos \delta_E}} \right)^2} \quad (7)$$

where δ is the friction angle between the pile shaft surface and the soil, assumed to be $-\phi/6$, which is also employed in practice for estimating the ultimate soil resistance stress for piles subjected to cyclic loading, and it is used hereafter in this study. Another method of estimating p_U is to use an analytical solution based on admissible plastic flows in the area in front of an underground pile. For example, Kishida and Nakai (1979a, 1979b, 1977), Reese (1958), Reese et al. (1974), Broms (1964a, 1964b), and Koda et al. (2000) provided analytical solutions with admissible plastic flow fields in the

surrounding soil in front of a horizontal loaded pile. Both methods will be compared in the following numerical examples.

The initial gradient k_H is set by multiplying the reference gradient k_0 by an empirical modifier, α_k , so that

$$k_H = \alpha_k k_0, \quad (8)$$

$$k_0 = \frac{E_0}{B_0} \times \left(\frac{B}{B_0} \right)^n, \text{ and} \quad (9)$$

where E_0 is the small strain deformation coefficient of soil, n is a constant that represents the loading width dependency of subgrade reaction coefficients, B is the foundation width (i.e. pile diameter in the case of piles), B_0 is the reference width in terms of the loading width dependency, and α_k is a modifier used to fit the reference subgrade reaction k_0 to the assumed bi-linear curve. The Japanese Specifications for Highway Bridges employ $B_0 = 0.3$ m and $n = -3/4$, based on the experimental study of Yoshida and Yoshinaka (1979). The value of α_k can also be determined by a load test of a single pile.

Equations (8) and (9) are similar to the subgrade reaction coefficient equation in the Japanese Specifications for Highway Bridges (Japan Road Association, 2002), however, Eqs. (8) and (9) are also reasonable in terms of dynamic interaction. Gazetas and Dobly (1984) and Kavvadas and Gazetas (1993) conducted comparative finite-element studies of harmonic pile-head loading and reported that as a first approximation, the subgrade reaction coefficient k_H can be considered to be frequency-independent and expressed as a multiple of the local soil stiffness E_s ,

$$k_H \approx a E_s, \quad (10)$$

in which a is a frequency-independent modifier.

With respect to foundation scale effects on the subgrade reaction, Koseki et al. (2001) used horizontal plate loading tests with plates with different sizes to demonstrate that the $-3/4$ power law can only be revealed when the subgrade reaction coefficients are estimated in terms of a certain displacement level of the plate; $n = -1$ can be observed when the subgrade reaction coefficients are estimated by referring to subgrade reactions at a certain strain level, which is defined by dividing plate displacement by the plate width. Further research is required to clarify the manner in which the loading width dependency should be dealt with in setting the initial gradient of the backbone curve.

Basic Hysteretic Rule without Loading Pattern Dependency

The hysteretic rule for fully-reversed cyclic loading is referred to herein as the basic hysteretic rule. The basic hysteretic rule is characterized by the following two major features, based on the above observations of experimental and analytical p - y curves.

1. The amplitude of soil resistance p during fully-reversed cyclic loading with a fixed displacement amplitude is constant.

2. A reversed p - y curve from the backbone curve trends toward a point corresponding to the absolute largest displacement $|y|_{\max}$ on the backbone curve in the other load and displacement direction.

An illustrative explanation of these two points is shown in Fig. 9. In Fig. 9, the p - y path runs through the origin $\rightarrow a \rightarrow b \rightarrow c \rightarrow d \rightarrow a \rightarrow e \rightarrow f \rightarrow g \rightarrow h \rightarrow e \rightarrow Y \rightarrow i \rightarrow j \rightarrow k \rightarrow l \rightarrow i \rightarrow m$. The paired points a and c , e and g , and i and k share the same absolute values of p and y .

The points corresponding to the absolute largest displacement $|y|_{\max}$ on the backbone curve are hereafter referred to as global control points. A pair of global control points is located symmetrically about the origin, with one point on the positive side and the other on the negative side. The global control point on the positive side is referred to as point C_1 , with that on the negative side referred to as point C_2 . The displacement amplitude of point C_1 is $y = y_{C1} \equiv |y|_{\max}$, while that of point C_2 is $y = y_{C2} \equiv |y|_{\max}$. The corresponding values of soil resistance stress, p , are referred to as p_{C1} and p_{C2} , respectively. For example, for the path $e \rightarrow f \rightarrow g$ in Fig. 9, $|y|_{\max}$ is the

Fig. 9. The basic hysteretic mechanism

absolute value of the displacement amplitude of point e , $|y_e|$. The corresponding negative global control point is defined as point g , i.e. $y_g = y_{C2} = -|y|_{\max} = -y_{C1} = -y_e$. The curves that connect the global control points, $C_i C_j$, are defined hereafter as the external curves for which i and j have a value of 1 or 2.

An unloading path that departs from the last displacement reversal point and trends toward $p=0$ is a straight line with an unloading gradient of k_{Hunl} . k_{Hunl} is provided by;

$$k_{Hunl} = \beta k_0. \quad (11)$$

As shown by pile experiments (Fig. 2) and analyses using the sand block-rigid body-sand block model (Fig. 5), the unloading gradient can be assumed to be independent of the displacement level and loading history. The unloading path is considered to be strongly associated with the unloading elastic modulus of soils; its gradient has the same order of small strain level elastic modulus. Ultimately, $\beta=1$ is assumed in Eq. (11).

When a displacement reversal occurs at a point other than a control point, the subsequent path trends towards the preceding displacement reversal point, as shown in Fig. 10. However, when strain reversal occurs numerous times on a path from one global control point to the other or back to the original control point, the coding of the computer program becomes cumbersome because the associated past displacement reversal points must be memorized and unfolded whenever displacement reversal occurs. Accordingly, additional rules are introduced, as illustrated in Fig. 11. As shown in Fig. 11(a), the displacement reversal point on an external curve is referred to as a local control point i , C_{Li} (p_{CLi} , y_{CLi}); the following p - y curve, $C_{Li} C_i$, that trends toward the preceding displacement reversal point (i.e. the global control point C_i) is referred to as the reference internal curve. As shown in Fig. 11(b), when a displacement reversal occurs at a point on the reference internal curve $C_{Li} C_i$, the displacement reversal point is defined as the other local control point

Fig. 10. Schematic diagrams of p - y behavior following a displacement reversal in the basic hysteretic rule (All p - y curves start from the origin)

Fig. 11. Behavior of internal loops of p - y in the basic hysteretic rule

C_{Lj} . Then, as shown in Fig. 11(c), the p - y loops that follow the displacement reversal from a local control point C_{Lj} always trend toward the local control points C_{Li} or C_{Lj} , depending on the loading direction. While p and y continue to respond in the area between $y = y_{CL1}$ and $y = y_{CL2}$, local control points are not renewed. For example, the p - y path trends from the last displacement reversal point R toward the local control point C_{L2} on the reference internal curve rather than the preceding displacement reversal point R' .

Loading Pattern Dependency

As shown in Fig. 7, the path of p - y from the last displacement reversal point does not always trend toward the preceding reversal point, especially in one-sided cyclic loading. We therefore introduce a special new rule called the deterioration rule to account for the p - y behavior as a function of eccentricity in cyclic loading direction.

When assuming different p - y cycles from the global control point C_1 , as illustrated in Fig. 12, the reversed loading paths from the external curve must trend toward points that are different from the global control point C_1 , depending on the degree of eccentricity in the loading direction. This is based on the behavior of p - y cycles modeled in Fig. 7. Accordingly, the proposed model introduces the target point T_1 and produces a reversed loading path from the external curve $C_1Z_1C_2$ that always trends toward the target point T_1 , as shown in Fig. 12. The reversed curves from the external curve comprise an unloading branch with a gradient of k_{Hunl} and a straight branch that trends toward T_1 from $p = 0$. The target point T_1 is defined as the intersection point of the reloading line from the perfectly unloaded point Z_1 of the global control point C_1 and the extension line of the external curve $C_2Z_2C_1$ (i.e. the extension line of Z_2C_1).

The gradient k_{Hr} is referred to as the reference reloading gradient, while the line that connects the perfectly unloaded point Z_i to the target point T_i with a reference reloading gradient is termed the reference reloading line. The reference reloading gradient is generalized as;

$$k_{Hr} = Mk_{Hunl} \quad (12)$$

where M is a modifier to adjust the degree of the deterioration of p . The value of M must also be given to satisfy the condition that the line of Z_2C_1 must have a cross point with the extension of the reference reloading curve in the

Fig. 12. Proposed hysteretic rule with loading pattern dependency (Part 1)

first quadrant. Therefore, we have the following equation:

$$M > \frac{\alpha_k}{2 - \alpha_k} \quad (13)$$

Figure 13 shows the p - y loops produced by the proposed model for fully-reversed cyclic loading and perfectly one-sided cyclic loading. As shown in Fig. 7(b), for perfect one-sided cyclic loading, the response should involve a horizontal shift of the backbone curve on the one-sided cyclic loading side. When we assume that the

Fig. 13. p - y hysteretic loops for two extreme loading patterns in the proposed model

Fig. 14. Proposed hysteretic rule with loading pattern dependency (Part 2)

reference reloading gradient k_{Hr} has a value of the same order as the initial gradient of the backbone curve k_H , the monotonic reloading path from point Z_1 increases with a gradient equivalent to k_H , reaches the backbone curve, and follows $p = p_u$. We therefore considered it appropriate that the value of M in Eq. (12) is taken to be the same as the value of α_k in Eq. (8). Our proposed hysteretic model is ultimately expected to be capable of accounting for the loading pattern dependency arising from stress-dilatancy behavior in the surrounding soil, as modeled in Figs. 7(a) and (b).

When an interaction spring is subjected to a fully-reversed cyclic displacement with gradually increasing or stationary amplitude, deterioration of p does not appear at peak points, regardless of M . We also note that for $M = 1.0$, the loading pattern dependency is expelled.

Figure 14 shows an example of setting the target point on the negative side of p . The target point T_2 can be defined in the same manner as the setting of T_1 , using the perfectly unloaded point Z_2 from the negative global control point C_2 , the gradient k_{Hr} , and the corresponding external curve $C_1Z_1C_2$.

The behavior of p - y prior to reaching the ultimate soil resistance p_u can also be specified by applying the above rules. An example is illustrated in Fig. 15; notations used in this figure are explained in the following explanation of special rules.

When an internal p - y path crosses the external curve and changes the direction of travel, the global control points C_1 and C_2 are renewed, as shown in Fig. 16. In Fig. 16, the prime denotes the values or points that have

Fig. 15. An example of the behavior of p - y , with consideration of the deterioration rule, prior to reaching the ultimate soil resistance

Fig. 16. Updating of global control points for the case in which an internal path crosses the external curve and changes its direction of travel

Fig. 17. Behavior of internal curves when the deterioration rule is considered

been renewed along with the most recent displacement reversal. When the reversal point is not on the backbone curve, C_i must be set as the intersection of the backbone curve and a line passing through the reversal point, with a gradient of $k_{H_{unl}}$. One way to include this rule into the subroutine for the proposed hysteretic rule in a computer code is that the subroutine continues to monitor the virtual perfectly unloaded point of the coordinate of an instantaneous displacement reversal point, z_c ($0, y_{zc}$). Then, the subroutine checks the relationship of y_{zc} with respect to the y -coordinates of the stored Z_1 ($0, y_{z1}$) and Z_2 ($0, y_{z2}$), in which $y_{z1} = -y_{z2}$. For $\Delta y > 0$ following the last displacement reversal, in which Δy is the instantaneous increment of y , all of the points that are necessitated to define the hysteretic rule, points C_i , Z_i , and T_i , are updated if y_{zc} is less than y_{z2} . In the case of $\Delta y < 0$ after the last displacement reversal, all of the necessary points are updated if y_{zc} is larger than y_{z1} . In addition to the update of C_1 and C_2 , external curves and points Z_1 and Z_2 are also updated, and the target points T_1 and T_2 are redefined, as shown in Fig. 16.

As shown in Fig. 17(a), for internal curves, a local target point T_{L1} is set on either the intersection of the local reference reloading line $Z_{L1}T_{L1}$ and the reference internal curve $C_{L1}Z_{L1}B_L$ or the extension thereof, or the intersection of the local reference reloading line $Z_{L1}T_{L2}$ and the external curve $C_1Z_1C_2$ or the extension thereof; the reloading branch that departs from point Z_{Li} is referred to as the local reference reloading line, with the reference reloading gradient k_{Hr} . The point Z_{Li} is the perfectly unloaded point of the last displacement reversal point from the reference internal curve $C_{L1}Z_{L1}B_L$ or external curve $C_1Z_1C_2$. The intersection point of a reference

Fig. 18. Updating of the reference internal curve when the deterioration rule is considered

internal curve to the backbone curve is referred to as point B_L . As shown in Fig. 17, the internal curves that follow a displacement reversal on a reference internal curve $C_{L1}Z_{L1}B_L$ trend toward the local target points T_{L1} and T_{L2} , in principle, depending on the traveling direction. When an internal p - y path reaches point T_{L1} , the following monotonic loading path is bound for the backbone curve from point T_{L1} to point B_L , followed by $p = p_U$.

Figure 18 provides an illustrative description of the response when an internal path crosses the curve $C_{L1}Z_{L1}$ followed by displacement reversal. In Fig. 18, the local control point C_{L1} , point Z_{L1} , and local target point T_{L1} are renewed in the same manner as the global target points; the reference internal curve $C_{L1}Z_{L1}B_L$ is also renewed.

NUMERICAL EXAMINATION OF THE PROPOSED HYSTERETIC RULE

Numerical Simulation of Experimental Results

The numerical model considered here provides a simulation of the experiments with single piles that are described above. The piles are modeled as beams in which the bending resistance, considering pile yielding, is defined by an elasto-perfectly plastic bilinear moment (M)-curvature (ψ) relationship, as shown in Fig. 19, in which M_p is the plastic moment of the cross-section of a pile. The elastic gradient of the M - ψ relationship is given by the flexural rigidity EI , where E is Young's modulus and I is the moment of inertia of the cross-section. The bottom sections of the piles are connected to the hinge boundary condition, in which the horizontal and vertical displacements are fixed and the rotation is free. The element length is set to 120 mm ($=0.38D$, in which D is the diameter of the pile) except for several elements around both ends of the pile.

The pile is assumed to be supported by distributed

Fig. 19. Moment-curvature relationship for steel pipe piles

springs, with the behavior of the springs described by the proposed hysteretic p - y curve; however, the distributed lateral springs are integrated into discrete lateral springs at nodes because of the specifications of the software used in the simulation. The load-displacement relationship of each integrated lateral spring is assigned on the basis the p - y relationship at the depth of the middle point between the node of interest and the neighboring deeper node, and is estimated by simply multiplying the width and length of the beam element by p of the corresponding p - y relationship.

Calibration for the Backbone Curve of the p - y Relationship

First, the backbone curve of p - y is set via comparisons of the results of monotonic loading calculations with the results of the fully-reversed cyclic loading experiment. The experimental result of Case S1 with fully-reversed cyclic loading is used to estimate the values of parameters α_p and α_k in Eqs. (6) and (8) such that the numerical simulation provides the best match with the measured load-displacement curve up to the final displacement level of 16δ ($240 \text{ mm} = 0.75D$).

The internal friction angle ϕ' of the tested sand deposit used for estimating earth pressure coefficients is assumed to be 39° , based on the results of the triaxial compression test described above. The small strain shear modulus G_0 at each depth used for estimating k_H is evaluated using $E=2(1+\nu)G$ and assuming $\nu=0.5$ for simplicity. The confined stress σ'_c is replaced with the mean effective stress:

$$\sigma'_m = \frac{(1+2K_0)\sigma'_v(x_2)}{3} \quad (\text{kN/m}^2), \quad (14)$$

where σ'_v is the effective overburden stress at depth x_2 and K_0 is the coefficient of earth pressure at rest. K_0 is estimated by the typical empirical equation $K_0=1-\sin\phi'$.

Figure 20 shows comparisons of various load-displacement relationships derived from different pairs of α_p and α_k . The envelope curves of the experimental results in

Fig. 20. Comparison of load-displacement relationships at the load point

Fig. 21. Comparison of the ultimate soil resistance stress p_U calculated using $\alpha_p = 4.0$, $\alpha_p = 5.0$, and that obtained by the theoretical method of Kishida and Nakai (1979)

both the positive- and negative-load sides are also shown on the figure; the negative envelope curve is shown after reversing the signs for both load and displacement. The results are not sensitive to variation in α_k (i.e., the initial rigidity of the backbone curve), while loads at any displacement levels vary proportionally with variation in α_p (the ultimate soil resistance stress). These trends indicate that estimation of the strength parameters of surrounding soils governs the calculated ultimate lateral resistance of a pile, as the pile is subject to large displacement and soil resistance become plastic. From an engineering point of view, backbone curves of the hyperbolic function are feasible in hysteretic p - y curves, as the numerical results are not sensitive to the initial gradient of the bi-linear p - y curves. Eventually, we select a relevant value of $\alpha_k = 0.1$ in the range 10^{-2} to 10^{-1} , and select a value of $\alpha_p = 4.0$.

Interestingly, the inversed value of the ultimate soil resistance stress p_U , with $\alpha_p = 4.0$, is very close to a theoretical solution based on assumed admissible plastic flows in front of a pile, as shown in Fig. 21. Figure 21 compares the ultimate soil resistance stress p_U , obtained from Eq. (6), with $\alpha_p = 4.0$ and p_U provided by the theoretical

Fig. 22. Comparisons of load-displacement relationships at the load point calculated using values of p_U calculated from $\alpha_p = 4.0$ and from the theoretical method of Kishida and Nakai (1979a)

Fig. 23. Comparison of the experiment- and simulation-derived bending strain distributions with respect to depth ($\alpha_p = 4.0$, $\alpha_k = 0.1$): the numbers on the figure represent the displacement levels in terms of δ

equations of Kishida and Nakai (1979a). Figure 22 shows the relationships between horizontal load and horizontal displacement at the loading point, which were calculated with the values of p_U obtained using both methods. Experimental results are also plotted for comparison. The two sets of calculated results are in good agreement. We therefore consider that theoretical solutions for the ultimate soil resistance stress may also work well.

We now briefly compare the calculated and measured deformation of the pile and soil resistance distributions with depth. Figure 23 shows the calculated and measured bending strain distributions versus depth at amplitudes of displacement levels of 1, 3, 5, 7, and 9δ . Figure 24 shows the soil resistance stress distributions with respect to depth at the amplitudes of displacement of 1, 3, and 5δ ,

Fig. 24. Comparison of the experiment- and the simulation-derived soil resistance p distribution with respect to depth ($\alpha_p = 4.0$, $\alpha_k = 0.1$): the numbers on the figure represent the displacement levels in terms of δ

in which the pile behaved in the elastic range. Figures 23 and 24 also show the measured values in the first cycles of the same displacement amplitudes, in which the measured values for positive loading displacement are shown after reversing the signs. The calculated and measured distributions averagely are in good agreement, especially for negative loading displacement.

There is an inconsistency between the calculated and measured results in terms of the depth at which the maximum bending strain and soil resistance stress appear. The calculated depths gradually move downward from shallow to deep positions as the interaction springs become plastic. The same trend is observed in the experimental results when the pile was displaced in the negative direction; but not when the pile was displaced in the positive direction. This inconsistency is inferred to result from the lack of consideration of soil densification. In the experiment, the densification effect of the surrounding sand with repeated loading leads to an increase in the mobilized internal friction angle of ϕ ; this in turn leads to an increase in the passive resistance of the sand. However, the mobilized soil resistance is considered to be simultaneously affected by the softening effect of the evolutions of shear strain and negative dilatancy in the soil. The loading pattern dependency of soil resistance will therefore cause a much greater change in pile behavior, as stated above. Therefore, we will not model the effects of soil densification on the backbone p - y curve.

Fully-reversed Cyclic Loading

The calculated responses to fully-reversed cyclic loading with the proposed hysteretic rule for p - y must be independent of M in Eq. (12). Accordingly, a fully-reversed cyclic loading experiment case S1 is simulated with the following two conditions: (1) When the loading pattern dependency in the hysteretic p - y curves is disre-

Fig. 25. Comparison of calculated load-displacement curves at the loading point for $M=1.0$ and $M=0.10$ and experimental results (Case S1)

Fig. 26. Comparison of calculated p - y curves for $M=1$ and $M=0.10$ and experimental results (Case S1)

garded and the hysteretic rule is made a peak-oriented rule, i.e., $M=1$, (2) when the loading pattern dependency was taken into account using $M=\alpha_k=0.1$, i.e., $k_{Hr}=k_H$, in Eqs. (12) and (8).

The calculated load-displacement relationships at the point of loading are compared in Fig. 25. For comparison, the experimental envelope curve is shown with dashed lines in Figs. 25(b) and (c). The results of the calculations and the experiment are almost identical; discrepancies resulting from the difference in M are barely discernible in the calculated results. Calculated and measured p - y curves at depths of -0.48 and -2.04 m (-1.51 and $-6.41D$) are shown in Fig. 26. The proposed hysteretic rule is capable of accounting for the measured p - y loops; there is no discernable difference between the

calculated curves with $M=1.0$ and $M=0.1$. We therefore conclude that the proposed hysteretic rule worked as expected.

One-sided Cyclic Loading

The value of M specifies the deterioration of soil resistance caused by one-sided cyclic loading. Back analyses with various values of M from 1.0 to 0.053 are conducted for the one-sided cyclic loading experiment case S3, in which $M=\alpha_k=0.1$ is considered to be the most likely value based on the phenomenological inference detailed above. Figure 27 shows the measured load-displacement curves at the point of loading and p - y curves at depths of -0.48 and -2.04 m.

The calculated curves are shown in Figs. 28, 29, 30 and

Fig. 27. Measured load-displacement curve and p - y curves (Case S3)

Fig. 29. Calculated load-displacement curve and p - y curves (Case S3, $M=0.25$)

Fig. 28. Calculated load-displacement curve and p - y curves (Case S3, $M=1.0$)

Fig. 30. Calculated load-displacement curve and p - y curves (Case S3, $M=0.10$)

31, in which the experimental envelope curves of Cases S2 (fully-reversed cyclic loading case) and S3 (one-sided cyclic loading case) are also shown for comparison. Note that only the p - y curves are shown up to a displacement level of 6δ , as the reliability of the strain gauge data was

degraded by shaft yielding.

When $M=1$ is used (Fig. 28), the calculated p - y curves follow a peak-oriented rule, the calculated load is overestimated, and the calculated load-displacement curve is close to the fully-reversed cyclic loading result of

Fig. 31. Calculated load-displacement curve and p - y curves (Case S3, $M=0.053$)

Case S2. The three loops at each displacement level are steady and indiscernible. The same tendencies are also observed in the calculated p - y curves. Even for $M=0.25$ (Fig. 29), the calculated load is still overestimated. The calculated load-displacement curves at the loading points are not sensitive to variation in M within the range 1.0 to 0.25.

$M=0.10$ provides the best match for the calculated results, as shown in Fig. 30. Comparison with the experimental results shows that the proposed p - y rule provides an excellent prediction of both the load-displacement curve and the p - y curves. When $M=0.053$ is used (Fig. 31), the load is underestimated. The difference in the numerical results for $M=0.25$ and 0.10 is larger than that for cases $M=1.0$ and 0.25. These trends indicate a possible transition zone in terms of M at approximately $M=0.10$; this is likely to be a relevant value in expressing the loading pattern dependency. The inferences described above, in the development of the new hysteretic rule, are therefore supported by the numerical results that the backbone curve of p - y is apparently shifted depending on the eccentricity of loading direction in one-sided cyclic loading, and that the value of k_{Hr} approaches k_H .

In terms of the relationship between load and displacement at the point of loading, the calculated accumulation of residual displacement when the load is unloaded to zero is less than that observed in the experiment. This is because, once the p - y curves cross $p=0$, the curves head straight for the target point on the corresponding side. In contrast, the actual p - y curves may have a more curved

shape. This difference in the shapes of the p - y curves can cause the discrepancy in the experimental and calculated residual displacement of the pile, however, it is speculated that this does not necessarily mean that the underestimation of the residual displacement of piles always occurs in dynamic analyses, because the proposed model has a tendency to underestimate hysteresis damping.

CONCLUDING REMARKS

In this study we proposed a new hysteretic rule for p - y curves that can be applied to problems within a framework of total stress computation. The proposed hysteretic rule satisfies the characteristics observed in lateral cyclic loading experiments of piles and soil element tests. In particular, the proposed rule takes into account the fact that the soil resistance intensity of piles varies with different loading patterns, as soil dilatancy behavior varies with cyclic loading patterns. Although the proposed model responds to fully-reversed cyclic loading in a similar way to the peak-oriented hysteretic rule, the original deterioration rule is introduced here using a target point; this is capable of accounting for the change in hysteresis with the degree of eccentricity in cyclic loading direction. The proposed hysteretic rule did a good job of simulating the responses of piles embedded in sand and subjected to either fully-reversed cyclic loading or one-sided cyclic loading at the pile top, including simulating the relationships of load versus displacement at the point of load, moment distributions in the piles, and p - y relationships that vary with load patterns; conventional peak-oriented hysteretic rules will never provide appropriate results that show the difference in pile behavior because of the variations in cyclic loading pattern.

The application of a model that uses Winkler-type interaction springs to the dynamic analysis of foundations can be carried out in two stages. In the first stage, far-field excitation is computed using a relevant method such as one-dimensional nonlinear or equivalent linear earthquake response analyses; these are relevant to the cyclic shear deformation mode of the far-field soil, as is expected to be excited during earthquakes. In the second stage, the far-field ground displacement at each depth is input into each end of the distributed interaction springs under the proposed hysteretic rule; this is pertinent to the cyclic compression-extension deformation mode of the near-field soil. Accordingly, the relevant soil deformation modes in the far- and near-fields can be taken into account, respectively, when the proposed hysteretic rule for p - y is used. It is also possible to perform the first and second stages simultaneously in a coupled system.

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