EFFECTS OF STRESS-DILATANCY BEHAVIOR OF SOIL ON LOAD TRANSFER HYSTERESIS IN SOIL-PILE INTERACTION

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ABSTRACT

This paper examines the nonlinear hysteretic characteristics of load transfer between soil and piles for random loading. First, we demonstrate that in past experiments the mobilized lateral soil resistance to piles varies with cyclic loading patterns. The soil resistance mobilized against one-sided cyclic loading is smaller than that mobilized against fully-reversed cyclic loading. We then establish that the stress-dilatancy behavior of soil causes such a loading pattern dependency, based on the mechanical property of soil elements subjected to cyclic compression-extension deformation. Finally, the hysteretic characteristics of load transfer between soil and pile associated with the stress-dilatancy effect is modeled as a function of cyclic loading pattern.

Key words: cyclic load, dilatancy, earthquake, interaction, pile, (*p-y* curves) (IGC: D6/E12/H1)

INTRODUCTION

Major codes of practice for foundation design in Japan adopt so-called nonlinear *p-y* curve models for piles as their usual design calculation models, in which *p-y* curves express lateral soil-pile interactions, with *p*=soil resistance stress to a pile and *y*=corresponding displacement of the pile relative to the far-field. The pile shaft is modeled as a beam, considering that the pile shaft may become plastic. As for *p-y* curves, for example, a monotonic bi-linear curve with non-zero hardening is used in the Japanese Specifications for Highway Bridges (Japan Road Association, 2002); this curve was calibrated from model experiments and *in situ* load tests on actual foundations (Fukui et al., 1997; Kimura et al., 1994; Kimura et al., 1995; Kosa et al., 1998).

In addition to these efforts, reasonable and practical nonlinear dynamic computational models of structures have been sought, especially following the disastrous consequences of the 1995 Hyogo-ken Nanbu (Kobe) earthquake (Public Works Research Institute, 1996; Editorial Committee for the Report on the Hanshin-Awaji Earthquake Disaster, JSCE, 1996). In line with this trend, extensive experiments involving piles and column-type foundations subjected to cyclic lateral displacement at the top of the foundation have been conducted in Japan, e.g., Nishiyama et al. (1998), Kondou et al. (1998), and Maki (2002), to develop and confirm nonlinear dynamic computational models of pile

foundations. Such experiments are useful for model calibration for a larger-scale displacement level, involving much larger pile displacement levels than those in conventional experiments. However, one of the limitations of such experiments is that they generally involve only symmetric fully-reversed cyclic loading. While it is widely recognized that fully-reversed cyclic loading is a standard procedure in experimental studies of the seismic design of structures, seismic situations commonly involve piles being subjected to random lateral loading. It is therefore important to scrutinize pile behavior under varying loading patterns to construct a numerical model that is more reliable in accounting for the behavior of piles during earthquakes.

Previous lateral load experiments of piles indicate a difference in the lateral soil resistance between a situation where piles are subject to monotonic loading and that where a cyclic loading process is included in the loading history on one side. In reloading paths following unloading, the mobilized load tends to become smaller than the previous peak load, even when it reaches the previous peak displacement level. This is demonstrated in experiments by, for example, Agaiby et al. (1992) and Mayne et al. (1992). These studies conducted experiments on the drained and undrained lateral behavior of drilled shafts subjected to lateral monotonic and one-sided cyclic loading in various densities of sand deposits and various consolidation conditions of clay deposits. The results demonstrate that the lateral resistance of the drilled

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Fig. 1. Test setup and pinned device attached to the base of the pile: All units are in mm

shafts in one-sided cyclic loading tends to be smaller than that in monotonic loading. In turn, it is inferred that there probably exists a general tendency for the lateral soil resistance to piles to vary with the cyclic loading pattern, independent of soil classification.

In the present paper, we re-examine a pile experiment conducted by Kimura et al. (1998) and Fukui et al. (1998), from the viewpoint of loading pattern dependency in the lateral soil resistance to piles. Although this experiment was conducted with very loose sand, it produced several noteworthy features. First, the experiment compared the difference in pile behavior between fully-reversed and one-sided lateral cyclic loading conditions that are unlikely to have been simultaneously involved in past experimental studies. These two different cyclic loading patterns can be considered as two extreme loading patterns in random loading situations such as seismic loading. Second, pile bending deformation data from the experiments are available, enabling an estimation of p - y curves and a direct investigation of the loading pattern dependency of lateral soil resistance.

We then interpret the loading pattern dependency of the lateral soil resistance to piles from the viewpoint of nonlinear soil behavior subjected to cyclic deformation. The possible soil deformation mode is simplified into an intuitive model, and we then study the characteristics of the load transfer hystereses between soil and pile. Both a stress-dilatancy relationship and a mobilized shear strength-shear strain relationship are considered to describe the behavior of the soil surrounding a pile.

Finally, many different patterns of loading will be given to the intuitive model. The loading pattern dependency is characterized on the basis of the simulation

results; these findings should be included in nonlinear dynamic computational models for pile foundations.

FULLY-REVERSED AND ONE-SIDED CYCLIC LATERAL LOADING EXPERIMENTS OF PILES

Summary of Experiments

Figure 1 illustrates the set-up of the experiment conducted by Kimura et al. (1998) and Fukui et al. (1998). A saturated sand deposit was made of Kashima sand in a deep test pit at the Foundation Engineering Laboratory of the Public Works Research Institute in Tsukuba, Japan. The deep test pit has a length of 4 m and a width of 3 m, and is 11 m deep. The initial soil deposit surface is referred to as the ground level. The set-up of the surrounding soil for each experimental case involved the following procedure:

1. The water level from the pit bed was set at 1 m.
2. Sand was thrown into the test pit from 0.5 m above the current water surface, using a grab bucket.
3. The sand surface was smoothed level.
4. The water surface was readjusted to a height 1 m above the smoothed sand surface.
5. Stages 2 to 5 were repeated.

Because this construction procedure did not work as expected, the constructed soil deposit became very loose, as described previously, with an average relative density of 17% and average dry unit weight of 15.1 kN/m^3 .

The sand was composed of 1.8% gravel, 97.4% sand, 0.8% silt, and 0% clay. The coefficient of uniformity, U_c , was 2.66, and the mean grain size, D_{50} , was 0.67 mm. The density of soil particles, ρ_s , was 2.64 g/cm^3 , and the maximum and minimum dry densities were $\rho_{d\max} = 1.79 \text{ g/}$

cm^3 and $\rho_{\text{dmin}} = 1.50 \text{ g/cm}^3$, respectively. The internal friction angle was estimated from drained triaxial compression laboratory tests at several confining stress levels and at a relative density D_r of 30% ($\rho_d = 1.53 \text{ g/cm}^3$). The estimated internal friction angles are shown in Table 1, in which the condition of cohesion c was assumed to be zero. Undrained cyclic triaxial compression laboratory tests at $D_r = 30\%$ determined a small strain shear modulus G_0 of:

$$G_0 = 199.63 (\sigma'_c/98)^{0.65} \times 10^3 \text{ kN/m}^2 \quad (1)$$

in which the unit of confining stress, σ'_c , is kN/m^2 . The water level was maintained at the ground surface level during each experiment.

Lateral cyclic loads were applied to the piles at an average loading rate of 60 mm/min at a height of 0.7 m above the initial ground surface. Positive displacement and positive jack load directions are indicated in Fig. 1.

Three steel pipe piles, a reinforced concrete (RC) pile, and a prestressed high-strength concrete (PHC) pile were used in the experiment. In the present study we consider just the steel pipe pile cases, because steel pile shafts generally behave linearly until the material reaches the yield stress, while concrete pile shafts behave nonlinearly because of cracking, even when the reinforcement has yet to reach the yield stress.

Each steel pipe had a diameter of 318.5 mm , an overall length of 9 m , and an embedded depth of 8 m . One pile had a wall thickness of 10.3 mm , while the other piles had a smaller wall thickness of 5.6 mm . The material element test results are listed in Table 2. Strain gauges were placed at both east and west sides of the pile surfaces, as illustrated with dots in Fig. 2. A pile with a pinned device, as shown in Fig. 1, was placed on the bottom soil layer in the middle of the soil preparation. The piles were not placed in the center of the test pit because of the length of the jack used in the experiment.

The experimental cases examined herein are also numbered in Table 2. Case S1 used a larger wall thickness pile, and the pile was subjected to fully-reversed cyclic displacement. The amplitude of the specified displacement was gradually increased from 1δ to $n \times \delta$ ($n = 2, 3, 4, \dots, 16$), in which $\delta = 15 \text{ mm}$, approximately 5% of the

pile diameter. Three cycles were repeated at each displacement amplitude from 1δ through 14δ , while only a single cycle was applied at the last two displacement amplitudes of 15δ and 16δ . Cases S2 and S3 employed smaller wall thickness piles and were subjected to fully-reversed cyclic loading and one-sided cyclic loading, respectively. In Case S2, the same pattern of fully-reversed cyclic loading as given in Case S1 was applied. In Case S3, one-sided lateral displacement cycles were applied, in which the amplitude of the specified displacement was gradually developed from 1δ to $n \times \delta$ ($n = 2, 3, 4, \dots, 19$), with three cycles at each displacement amplitude. In a loading phase, displacement was controlled and increased up to the specified displacement level. The load was then released down to zero in the unloading phases, followed by displacement control in the next loading phase.

Photo 1 shows the state of the test sand deposit after the Case S1 fully-reversed cyclic loading experiment. Photo 2 shows the state of the test sand deposit at the 9δ and 18δ cycles in the Case S3 one-sided cyclic loading experiment. In all experimental cases, average slumping of 1.5 to 4.0 cm was observed over the soil deposit surface, and large subsidence of approximately 35 to 50 cm was developed around the piles. These observations indicate that a gradual volume change in the soil deposit,

Fig. 2. Positions and spacing intervals of strain gauges along the pile: All units are in mm

Table 1. Strength data with respect to consolidation stress for Kashima sand with $D_r = 30\%$

σ'_c (kN/m^2)	29.4	49.0	68.6
ϕ' (deg.)	39	36	35

Table 2. Descriptions of experimental cases

Case	Cyclic loading pattern	Pile wall thickness	Yield stress	Young's modulus
S1	Fully-reversed	10.3 mm	562 N/mm^2	227 kN/mm^2
S2	Fully-reversed	5.6 mm	589 N/mm^2	238 kN/mm^2
S3	One-sided

Photo 1. State of the test soil deposit after the experiment of Case S1

Photo 2. State of the test soil deposit after the first cycles of 9δ (left-hand side) and 18δ (right-hand side) in the experiment of Case S3

Fig. 3. Load-displacement hysteresis at the load point of the Case S1 and Case S3 experiments

i.e. negative dilative deformation, evolved as the cyclic loading progressed. Slumping of the test sand deposit around a pile subjected to cyclic loading can also be seen in the experiment conducted by Maki (2002) and Maki and Mutsuyoshi (2002), although in this case a dense sand deposit was used.

The load-displacement curves at the loading point in Case S1 fully-reversed cyclic loading and Case S3 one-sided cyclic loading are shown in Fig. 3. In both cases, the measured strain at the pile surface reached the yield strain

Fig. 4. Envelope curves of load-displacement hysteresses in the positive region at the load point for all experiment cases

during the 7δ ($= 105$ mm) cycle; the two behaviors appear similar, and the hysteresses contain nonlinearities. Residual horizontal displacement at the unloaded states increases with applied displacement amplitude because of soil nonlinearity, even before the 7δ cycle.

Figure 4 compares the backbone curves of the load-displacement relationships in the positive load and displacement region among all experimental cases, in

which backbone curves are drawn by connecting neighboring peak points of the first loops at all displacement amplitudes. Note that a sudden loss of the load following the mobilized maximum strength in Case S2 is considered to stem from the failure of the pile shaft itself, as will be described below; the behavior before the maximum strength point will be considered hereafter. The lateral load in the Case S3, one-sided cyclic loading was clearly less than those in the Case S1 and S2 fully-reversed cyclic loading at every displacement level, even though Case S2 involved the same conditions of pile shaft and soil deposit as Case S3. Accordingly, the load-displacement curves lead to the hypothesis that the mobilized soil resistance to piles subjected to one-sided cyclic loading becomes smaller than that for piles subjected to fully-reversed cyclic loading.

The depths of maximum residual deflection in the pile shafts were measured after each experiment. Because all the pile shafts became plastic, plastic hinge regions can be considered to have formed around the depths of large residual deformation in the piles. In a fully-reversed cyclic loading of Case S1, a small residual deflection was observed at a depth of 2.45 m below the initial ground level. In the other fully-reversed cyclic loading of Case S2, the pile shaft ruptured at a depth of 1.55 m. In the one-sided cyclic loading of Case S3, small residual deflection was found at a depth of around 2.5 m. The maximum residual deflections in Case S1 (fully-reversed cyclic loading case) and Case S3 (one-sided cyclic loading case) developed at approximately the same depths, although the pile shaft in Case S1 had a larger bending stiffness. In addition, the positions of rupture or residual displacement in Case S2 (fully-reversed cyclic loading case) and Case S3 (one-sided cyclic loading case) were different, although the same pile shafts and soil conditions were employed in the two experiments. These inconsistencies also indicate that the mobilized lateral soil resistance to piles varies with the pattern of cyclic loading, and that the soil resistance in the Case S3 one-sided cyclic loading was smaller than that in the fully-reversed cyclic loading of Cases S1 and S2. Therefore, we need to examine p - y curves to directly investigate the load transfer between soil and pile during the experiments.

Estimation of p - y Curves from Strain Gauge Data

The x_1 - x_2 Cartesian coordinate system is set as described in Fig. 1. A total x_1 -direction displacement of a pile at a depth of x_2 , $y_f(x_2)$, can be decomposed into a rigid rotation component around the bottom hinge, $y_{f1}(x_2)$, and a substantial small deformation component in the pile, $y_{f2}(x_2)$ (Iwakuma et al., 1987; Saiki et al., 1998).

$$y_f(x_2, t) = y_{f1}(x_2, t) + y_{f2}(x_2, t), \quad (2)$$

in which t =time. The rotational component y_{f1} can be obtained by

$$y_1(x_2, t) = l \sin \Theta(t), \quad (3)$$

in which $\Theta(t)$ =rotation angle of the pile base at time t , and l =distance from the pile base to the considered

Fig. 5. Schematic diagram of the physical meaning of numerical differentiation

section. The substantial deformation component, $y_{f2}(x_2)$, can be obtained using the double straightforward trapezoidal integration of bending moment values along the x_2 -direction, in which the infinitesimal displacement theory and Bernoulli-Euler assumption are employed.

The bending moment distribution along the x_2 -direction is expressed as;

$$M(x_2, t) = -EI \frac{d^2 y_{f2}(x_2, t)}{dx_2^2} = EI \frac{\varepsilon_E(x_2, t) - \varepsilon_W(x_2, t)}{D}, \quad (4)$$

in which ε_E and ε_W =measured strains on the east and west sides of the pile, respectively, and D =pile diameter. There are three unknowns: the rotation angle at the bottom, Θ , for estimating the rotational displacement component, y_{f1} , and the two integration constants for estimating the substantial deformation component, y_{f2} . These are determined with the following boundary conditions:

$$y_f = y_{f1} = y_{f2} = 0$$

at the bottom of pile, and

$$y_f = y_{f1} + y_{f2}$$

that can be given from the displacement transducer record at a height near the ground level. In addition, the deflection angle at the bottom of the pile can be considered to be zero in the substantial deformation component because it has already been considered in Eq. (3) as the rotation of the bottom of the pile. Finally, the relative displacement of the pile to the soil, y , is introduced. In the present experimental cases, $y = y_f$, because the present experiment was conducted in a static condition.

The soil resistance stress to the pile, p , can be obtained through the double finite differentiation of pile bending moment values along the x_2 -direction.

$$p_i = \frac{1}{D} \frac{\frac{M_{i+1} - M_i}{l_i} - \frac{M_i - M_{i-1}}{l_{i-1}}}{l_i + l_{i-1}} \quad (5)$$

Figure 5 shows a schematic diagram of p and the measured bending moments M_{i-1} , M_i , and M_{i+1} at three consecutive cross sections of the $(i-1)$ -th, i -th, and $(i+1)$ -th with strain gauges, respectively. The discretized Eq. (5) yields the value of p_i at the mid-height point of the

$(i+1)$ -th and $(i-1)$ -th points. The representative depth for pile displacement corresponding to the value of p_i is the mid-height point of the $(i+1)$ -th and $(i-1)$ -th points. However, because high-order fluctuations within the bending moment distribution because of measurement errors were strongly exposed and amplified via the double differentiation process, such that the obtained distribution of p became too bumpy to be considered realistic, when the measured point bending moment values were directly substituted into Eq. (5). Eventually, before the differentiation process, measured bending moments in piles were subject to a data smoothing process.

Interpolating functions of measured bending moments are often manipulated with double mathematical differentiation for estimating the soil resistance stress distribution to piles. However, it is not certain that an interpolated spline moment distribution will stably induce a good approximation of soil resistance stress distribution when the derived moment distribution function is manipulated with double mathematical differentiation. In addition, as reported by Kikuchi (2002) and others, many issues have yet to be resolved in the use of a spline interpolation. Certain measures can be taken to avoid such difficulties. For example, Kikuchi (2002) employed the following technique: (a) a smoothing process using a smoothing spline algorithm with a third-order polynomial was applied to a measured bending moment distribution, (b) the obtained smoothing function was mathematically differentiated one-time, (c) the values at the cross sections with strain gauges taken from the function derived in (b) were interpolated using a separate typical spline process with a third-order polynomial, which is not a smoothing spline, (d) the second mathematical differentiation process was conducted for the interpolating spline function obtained in (c), and (e) the values at several selected points on the function derived in (d) were also interpolated using a separate typical spline process with a third-order polynomial, which is not a smoothing spline.

Although no specific resolution has been established in terms of the authors' findings, and alternatives may be available, in the present study we adopt the simpler method as described above, that is: (a) bending moment distributions in piles were subject to a smoothing process using a smoothing spline algorithm with a third-order polynomial, and (b) the point bending moment values at the cross sections with strain gauges were taken from the smoothing function and substituted into the double finite differentiation process shown in Eq. (5).

A relevant value of the smoothing parameter was determined by trial-and-error visual inspections for several bending moment distributions, so that the distributions of pile bending moment, M , and soil resistance stress, p , versus depth satisfied the following conditions:

1. In terms of bending moments, the predominant trend in the original distribution, prior to the smoothing process, remained visible.
2. A bumpy high-order fluctuation was not evident in the distribution of soil resistance stress, p ,

Fig. 6. p - y relations estimated from strain gauge data (Case S1)

obtained via Eq. (5).

Estimated p - y Curves

To estimate p - y curves, we used strain gauge data measured until the pile reached a displacement amplitude of 7δ , because the pile shafts could be regarded as being elastic within the displacement amplitudes smaller than 7δ . The derived p - y curves for a fully-reversed cyclic loading experiment, Case S1, are illustrated in Fig. 6. The loops expand with an increase in the peak displacement level. The soil resistance stress, p , at depths of GL -0.48 and -0.60 m, appears to approach their ultimate values. In addition, there is little variation in the loop paths during cycles at each displacement amplitude.

The rigidities of the p - y envelope curves that are derived by connecting neighboring peak points of the first loops at all displacement levels increase as the depth increases. This is considered to stem from the increase in the shear stiffness of the test sand deposit versus depth with increasing confining pressure.

Unloading branches from the peaks of loops to $p=0$ appear to be linear. Their gradients are stiffer than those of the ascending branch of backbone curves, and do not vary greatly with the level of y . Herein, "unloading" indicates that the increment of $|p|$ is negative, while "loading" indicates that the increment of $|p|$ is positive. After reaching $p=0$ in an unloading phase, the following p - y branch trends approximately linearly toward the reversed peak point of the preceding peak point (or the point placed in the reversed side and corresponding to a

Fig. 7. p - y relations estimated from strain gauge data (Case S3)

point that is obtained by multiplying (-1) by p and y at the preceding peak point), with a lesser gradient than the preceding unloading gradient. Finally, it seems like the loop shape of p - y approaches a parallelogram. It follows that the basic hysteretic mechanism of p - y is likely to be modeled with a conventional peak-oriented mechanism.

Figure 7 shows p - y curves observed in the Case S3 one-sided cyclic loading experiment. As also observed in the Case S1 fully-reversed cyclic loading experiment, the unloading gradient does not vary with displacement level. There is also a tendency for the reloading paths of p - y to approach the preceding peak points; however, the peak values of p in the second and the third cycles are clearly smaller than those in the first cycle at a certain displacement amplitude, for example, as indicated by arrows in the first figure in Fig. 7. In particular, the difference in p values between the first and second cycle is much larger than the difference between the second and third cycle.

Figures 8 and 9 compare p - y curves observed at two different depths between Case S1 fully-reversed cyclic loading and Case S3 one-sided cyclic loading, and differences between Cases S2 and S3, respectively. The soil resistance mobilized in the one-sided cyclic loading case was entirely smaller than that in the fully-reversed cyclic loading cases. This observation is consistent with the other experimental findings concerning differences in the depths to maximum deformation and the load-displacement curves at the loading point among these experimental cases.

Based on the results of these three experimental cases,

Fig. 8. Comparison of the Case S1 and Case S3 p - y hysteresses

Fig. 9. Comparison of the Case S2 and Case S3 p - y hysteresses

it is expected that the load transfer characteristics of soil-pile interaction can be considered as expected in conventional peak-oriented hysteretic rules; however, conventional hysteretic rules cannot account for the observed difference in soil resistance intensity between fully-reversed cyclic loading and one-sided cyclic loading. Therefore, it is important to examine characteristics in p - y hysteresses that vary with cyclic loading patterns, because piles are subjected to arbitrary cyclic loads during earthquakes.

ANALOGY OF SOIL RESISTANCE BETWEEN LATERAL CYCLIC LOADING EXPERIMENTS OF PILES AND SOIL ELEMENTS

Figure 10 illustrates a model of the deformation modes of surrounding soil during pile deflection, in which σ'_v = effective overburden stress, K_0 = coefficient of earth pressure at rest, and Δp_+ and Δp_- = variations of earth pressures on the positive and negative sides, respectively, of x_1 from the at-rest state. A compression field is induced in a soil element at depth z in the passive side of the ground, while an extension field is induced in a soil element in the active side because of the reduction in the horizontal stress. Accordingly, during an earthquake, the surrounding soil will chiefly sustain a random cyclic compression-extension deformation history with variation in the displacement of the pile relative to the far-field of the ground. Based on these hypotheses, a very simple model, as shown in Fig. 11, can be assumed to examine as basically as possible the soil resistance mobilized in the near-field. The x_1 - x_2 Cartesian coordinate system is assumed, and the x_1 -direction is the loading direction.

The simplified model shown in Fig. 11(c) consists of a rigid body sandwiched between two plane strain sand

Fig. 10. Schematic diagram of the deformation of two typical local soil elements within the ground

Fig. 11. Soil elements surrounding a pile (a, b) and accompanying simplified model (c)

blocks, which is assumed that a soil-pile system is sliced thin with two vertical and two horizontal planes as shown in Figs. 11(a) and (b), such that the center axis of the pile can be contained. The model is designed in an attempt to

Fig. 12. σ_{11} - ϵ_{11} , σ_{11} - γ , $\sin \phi_{mob}$ - γ , and ϵ_{vol} - γ relationships obtained via a plane strain cyclic test and plane strain compression (PSC) and plane strain extension (PSE) tests of isotropically consolidated specimens of Toyoura sand, as performed by Masuda et al. (1999)

involve an intensive plastic deformation zone in the soil close to the pile in the loading plane. The soil close to the pile is considered to become plastic to a much larger extent because of the cyclic compression-extension deformation, while the strain level in the further-field soil remains small. It can therefore be assumed that the nonlinear behavior of the soil close to the pile strongly influences the nonlinear hysteresis of the soil resistance to the pile at every depth, and that soil resistance to the rigid body in the simplified model in Fig. 11(c) can be expected to show similar characteristics of nonlinear hysteresis in p - y curves as those in nonlinear soil-pile interaction in the field; this is likely to change with cyclic loading pattern. A sand block length, L , is considered to be conceptual, representing the zone that becomes largely plastic in the surrounding soil. Although it is impossible to specify the sand block length, L , different loading patterns retain their differences despite the different lengths of L . Therefore, L should be considered to be introduced solely for convenience to calculate the averaged nonlinear compression-extension response in the soil close to the pile at a certain depth. Ultimately, we disregard three-dimensional effects and seek only the relative differences in the hysteretic loops that are related to differences in the cyclic loading patterns, all from a qualitative viewpoint.

Masuda et al. (1999) conducted a series of special plane strain tests for saturated specimens of air-pluviated Toyoura sand, in which cyclic compression and extension deformation histories were applied as well as monotonic compression and extension histories. A typical result from their tests is plotted in Fig. 12. The major and minor principle stress axes are x_1 and x_2 , respectively, and the principal stresses and strains are considered to be positive in compression. When a pile in the field is assumed to move laterally, as shown in Fig. 10, the vertical principal stress σ_v in the field corresponds to the confining pressure σ_{22} in the specimen, while the horizontal principal stress σ_h in the field corresponds to the variable component of cyclic axial stress σ_{11} in the speci-

men, in which σ_{ij} and ε_{ij} are the (i, j) components of stress and strain, respectively. In Masuda et al.'s tests (1999), the axial principal stresses at the initial states were not larger than the confining pressures; this was adopted to reproduce typical initial stress conditions in the field. In addition, the test specimens were subjected to axial loading in the direction parallel to the bedding plane. ϕ_{mob} is the mobilized internal friction angle, and is expressed by;

$$\sin \phi_{\text{mob}} = \frac{R-1}{R+1} \quad (6)$$

in which R = ratio of principal stresses, σ_{11}/σ_{22} , and γ and ε_{vol} = shear and volumetric strains, respectively, as defined by;

$$\gamma = \varepsilon_{11} - \varepsilon_{22} \quad \text{and} \quad \varepsilon_{\text{vol}} = \varepsilon_{11} + \varepsilon_{22} \quad (7)$$

in the plane strain condition. It should be noted that changes in volume, i.e. an increment of ε_{vol} , $\Delta\varepsilon_{\text{vol}}$, are negative for volume dilation and positive for negative volume dilation (or volume contraction).

The following features are apparent from the σ_{11} - ε_{11} relationships in Fig. 12. In cyclic loading tests, the σ_{11} - ε_{11} loops are non-symmetric for compression and extension states and analogous to a kind of triangular shape, in which $\sigma_{11} > \sigma_{22}$ in compression (or passive) states and $\sigma_{11} < \sigma_{22}$ in extension (or active) states in the case of initial isotropic consolidations. Variation in the principal stress ratio, σ_{11}/σ_{22} , in the element is indeed associated with that in the earth pressure coefficient in the field. During the transition from a passive compression state to an active extension state, soil resistance stress, i.e., σ_{11} , decreased sharply before maintaining a small resistance. In contrast, during the transition from an active extension state to a passive compression state, the σ_{11} - ε_{11} path progressed almost immediately toward the previous peak point, without making any sort of spindle-shaped path. These features are similar to parts of the p - y curves, either on the positive or negative side of p , as observed in the cyclic lateral loading experiments of piles that are shown in Fig. 6. It follows that the superposition of the σ_{11} - ε_{11} relationship with the $(-\sigma_{11})$ - $(-\varepsilon_{11})$ relationship obtained in the soil element test results in a similar parallelogram hysteretic loop shape to the p - y loop shape observed in the experiment of piles subjected to fully-reversed cyclic loading that is shown in Fig. 6; this experiment shows clear break points around the peaks and $p=0$. The above analogy supports the validity of following the simplified modeling process shown in Fig. 11.

SIMULATION OF THE INFLUENCE OF STRESS-DILATANCY BEHAVIOR ON THE LOADING PATTERN DEPENDENCY OF p - y

Calculation Model

Here we use the simplified model shown in Fig. 11 to carry out analytical observations to determine the characteristics of the hystereses in the load transfer between soil

and pile. Each sand block is modeled as one element, and can be freely deformed in the x_1 - and x_2 -directions. A plane strain sand block on the positive side with respect to $y=0$ is hereafter referred to as a positive-side sand block, while a block on the negative side with respect to $y=0$ is hereafter referred to as a negative-side sand block. For simplicity, the plane strain sand blocks are assumed to be initially under an isotropic consolidation state of $\sigma_{11} = \sigma_{22}$, and their nonlinear behavior is given based on the element test results conducted by Masuda et al. (1999). In our calculations, principal stresses and strains are also assumed to be positive in compression. Only the displacement of the central rigid body, $y(t)$, is controlled, and we examine the response of soil resistance stress, $p(t)$, per unit area that acts on the central rigid body at time t . The response is derived as follows:

1. Calculate the axial stress response in the positive-side sand block, $\sigma_{11}^+(t)$, in a step-by-step manner for a given axial strain history $\varepsilon_{11}^+ = y(t)/L$;
2. Derive the axial stress response of $\sigma_{11}^-(t)$ in the negative-side sand block in a step-by-step manner for the corresponding axial strain history $\varepsilon_{11}^- = -y(t)/L$ symmetric to ε_{11}^+ ;
3. The sum of σ_{11}^+ and σ_{11}^- , with respect to the same time t , yields the response of $p(t)$; in which $y(t)$ is the displacement of the central rigid body at time t .

Calculations without Consideration of Stress-Dilatancy Laws

Based on the element test results of stress-strain relationships in sand subjected to cyclic loading under plane strain conditions, as shown in Fig. 12, the behavior of the sand block can be described using a combination of a shear component relationship and a volumetric component relationship, as in the constitutive model proposed by Tatsuoka et al. (2003). For the shear component relationship, a $\sin \phi_{\text{mob}}-\gamma$ relationship is modeled. $\sin \phi_{\text{mob}}-\gamma$ curves observed in the element tests tended to become approximately symmetric about the initial states. We therefore employ a typical hyperbolic function as the backbone curve and Masing's rule as the hysteretic rule. The backbone curve can be expressed by:

$$\sin \phi_{\text{mob}} = \frac{\gamma/\gamma_r}{1 + \gamma/\gamma_r} \times \sin \phi_{\text{max}} \quad (8)$$

in which ϕ_{max} is the ultimate internal friction angle and γ_r is the reference strain defined using ϕ_{max} and the tangent gradient s_0 at the origin for the following equation:

$$\gamma_r = \frac{\sin \phi_{\text{max}}}{s_0} \quad (9)$$

Equation (8) can be rewritten as;

$$\sin \phi_{\text{mob}} = \frac{s_0 \gamma}{1 + \frac{s_0 \gamma}{\sin \phi_{\text{max}}}} \quad (10)$$

The value of γ_r is assumed to be 0.0002, and the value of ϕ_{max} is assumed to be 30 deg. The internal friction angle of the sand deposit in the pile load experiment, as

Table 3. Numerical analyses of the behavior of plane strain sand blocks under a constant Poisson's ratio

Case	Cyclic loading condition
1	Fully-reversed (smaller)
2	Fully-reversed
3	One-sided (repeated)

obtained from a triaxial compression test, was larger than 30 deg., even though the soil deposit was very loose. In addition, the mobilized internal friction angle in the plane strain compression condition is generally considered to be larger than that in the triaxial compression condition (Kulhawy and Mayne, 1990). Therefore, the value of 30 deg. can be regarded as a representative value for loose to moderately dense sand.

The volumetric component is needed to translate the $\sin \phi_{\text{mob}}-\gamma$ relationship into a $\sigma_{11}-\varepsilon_{11}$ relationship. For the first case study, the most typical linear relationship is set as follows:

$$\varepsilon_{22} = -\nu \varepsilon_{11}, \quad (11)$$

in which ν is the Poisson's ratio. A published soil properties manual (Kulhawy and Mayne, 1990) indicates that Poisson's ratio ranges between 0.1 and 0.3 for loose sand and 0.3 to 0.4 for dense sand. Therefore, a constant Poisson's ratio of 0.3 is adopted as a representative value for sand with a wide range of densities. However, Eq. (11) cannot express a nonlinear volumetric strain-shear strain relationship and the accumulation of an irreversible component in the volumetric strain, ε_{vol} , such as those observed in Fig. 12.

Three cases are firstly calculated, as listed in Table 3. Cases 1 and 2 have the same conditions except for the given displacement amplitudes. In a "fully-reversed" cyclic loading condition, we apply displacement cycles that have symmetric displacement amplitudes with respect to $y=0$. The term "smaller" for Case 1 in Table 3 indicates that smaller amplitudes of displacement, y , are applied; it is therefore expected that the mobilized strain level in the sand blocks will be one order of magnitude smaller than those in the other calculation cases. Case 3 involves a "one-sided" cyclic displacement history that consists of displacement cycles with an eccentricity of displacement, y , on a particular side. A "repeated" cyclic loading means that multiple cycles are given at each displacement amplitude.

The result of Case 1 is shown in Fig. 13, in which Case 1 is the fully-reversed cyclic case with smaller displacement amplitudes. For comparison, the result for monotonic loading is also shown on the figure with thick dashed lines. The intensity of p at each peak is placed on the monotonic loading curve. The result of Case 2, which is also a fully-reversed cyclic loading case, but not with smaller amplitudes of displacement, is analogous to that of Case S1, as shown in Fig. 14. The simplified model used herein can approximately predict the shapes of loops of $p-y$ that are observed in pile load experiments, as expected; however, the predicted loops have slightly

Fig. 13. Behavior of a rigid body supported by plane strain sand blocks (Case 1): (a) $p-y$ curve of the rigid body between the sand blocks, (b) $\sigma_{11}-y/L$ curves of sand blocks, (c) $\sin \phi_{\text{mob}}-\gamma$ curve of the negative-side sand block and (d) $\sin \phi_{\text{mob}}-\gamma$ curve of the positive-side sand block

larger curvatures than the $p-y$ hystereses derived from the pile load experiments. While the calculated $\sin \phi_{\text{mob}}-\gamma$ paths during compression and extension in a fully-reversed cycle are symmetric about the origin, the measured paths do not appear perfectly symmetric, and the measured paths during compression (i.e. the increment of shear strain, γ , is positive) tend to be less curved than those during extension, as evident in the measured curve shown in Fig. 12. Therefore, the calculated $\sigma_{11}/\sigma_{22}-(y/L)/\gamma_r$ paths during compression tend to be curved. Simplifications involved in the calculation model are considered to cause the differences in the $p-y$ loop shapes derived from calculations and from the pile experiment.

The calculated $p-y$ curve for the case of one-sided cyclic loading with three-times repetition at each displacement amplitude, Case 3, is shown in Fig. 14. The peaks in the calculated $p-y$ curve are also located on the monotonic loading curve. Eventually, the tendency of the degradation in soil resistance is unable to be detected.

Calculations with Consideration of Stress-Dilatancy Laws

We now seek to improve the stress-strain relationships. First, a minor change is made to the shear component of the $\sin \phi_{\text{mob}}-\gamma$ relationship. The relationship is extended so that different ultimate internal friction angles, ϕ_{max} , in both compression and extension can be involved in Eq. (10), as observed in the measured $\sin \phi_{\text{mob}}-\gamma$ relationship in Fig. 12. In the calculation cases below, the ultimate internal friction angles are assumed to be 30° in

Fig. 14. Behavior of a rigid body subjected to a fully-reversed cyclic displacement (Case 2, thin line), and a one-sided cyclic displacement (Case 3, thick line): (a) p - γ curve of the rigid body between the sand blocks, (b) σ_{11} - γ/L curves of sand blocks, (c) $\sin \phi_{\text{mob}}$ - γ curve of the negative-side sand block and (d) $\sin \phi_{\text{mob}}$ - γ curve of the positive-side sand block

compression and 45° in extension. Because the Masing's rule cannot be applied to non-symmetric backbone curves, we adopt its extended version: the proportional rule proposed by Tatsuoka et al. (2003). Nevertheless, for continuity, we set the initial gradient s_0 used in Eq. (10) to be identical for both positive and negative increments of γ from the origin. The same value of s_0 , with $\phi_{\text{max}} = 30^\circ$, is applied to both compression and extension. Note that a companion paper, Shirato et al. (2006a), provides several examples of the calculation results with the same ultimate internal friction angle, ϕ_{max} , in compression and extension.

The critical change in stress-strain relationships is applied to the volumetric component. The stress-dilatancy law proposed by Rowe (1962) is adopted rather than using Eq. (11). Masuda et al. (1999) have confirmed empirical stress-dilatancy relationships for their plane strain cyclic compression-extension element test results as follows:

$$R_1 = K_1 D_1, \quad R_1 = \frac{\sigma_{11}}{\sigma_{22}}, \quad D_1 = -\frac{d\varepsilon_{22}^p}{d\varepsilon_{11}^p} \quad (\text{when } \Delta\sigma_{11} > 0), \quad (12)$$

and

$$R_2 = K_2 D_2, \quad R_2 = \frac{\sigma_{22}}{\sigma_{11}}, \quad D_2 = -\frac{d\varepsilon_{11}^p}{d\varepsilon_{22}^p} \quad (\text{when } \Delta\sigma_{11} < 0), \quad (13)$$

Table 4. Numerical analyses of the behavior of plane strain sand block under stress-dilatancy law

Case	Cyclic loading condition
4	Fully-reversed (smaller)
5	Fully-reversed
6	Fully-reversed (repeated)
7	One-sided
8	One-sided (repeated)

in which ε_{ij}^p is the (i, j) component of plastic strain, and K_1 and K_2 are constants. Unlike Eq. (11), the stress-dilatancy law is capable of accounting for the gradual accumulation of irreversible positive volumetric strain with increasing numbers of loading cycles, as can be seen in the volumetric strain-shear strain relationship in the element test (Fig. 12). Negative dilatancy, in which volumetric strain, ε_{vol} , is positive, generally appears in sand at every strain reversal.

For simplicity, the elastic component of strain in sand is neglected in the calculations below, and ε_{ij}^p is replaced with ε_{ij} in Eqs. (12) and (13). Typical values of K_1 and K_2 in Eqs. (12) and (13) for Toyoura sand under plane strain deformation have been fitted by Masuda et al. (1999) with their element test results as;

$$K_1 = 3.5 \quad \text{and} \quad K_2 = 1.8 \quad (14)$$

and these values are adopted in the calculations below. However, while these values were determined based on element tests using dense sand, the pile load experiments scrutinized above were carried out with loose sand. Although the amount of mobilized dilative strain is highly dependent on soil density, the values of K_1 and K_2 are unlikely to vary significantly with density (Pradhan and Tatsuoka, 1989). Therefore, the overall characteristics that will be obtained in the calculations below using Eq. (14) are considered to also be valid for loose sand. Note that a companion paper, Shirato et al. (2006a), shows several examples of the calculation results with the same value of K_1 and K_2 .

Table 4 lists the cases that are calculated below. The result of Case 4, in which the same displacement history as Case 1, i.e. a fully-reversed cyclic displacement history with smaller displacement amplitudes, is applied, is shown in Figs. 15 and 16. In the graphs of calculated results hereafter, the numbers 1, 2, 3, ... and 1', 2', 3', ... correspond to the points at which the loading direction is reversed and depict the orders of traveling after the loading histories begin from the point of Number 0. The numbers with and without primes are associated with the sand blocks on the positive and negative sides, respectively. The reversed loading path following unloading in the calculated p - γ curve trends towards the opposite peak point, as observed in the calculation in Case 1; however, the intensity of p at each peak point is less than that of the monotonic loading curve (thick dashed line).

In effect, stress-dilatancy behavior plays a salient role in the differences in the calculated intensities of soil resistance stress, p , between Case 1 and Case 4. As

Fig. 15. Behavior of a rigid body supported by plane strain sand blocks (Case 4): (a) p - y curve of the rigid body between the sand blocks, (b) σ_{11} - y/L curves of sand blocks, (c) $\sin \phi_{mob}$ - γ curve of the negative-side sand block and (d) $\sin \phi_{mob}$ - γ curve of the positive-side sand block

Fig. 16. Development of strain in the sand blocks (Case 4): (a) ϵ_{11} - ϵ_{vol} relationship and (b) ϵ_{11} - γ relationship

evident from Fig. 16, both the shear and volumetric strains develop greatly and accumulate, because any two strain components have a nonlinear and irreversible relationship under stress-dilatancy law. For example, in the path from point 2 to point 3, negative dilatant deformation evolves in the sand block. Eventually, the negative volume dilatation is gradually increased along with cyclic loading, and the irreversible component in shear strain, γ , is also increased. Therefore, the center points of loops of the $\sin \phi_{mob}$ - γ curves shown in Fig. 15 shift to the negative side of γ . The internal friction angle in compression (i.e. passive deformation) is less mobilized than that in Case 1, where Case 1 considers a constant Poisson's ratio instead of the stress-dilatancy law, while the given displacement histories of y are the

Fig. 17. Behavior of a rigid body supported by plane strain sand blocks (Case 5): (a) p - y curve of the rigid body between the sand blocks, (b) σ_{11} - y/L curves of sand blocks, (c) $\sin \phi_{mob}$ - γ curve of the negative-side sand block and (d) $\sin \phi_{mob}$ - γ curve of the positive-side sand block

Fig. 18. Development of strain in the sand blocks (Case 5): (a) ϵ_{11} - ϵ_{vol} relationship and (b) ϵ_{11} - γ relationship

same in Case 1 and Case 4. In Cases 1 and 2 that do not consider stress-dilatancy law, the center points of the calculated $\sin \phi_{mob}$ - γ loops remained at the origin, as shown in Figs. 13 and 14, even though the number of cycles increased. It should be noted that because the ultimate internal friction angles in Case 1 and Case 4 are identical in compression, that is, passive soil resistance, the introduction of stress-dilatancy law is considered to be essentially responsible for differences in the calculation results between Case 1 and Case 4.

Figures 17 and 18 show the calculated result for Case 5, another fully-reversed cyclic loading case with the same displacement history as that in Case 2. In comparison with the calculated result of Case 4, a fully-reversed cyclic loading case with smaller displacement amplitudes, the

Fig. 19. Behavior of a rigid body supported by plane strain sand blocks (Case 6): (a) p - y curve of the rigid body between the sand blocks, (b) σ_{11} - y/L curves of sand blocks, (c) $\sin \phi_{mob}$ - γ curve of the negative-side sand block and (d) $\sin \phi_{mob}$ - γ curve of the positive-side sand block

Fig. 21. Behavior of a rigid body supported by plane strain sand blocks (Case 7): (a) p - y curve of the rigid body between the sand blocks, (b) σ_{11} - y/L curves of sand blocks, (c) $\sin \phi_{mob}$ - γ curve of the negative-side sand block and (d) $\sin \phi_{mob}$ - γ curve of the positive-side sand block

Fig. 20. Development of strain in the sand blocks (Case 6): (a) ϵ_{11} - ϵ_{vol} relationship and (b) ϵ_{11} - γ relationship

Fig. 22. Development of strain in the sand blocks (Case 7): (a) ϵ_{11} - ϵ_{vol} relationship and (b) ϵ_{11} - γ relationship

values of p at the peak points are much closer to those of the monotonic loading curve, especially with a larger amplitude of displacement y . As the displacement amplitude increases, the mobilized shear strain amplitude in compression becomes larger and the mobilized internal friction angle, ϕ_{mob} , approaches the ultimate value. Eventually, the value of ϕ_{mob} reaches a flat region of the ϕ_{mob} - γ relationship.

Figures 19 and 20 show the result for Case 6, which involves fully-reversed cyclic loading with a three-times repetition of cycles at every displacement amplitude. In Fig. 19, the calculated p - y curve for Case 5 is also shown for comparison; a single cycle was applied to Case 5 at every displacement amplitude. Discrepancies in the values of p at the peak points in Case 5 and Case 6 are

minor. If the normalized displacement level, $y/L/\gamma_r$, is less than 6.0, degradation in the soil resistance stress, p , appears during the cycles at each displacement amplitude. Following this, degradation of p during the cycles at a particular displacement amplitude is reduced, while the development of irreversible volumetric dilation during the cycles is simultaneously slowed, as shown in Fig. 20.

The calculated results of the two one-sided cyclic loading histories for Cases 7 and 8 are shown in Figs. 21, 22 and 23. For comparison, responses against monotonic loading are again plotted with thick dashed lines. It is a crucial fact that the reloading paths with positive increments of y in the calculated p - y curves have smaller gradients and p -values than those in the previous cycles.

Fig. 23. Behavior of a rigid body supported by plane strain sand blocks (Case 8): (a) p - y curve of the rigid body between the sand blocks, (b) σ_{11} - y/L curves of sand blocks, (c) $\sin \phi_{mob}$ - γ curve of the negative-side sand block and (d) $\sin \phi_{mob}$ - γ curve of the positive-side sand block

The p - y curves seem to gradually trend toward to the right-hand side of the plot as the cyclic loading develops. In particular, three cycles are repeated in Case 8 at each displacement amplitude. For a displacement cycle leaving from a positive peak point, the corresponding p - y loop does not return to the original peak point, but returns instead to a point with a smaller soil resistance stress, even when the displacement amplitude is larger than $y/L/\gamma_r = 6.0$.

In conclusion, our simplified model is capable of accounting for the changes in soil resistance with different cyclic loading patterns, provided that stress-dilatancy law is taken into account. The calculated results reproduced the same characteristics that were observed in the pile load experiment. For example, the soil resistance in one-sided cyclic loading decreases compared to the soil resistance expected in fully-reversed cyclic loading, while the calculated p - y curves generally tend to return toward the previous peak points in fully-reversed cyclic loading. The calculation results suggest that dilatant behavior plays a key role in the loading pattern dependence of soil resistance.

DISCUSSION ON THE EFFECT OF STRESS-DILATANCY LAW ON p - y HYSTERESES

Both the thin and thick solid curves in Fig. 24 show the schematic stress-strain response of a sand block subjected to a cyclic axial strain that involves the transition from a compression state to an extension state and back to a

Fig. 24. Schematic diagram of the behavior of a sand specimen in an axial compression state subjected to a sequence of extension-compression deformations

compression state, in which ε_{11} is controlled. The thin solid curves show responses to fully-reversed cyclic loading, and the bold solid curves show responses to one-sided cyclic loading. The dashed lines show an example of the monotonic loading result. The numbers 1, 2, 3 and 1', 2', 3' in the diagrams correspond to the orders of the loading direction reversals, with the primed numbers representing fully-reversed cyclic loading. The point 1' is identical to point 1, and the first strain reversal occurs at point 1 or 1' from the monotonic loading.

The soil resistance stress to piles in the field, p , is equated with compressive (or passive) soil behavior. In Fig. 24, the path from point 2 to point 3 corresponds to a compressive (or passive) path, following the strain reversal at point 2. Based on the above calculation results using the simplified model shown in Fig. 11, we consider that an irreversible negative dilatancy (i.e. the positive irreversible components in ε_{vol}) and a shear strain are generated along the path from point 2 to point 3. Eventually, the mobilized values of γ before and after the ε_{11} cycle diverge from each other, and $\sin \phi_{mob}$ is simultaneously reduced following the ε_{11} cycle.

For cyclic alternating deformation in both compression and extension, a large part of the incremental negative dilatant strain that is generated in a compression deformation phase tends to be canceled out in the following extension deformation phase. As shown in the ε_{11} - ε_{vol} relationship of calculation Cases 5 and 6 in Figs. 18 and 20, as the amplitude of axial extension strain increases, the increment of irreversible volumetric strain and shear strain during a cycle decreases. The increment of irreversible volumetric strain is considered as the difference in the values of ε_{vol} at $\varepsilon_{11} = 0$ before and after the ε_{11} cycle. Almost all of the incremental negative dilatant strain ($\varepsilon_{vol} > 0$) that is generated in a compression deformation phase is canceled out in the following extension deformation phase. Therefore, the effects of dilatancy on p - y curves tend to diminish, and it can be assumed that the soil resistance in the fully-reversed cyclic loading becomes approximately the same as that mobilized in monotonic loading. For one-sided cyclic deformation, the residual negative dilatant strain accumulates such as shown in Fig. 22, because it is not

Fig. 25. Unloading and loading from a point on a p - y curve: (a) One-time reloading in the original p increment direction and (b) Multiple reloading in the original p increment direction

canceled during a reversed phase. Therefore, in Fig. 24, a larger axial deformation is required to mobilize a large shear strain and shear strength. Although it is considered that dilatancy can generally affect the intensity of the soil resistance to the pile for both fully-reversed and one-sided cyclic loading, the influence of dilatancy will prevail to a larger degree in one-sided cyclic loading.

p - y HYSTERESES AS A FUNCTION OF CYCLIC LOADING PATTERN

Based on Parametric Calculations

On the basis of the experimental results of the piles and the analytical results at the soil element level, the load transfer between soil and pile is considered to be expressed as a function of the loading pattern. Here, we calculate the load-displacement responses of the central rigid body shown in Fig. 11 to several displacement histories, and we derive more specific characteristics of the loading pattern dependency.

In Fig. 25, point $R(p_r, y_r)$ is the last reversal point, and point $R'(p_r', y_r')$ is the preceding reversal point in a p - y hysteresis. Then, point $Q(p_q, y_q)$, which has the same displacement level as point R' , is conceived as the end point of the p - y path following the last displacement reversal. Namely, y_r' is equal to y_q . We define:

$$\begin{aligned}\Delta p_{R'R} &= p_r' - p_r = \Delta p_{R'Q} + \Delta p_{QR}, \\ \Delta p_{R'Q} &= p_r' - p_q, \\ \Delta p_{QR} &= p_q - p_r\end{aligned}\quad (15)$$

$\Delta p_{R'R}$ becomes equal to $2p_r$ in the case of fully-reversed cyclic loading. We define:

$$\xi = \frac{\Delta p_{R'R}}{2p_r}, \quad \eta = \frac{\Delta p_{QR}}{\Delta p_{R'R}} \quad (16)$$

in which ξ is associated with the degree of eccentricity in the one-sided cyclic loading, and η is associated with the degree of degradation in the soil resistance stress, p . Eventually, the ξ - η relationship indicates the degree of the degradation of soil resistance stress, p , as a function of eccentricity in cyclic loading.

We analyze the ξ - η relationship based on the p - y curves obtained in above calculation Cases 7 and 8 and the new calculation Cases 9 to 14. Figure 26 shows the calculation

Fig. 26. Comparison of p - y curves of the rigid body between the sand blocks in Cases 9 to 14

Fig. 27. Relationship between ξ and η

results for Cases 9 to 14, in which the numbers in the result of Case 13 depict the order of traveling of p - y . In the determination of points R' , R , and Q , when cyclic loading leaving the same displacement level is repeated several times, points R' , R , and Q are always renewed where a new loop begins.

The analyzed ξ - η relationship is shown in Fig. 27. In Cases 8, 10, and 12, the results for the loops starting from the same displacement level and having a stationary

Fig. 28. Tendency of the deterioration of p with loading pattern

displacement variation are distinguished in Fig. 27 as the results of the first loop, the second loop, and the third loop, respectively. Figure 27 shows that the first approximation of the ξ - η relationship can be estimated as:

$$\eta = \xi \quad (17)$$

Equation (17) can be transformed as follows:

$$\begin{aligned} \frac{(\Delta p_{R'R})^2}{2p_{r'}} &= \Delta p_{QR} \\ \left(\frac{\Delta p_{R'R}}{2p_{r'}} \right)^2 &= \frac{\Delta p_{QR}}{2p_{r'}} \\ \xi^2 &= \frac{p_q - p_r}{2p_{r'}} \end{aligned} \quad (18)$$

ξ can be also rewritten as:

$$\xi = \frac{1}{2} \left(1 - \frac{p_r}{p_{r'}} \right) \quad (19)$$

Finally, substituting Eq. (19) into Eq. (18) yields

$$\frac{p_q}{p_{r'}} = 2 \left(\xi - \frac{1}{2} \right)^2 + \frac{1}{2} \quad (20)$$

This is shown graphically in Fig. 28.

Because Eq. (20) was obtained using the simplified model shown in Fig. 11 and an assumption of plane strain conditions, it is more important to extract the essential tendency from Eq. (20) than to directly accept the mathematical expression. Equation (20) shows that when a displacement reversal takes place at $p=0$ (i.e. $p_r=0$ and $\xi=1/2$), the most degraded case for the mobilized soil resistance stress, p , occurs. Moreover, when $\xi=0$ or $\xi=1$, little degradation of p occurs. The condition of $\xi=0$ corresponds to the situations in which a series of unloading and reloading occurs in the vicinity of point R' , because $\Delta p_{R'R} = p_{r'} - p_r \rightarrow 0$. The condition of $\xi=1$ corresponds to the situations in which point R exists in the vicinity of the point $p = -p_{r'}$.

Finally, an illustrative interpretation of the relationships indicated by Eq. (20) is summarized in Fig. 29. In a p - y loop ((a) in Fig. 29), the p - y curve returns to a point near the preceding reversal point when the plus and minus intensities of p at the preceding and last reversal points are almost equal. As the difference in $|p|$ of the preceding and last reversal points increases, the reversed p - y curve

Fig. 29. Degradation of p with different values of $|p|$ at the last and next-to-last reversal points

Fig. 30. Schematic diagrams showing differences in the behavior of p - y in reloading between the cases of fully-reversed cyclic loading and one-sided cyclic loading

runs lower ((b) and (c) in Fig. 29), and if the last reversal occurs in the vicinity of the preceding reversal point, the p - y curve returns to the vicinity of the preceding reversal point ((d) in Fig. 29). Note that the experimental results for the piles shown in Fig. 7 and the calculation results shown in Fig. 27 indicate that the peak values of p in the second and third loops of the repeated cycles of identical displacement amplitude degrade less than that in the first loop; this is not taken into account in the tendency summarized in Fig. 29.

Phenomenological Modeling

We first examine the calculated $\sin \phi_{\text{mob}}-\gamma$ relationships in the fully-reversed cyclic loading cases shown in Figs. 15 to 20. The center point of a loop of $\sin \phi_{\text{mob}}-\gamma$ is hereafter referred to as the neutral point of γ . In the fully-reversed cyclic loading cases, the degrees of drift of the neutral points of γ in both the positive-side and negative-side sand blocks are almost identical, as both sand blocks are alternately and equally subjected to compression and extension. Accordingly, every time the simplified model returns to the unloaded state following a completed cycle, it appears to resume a pseudo-initial state that involves a symmetric stress and strain state between the sand blocks.

Next, we consider the calculated relationships of $\sin \phi_{\text{mob}}-\gamma$ in the one-sided cyclic loading cases shown in Figs. 21 to 23. A gap appears in the drifts of the neutral point of γ between the positive-side and negative-side sand blocks; this occurs because the dilatant deformation differs when the soil elements are subjected to cyclic

compression deformation compared with cyclic extension deformation, as shown in Fig. 22. Therefore, the shift of the center points of loops of $\sin \phi_{\text{mob}} - \gamma$ that occurs with negative dilatant deformation in the sand blocks is also considered to occur synchronously with the degradation of soil resistance.

Finally, the important characteristics of p - γ hystereses that vary with cyclic loading patterns can be expressed by schematic diagrams, as shown in Fig. 30. For fully-reversed cyclic loading, the resistance of the sand blocks is likely to be mobilized in the same manner as at the initial state. In contrast, for one-sided cyclic loading the reloading path following a one-sided p - γ cycle drifts toward the corresponding side as though the origin and the backbone curve have shifted toward this side, dependent on the degree of eccentricity of cyclic loading direction.

CONCLUDING REMARKS

This study examines the characteristics of the hystereses in the load transfer between soil and pile for seismic random loading. First, we analyzed the load transfer between soil and pile based on experiments of single piles embedded in sand and subjected to lateral cyclic loading. The experimental results show that the soil resistance hystereses have a tendency to move toward the previous peak point in fully-reversed cyclic loading, as would be expected in conventional peak-oriented hysteretic rules; however, the soil resistance hysteresis varies with the loading pattern, and the mobilized soil resistance in one-sided cyclic loading clearly diminished compared to that in fully-reversed cyclic loading.

Second, an analytical study using a very simple model consisting of a sand block-rigid body-sand block arrangement shows that the loading pattern dependency in the mobilized soil resistance is caused by the dilatancy behavior of soil. As the development of irreversible strain resulting from negative dilatancy in the surrounding soil varies with cyclic loading patterns, the mobilized soil resistance also varies.

Finally, parametric calculations with various loading patterns lead to the conclusion that the mobilized soil resistance to a pile is assumed to be dependent on the degree of eccentricity of cyclic loading direction. The twofold interpretations shown in Figs. 29 and 30 are concluded to represent the key functions in the loading pattern dependency of load transfer between soil and pile.

Based on these results, we propose that the influences of stress-dilatancy behavior on load transfer between soil and pile should be incorporated into modeling of soil-pile interaction under random cyclic loading. For example, a new hysteretic rule for p - γ curves that is capable of accounting for the key functions shown in Figs. 29 and 30 has been proposed and tested elsewhere (Shirato et al., 2006a, 2006b, 2005) for dynamic analyses of pile foundations employing the Beam on Nonlinear Winkler Foundation concept. When the soil surrounding a pile is

modeled with solid elements in a finite element analysis, it is considered to be desirable that the constitutive relation used for the soil is capable of accounting for stress-dilatancy behavior.

The simplified model used for calculations in this paper achieves this aim, as it succeeded in describing the soil resistance as a function of cyclic loading patterns resulting from the stress-dilatancy behavior in the surrounding soil. Simplification of the model involved neglecting several factors from the actual phenomena. For example, the simplified model disregards three dimensional effects in the distribution of stress and strain in the surrounding soil in the field. In addition, the soil material parameter values adopted in our calculations did not completely cover a wide range of densities. The values didn't include clay, while a similar loading pattern dependency has been observed in other experiments of laterally cyclic loaded piles embedded in both cohesive and cohesionless soils with various densities, as mentioned in the Introduction. We encourage future investigations that seek to confirm the results of this paper in terms of these remaining issues.

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