UPLIFT CAPACITY OF PILE GROUPS EMBEDDED IN SANDS: PREDICTIONS AND PERFORMANCE

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ABSTRACT
The paper pertains to the development of a simple semi-empirical method of analysis for predicting the uplift capacity of pile groups embedded in sand assuming an inverted truncated rectangular pyramidal failure surface. Various pile and soil parameters such as length, diameter of the pile, group configuration, spacing of the piles and the soil properties such as density, angle of internal friction and the pile-soil interface friction angle having direct influence on the uplift capacity of the pile group are incorporated in the analysis. The predicted values of uplift capacity of pile groups with different configuration and length to embedment ratio are then compared with model test results carried out as a part of the present investigation and also with the values reported in literature. The predictions are found to be in good agreement with the measured values validating the developed method of analysis.

Key words: limiting equilibrium, model test, pile, pile group, sand, uplift (IGC: E4)

INTRODUCTION
Prediction of uplift capacity of single piles and especially of pile groups is one of the most interesting areas of research in geotechnical engineers. Some of the studies conducted on the behavior of single piles under uplift loads are due to Sowa (1970), Vesic (1970), Meyerhof (1973), Das and Seeley (1975), Ismael and Klym (1979), Das (1983), Levacher and Sieffert (1984), Rao and Venkatesh (1985), Chattopadhyay and Pise (1986), and Ramasamy et al. (2004). These studies are very helpful in understanding the behavior of piles under tensile loads and predicting the value of uplift capacity of single piles. But, literature on such studies on the uplift capacity of pile groups embedded in sand are scanty (Das et al., 1976; Siddamal, 1989; Chattopadhyay, 1994; Patra and Pise, 2003). The theories to predict the uplift capacity of piles were developed mostly by extending the analysis of horizontal plate anchors under uplift loads assuming development of failure surfaces starting from the edges of the anchor. Based on experimental observation and test data, Meyerhof and Adams (1968) proposed a general theory of uplift resistance for a strip footing in soils with the assumption that soil mass having approximately truncated pyramidal shape is lifted up and for shallow footing the failure surface extends up to the ground surface. The theory was further modified to analyze circular footings and square and rectangular pile groups. Das et al. (1976) and Chattopadhyay (1994) concluded that the efficiency of a pile group varies with embedment length, number of piles in a group and spacing of piles in a group. Madhav (1987) studied the interaction between two identical piles in tension using boundary integral technique. The reduction in individual pile capacity due to the existence of another pile is quantified and it is found to depend on the spacing and length to diameter ratio of piles.

It is evident from the literature as cited above that most of the available studies are limited to single piles only. But increasing use of group of piles to resist and sustain uplift loads requires accurate assessment of uplift resistance for safe and economical design of pile foundations. As such, there is a need to develop methods to predict the uplift capacity of pile groups and such an analysis is presented in this study. The uplift resistance of pile groups depends on several variables like group size, shape, spacing, embedment length to diameter ratio of piles, soil type and its density and soil-pile friction angle. Considering some of the above parameters a simplified analysis based on limiting equilibrium is proposed to predict the uplift resistance of the group of piles. Laboratory model tests on group of piles were conducted in medium dense sand under axial uplift loads at different pile spacing and with varying embedment length to diameter ratio. The predicted values of uplift capacity were compared with the model test results so obtained and with other published experimental data to check the validity of the developed method of analysis.

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STATEMENT OF THE PROBLEM

Figure 1 shows a typical general pile group (a 2 x 2 pile group is shown here) of diameter \( d \) and length \( L \) embedded in soil. The plan dimensions of the pile group are \( a \) and \( b \). The friction angle of the soil is \( \phi \) and the pile-soil interface friction angle is \( \delta \). The object is to determine the ultimate uplift capacity of the pile group.

ANALYSIS

In studying the performance of piles with enlarged base, Dickin and Leung (1990) critically discussed the various possible types of slip surfaces namely the vertical slip surface model, inverted truncated cone or pyramidal model, curved slip surface model considered by several investigators in estimating the pile capacity. It is observed that the analysis is relatively simpler if the slip surface is assumed to be linear making an angle with the vertical that depends on the factors like friction angle and angle of dilatancy (\( \psi \)), which is a function of the relative density of the soil. From literature it is found that for single piles this angle has been assumed to be equal to any one of the following namely the dilatancy angle (\( \psi \)) (Vermeer and Sutjiadi, 1985), \( \phi/2 \) (Clemence and Veesaert, 1977), \( \phi \) (Murray and Geddes, 1987) or a function of \( \phi \) (Sutherland et al., 1982).

Based on the above discussion, in the present analysis an expression for the uplift capacity of pile group is derived based on the assumption that under limiting condition, soil mass around the pile group fails as an inverted truncated pyramidal solid body having similar cross section as that of the pile group extending up to the ground surface passing through the tip of the pile at an angle \( \beta \) from the vertical.

Derivation

In the limiting equilibrium condition, ultimate capacity of the pile is attained when the mobilized shear strength along the failure surface and the weights of the soil and piles balance the applied forces. A wedge of thickness \( \Delta Z \) at a height \( Z \) above the tip of the pile is considered; the wedge and its free body diagram are shown in Fig. 1.

The mobilized shear resistance \( \Delta T \) along the failure surface of length \( \Delta L \), at limiting condition is,

\[
\Delta T = \Delta R \tan \phi
\]  

(1)

Where \( \Delta R \) = Normal force acting on the failure of the wedge

\[
\Delta R = \Delta Q \cos \theta + K \Delta Q \sin \theta
\]  

(2)

Where \( \Delta Q = \gamma (L - Z - \frac{\Delta Z}{2}) \Delta L \)

(3)

Following Chattopadhyay and Pise (1986) the coefficient of lateral earth pressure within the wedge is taken as,

\[
K = (1 - \sin \phi) \frac{\tan \delta}{\tan \phi}
\]  

(4)

Substituting Eq. (3) and Eq. (4) into Eq. (2) we get,

\[
\Delta R = \gamma (L - Z - \frac{\Delta Z}{2}) (\cos \theta + K \sin \theta) \frac{\Delta L}{\sin \theta}
\]  

(5)

Substituting Eq. (5) into Eq. (1), we get

\[
\Delta T = \gamma (L - Z - \frac{\Delta Z}{2}) (\cos \theta + K \sin \theta) \frac{\Delta Z \tan \phi}{\sin \theta}
\]  

(6)

Considering the vertical equilibrium of the wedge and assuming that the weight of the pile group of the length \( \Delta Z \) is equal to the total weight of the soil corresponding to the volume occupied by each pile in the group for the length \( \Delta Z \).

\[
(P + \Delta P) = P + q(a + 2x)(b + 2x)
\]

\[-(q + \Delta q)(a + 2x + 2\Delta x)(b + 2x + 2\Delta x)
\]

\[-\Delta W - 2(a + b + 4x + 2\Delta x)\Delta T \sin \theta = 0
\]  

(7)

Substituting Eq. (6) into Eq. (7) and on simplification we get

\[
\frac{dP}{dZ} = 2q(a + 2x) \frac{dx}{dZ} + 2q(b + 2x) \frac{dx}{dZ}
\]

\[\quad + (a + 2x)(b + 2x) \left( \frac{dq}{dZ} + \frac{dW}{dZ} \right) + 2(a + b + 4x)\gamma(L - Z)M \]

(8)

where \( M = (\cos \theta + K \sin \theta) \tan \phi \)

Referring to Fig. 1 the following relations are obtained,

\[
\frac{dx}{dZ} = \cot \theta \quad x = Z \cot \theta
\]

\[
q = \gamma(L - Z) \frac{dq}{dz} = -\gamma
\]

This on substitution into Eq. (8) yields,

\[
\frac{dP}{dZ} = 2q(a + 2Z \cot \theta) \cot \theta + 2q(b + 2Z \cot \theta) \cot \theta
\]

\[\quad + (a + 2Z \cot \theta)(b + 2Z \cot \theta)(-\gamma)
\]

\[\quad + \frac{dW}{dZ} + 2(a + b + 4Z \cot \theta) \gamma(L - Z)M \]

(9)
where \( \frac{dW}{dZ} = (a + 2x)(b + 2x) \gamma \)

Setting \( q = \gamma (L - Z) \) and \( K_1 = a + b \) and substituting Eq. (10) into Eq. (9) the Eq. (9) can further be simplified to a form as given below

\[
\frac{dP}{dZ} = 2\gamma L K_1 \cot \theta + 8\gamma LZ \cot^2 \theta - 2\gamma ZK_1 \cot \theta
- 8\gamma Z^2 \cot^2 \theta + 2K_1 \gamma LM - 2K_1 \gamma ZM
+ 8\gamma ZLM \cot \theta - 8\gamma MZ^2 \cot \theta
\]

\[
\frac{dP}{dZ} = 2\gamma L^2 K_1 \cot \theta + 4\gamma \cot^2 \theta L^2 - \gamma K_1 \cot \theta L^2
- 8\gamma \cot^2 \theta \frac{L^3}{3} + 2K_1 \gamma L^3 M - K_1 \gamma ML^3
+ 4\gamma M \cot \theta L^2 - 8\gamma M \cot \theta \frac{L^3}{3}
\]

\[
P_o = K_1 \gamma (\cot \theta + M) L^2 + \frac{4}{3} \gamma \cot \theta (\cot \theta + M) L^3
\]

The net uplift capacity of pile group is,

\[
P_m = \gamma L^2 \left( C_1 K_1 + C_2 L \right) - W_s
\]

Where, \( C_1 \) and \( C_2 \) are the dimensionless constants equal to \( (\cot \theta + M) \) and \( \frac{4}{3} \cot \theta (\cot \theta + M) \) respectively.

\[
W_s = \text{the self weight of the pile group} = \frac{n \gamma \pi d^2}{4}
\]

Where, \( n \) is the number of piles in a group.

For \( L/d > 20 \) the failure surface is assumed tangential to the pile surface up to 0.3L from the tip of the pile. Hence Eq. (11) is integrated between the limits 0 to 0.7L and added to the skin friction developed in the remaining length.

It is to be noted that the pile group capacity will be the minimum of the values predicted by considering group action or the capacity of an individual pile multiplied by the number of piles in the group. However, it is observed that only for spacing beyond 6d piles in a group acts individually. The present paper is concerned with the group action only that is consistent with experimental results. In recommending the capacity that is to be adopted in design one must consider which one of the above two mechanisms gives the minimum value and adopt the same.

The above predictive model is validated by experimental studies made on model piles, which are as follows.

**EXPERIMENTAL DETAILS**

Tests on model pile groups were conducted in a steel tank (size 990 mm x 975 mm x 970 mm). The tank was sufficiently large enough to take care of the effect of the edges of the tank on the test results as the zone of influence of the piles and loading there on is reported to be in the range of 3-8 pile diameters (Kishida, 1963).

Model piles were prepared from mild steel rod of 20 mm x 20 mm cross section. The length of embedment of pile, \( L \) in sand bed was 400 mm, 600 mm and 800 mm resulting \( L/d \) as 20, 30 and 40 respectively. The pile-soil interface friction angle \( (\phi) \) was found to be 26° from the direct shear test. Pile caps were prepared for 2 x 1, 3 x 1, 2 x 2, 3 x 2 and 3 x 3 pile groups (at 3d, 4d and 6d spacing) using 12 mm thick mild steel plate. As most of the previously studied model test were conducted for \( L/d \) ratio ranging from 12 to 20 with only a few tests conducted with higher \( L/d \) values, in the present study the same has been taken on higher side i.e 20, 30 and 40. With the addition of data with higher \( L/d \) ratio the data bank represented short, intermediate and long piles. The spacing ranging from 3d to 6d is generally adopted in design. As such, the same was chosen representing piles with closer to large spacing.

The model piles were embedded in homogeneous dry sand bed (made by rainfall technique) composed of uniformly graded Ennore sand having uniformity coefficient of 1.71 and specific gravity of 2.69. The maximum and minimum dry densities of the sand were found to be 16.2 and 14.74 kN/m³ respectively. Sand was poured uniformly into the tank through the slot hopper keeping the height of fall of 300 mm. By using above technique medium dense bed was prepared with a dry unit weight \( (\gamma) \) of 15.8 kN/m³, corresponding to relative density \( (D_r) \) of 54.3% and angle of shearing resistance \( (\phi) \) of 38°.

Piles were subjected to tensile loading through a pulley arrangement with a flexible wire whose end was attached with the pile cap and the other end with a loading pan over which dead loads are gradually placed in stages. A schematic diagram of the complete experimen-
tal set-up with the loading system and pile in place and ready for test is shown in Fig. 2. Two dial gauges with magnetic base having sensitivity of 0.01 mm were used to measure the displacement placing them on the pile cap at 180° apart and equidistant from load axis.

RESULTS AND DISCUSSION

Series of tests were conducted in medium dense soil to study the effect of pile spacing (s), length to diameter ratio (L/d) and number of piles in a group (n) on the uplift capacity of model pile groups. The load versus displacement curves were plotted for all the pile groups and some of them presented in Figs. 3(a), 3(b) and 3(c). The load-displacement curves show similar behavior for different spacing and length to diameter ratio. By using the double tangent method the gross ultimate uplift load was found. The net ultimate uplift load of a pile group was reworked by subtracting the corresponding self weight of piles and cap.

Variations of group efficiency with respect to pile spacing for different pile groups with L/d ratio being 20, 30 and 40 are presented in Figs. 4(a), 4(b) and 4(c) respectively. From these figures it is observed that there is a definite trend of increase in the group efficiency with the increase of pile spacing. From Fig. 4(b) for a 3 x 3 pile group the efficiency is increasing from 49% to 60% as the spacing changing from 3d to 6d. Similar trend has been observed for other pile groups. Further, it is observed that the group efficiency is decreasing with the increasing number of piles in a group. For any given spacing the maximum efficiency has been observed for 2 x 1 pile group and minimum efficiency for 3 x 3 pile group. From Figs. 5(a) and 5(b) it is observed that the group efficiency is significantly affected by length to diameter ratio of the pile group. For any given pile group and spacing the group efficiency is decreasing as the length to diameter ratio of the pile group increasing. From Fig. 5(b), for a 3 x 1 pile group at 3d spacing as the L/d ratio increases from 20 to 40 the reduction in the group efficiency is around 14%.

The theoretical values of the net uplift capacities for different group of piles considered for experimental studies were estimated from Eq. (14). Different trial values of β, the angle that the slip surface makes with the vertical were taken to estimate the theoretical pile group capacity and compared the same with experimental observations. It was observed that an angle equal to φ/4 gives values that are in good agreement with experimental results. The same is adopted for further predictions. A comparison of the predicted and measured values as shown in Figs. 6(a), 6(b) and 6(c) demonstrates a close agreement as most of the data points lie very close to the ideal line. A quantitative comparative study was conducted to estimate the deviation of the predicted uplift capacity from the measured one. It was observed that for 89% of the data (40 out of 45) the deviation was within 30% and for 55% of the data (26 out of 45) the error was even less than 10%. Thus the absolute relative errors between the predicted and measured values lie in general in a range which may be considered to be well within the range of experimental error and errors inherent to the models. However, for the sake of space and brevity these computations are not presented here.

While doing the model testing, an attempt was made to check the validity of the assumption of plane failure
surface by introducing wet soft tissue papers at different levels in the soil strata. However, the adopted technique did not work and the verification of the assumption regarding the slip surface could not be made.

After comparing the predictions made by using the present method with the experimental data obtained from the present investigation, the correctness of the model were further checked with the experimental data reported by various investigators as follows.

Das et al. (1976) reported net uplift capacity in terms of efficiency of pile groups of size $2 \times 1$, $3 \times 1$, $4 \times 1$, $2 \times 2$, $3 \times 2$, $3 \times 3$. Rough wooden piles of diameter 12.7 mm and $L/d = 24$ were used as model piles. Medium condition of sand having $\gamma = 15.10$ kN/m$^3$, $\phi = 31^\circ$ and $D_r = 21\%$ was used as the soil media. Pile-soil interface friction angle $\delta$ was taken as $19.3^\circ$ (Das, 1983). The measured values of the net uplift capacity of pile groups were indirectly evaluated from the reported efficiency diagram. The measured values of the net uplift capacities for different groups and the corresponding predicted uplift capacities using Eq. (14) are plotted in Fig. 7. Most of the data lie very close to the ideal line. The percentage of error in the predicted values from the measured one is found to be less than 30% for 83% of the total data (10 data points out of 12 data points).

Siddamal (1989) reported axial uplift test results on model mild steel pile groups of size $2 \times 1$ and $2 \times 2$ having, $L/d = 20$ and 40. The spacing of 2, 4 and 6 times the pile diameter for $2 \times 1$ pile groups and 2 and 4 pile diameter for $2 \times 2$ pile groups were used. The diameter of the pile
was 20 mm. Dry sand having $\gamma = 16.1$ kN/m$^3$, $\phi = 40.5^\circ$ and $\delta = 23^\circ$ was used as the foundation medium. Figure 8 shows the comparison between the predicted values of uplift resistance by the present theory with the measured values of uplift test results. It is seen that the predicted values are in reasonable agreement with the experimental values with 70% of the data having error, less than 30%.

Chattopadhyay (1994) reported limited uplift test results for model pile groups of size $2 \times 1$ and $2 \times 2$ and $L/d = 15.78, 23.68$ and 31.57. Aluminium tubes of outer diameter 19 mm were used as piles. The groups were tested for spacing 2.3d, 4d, 5d and 6d. Coarse to medium sand with $D_{10} = 0.95$ mm, $D_{50} = 48$ mm, $C_0 = 1.98$, $\phi = 40^\circ$ and $\delta = 25^\circ$ ($2/3\phi$) and unit weight of $\gamma = 17.00$ kN/m$^3$ were used. From the load displacement diagrams, the net
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Fig. 9. Comparison with Chattopadhyay (1994) model test results

Fig. 10. Comparison with Patra and Pise (2003) model test results

uplift capacity of pile and pile groups were measured. The theoretical and measured net uplift capacities are plotted in Fig. 9 showing the predictions to be in close proximity to the ideal line with 67% of the data (4 out of 6) having error, less than 30%.

Patra and Pise (2003) conducted model test on pile groups of size 2 x 1, 3 x 1, 2 x 2, 3 x 2 using dry Ennore sand as foundation medium. The specific gravity and uniformity coefficient of the sand were 2.64 and 1.6 respectively. The unit weight of the sand during testing was 16.4 kN/m³ (Dₚ = 80%). The corresponding angle of shearing resistance φ = 37°. Aluminum alloy tubes of 19 mm outer diameter, 0.81 mm wall thickness were used as model piles. To increase the wall friction of pile, fine Ennore sand was glued around the pile by adhesive. The lengths to diameter ratios of piles were 12 and 38. The pile-soil interface friction angle δ between smooth and rough surfaces of piles and sand was evaluated from the direct shear test results and reported as 20° and 31° respectively. The theoretical and measured net uplift capacity of rough pile groups for L/d ratio of 12 are plotted for comparison in Fig. 10. Most of the data (9 out of 12 i.e 75%) is very close to the ideal line with error, less than 30%. Thus, it is seen that the semi-empirical model developed presently acquires itself quite well in predicting the values of uplift capacity of pile groups that are in close agreement with measured values.

CONCLUSIONS

Based on the studies reported in the previous sections it can be concluded that the proposed model has excellent potential in predicting the uplift capacity of pile groups embedded in sand. The error margin between the predicted values and the experimentally observed values in general is well within 30% (which may attribute to either experimental error or error due to the model) and may be considered to be permissible in such predictions. However, the predictive model needs to be verified with field tests and for spacing beyond 6d.

NOTATION

The following symbols are used in this paper.

- a and b = plan dimensions of the pile group
- C₁ and C₂ = dimensionless constants
- Dₚ = relative density
- d = pile diameter
- K = lateral earth pressure coefficient
- L = embedded length of pile
- L/d = ratio of embedded length to diameter of pile
- Pᵤ = net ultimate uplift capacity of pile
- Wₑ = self weight of the pile group
- Z = thickness of wedge element
- θ = angle of failure surface with horizontal
- β = angle of failure surface with vertical
- ϕ = angle of internal friction of the soil
- δ = pile-soil interface friction angle
- γ = unit weight of the soil

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