SMALL STRAIN STIFFNESS AND NON-LINEAR STRESS-STRAIN BEHAVIOUR OF CEMENT-MIXED GRAVELLY SOIL

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ABSTRACT

The stiffness at small strains and non-linear stress-strain relation of compacted cement-mixed well-graded gravelly soil as well as the ageing effects were evaluated by drained triaxial compression tests on compacted moist specimens cured for different periods at isotropic and different anisotropic stress states. In all the tests, the initial stress-strain relation at small strains less than about 0.001 \( \varepsilon \) was essentially elastic and the initial Young’s modulus, \( E_0 \), was essentially the same as the \( E_{eq} \) value evaluated by applying unload/reload cycles under otherwise the same conditions. The \( E_{eq} \) values were rather independent of strain rate. The \( E_{eq} \) value from the first unload/reload cycle applied during otherwise continuous ML became noticeably lower than the elastic modulus evaluated at the same stress state, more as approaching the peak stress state. After a number of small unload/reload cycles and long sustained loading, the \( E_{eq} \) value became closer to the elastic modulus due to a decrease in the viscous effects. The ratio of \( E_0 \) to the compressive strength (\( q_{\text{max}} \)) was similar to that of concrete but noticeably larger than those of uncompacted cement-mixed soil, sedimentary softrock and unbound gravelly soil. Both \( E_0 \) and \( q_{\text{max}} \) increased with time by ageing, while the \( E_0/q_{\text{max}} \) ratio decreased with time. When ML was restarted at a constant strain rate after ageing with a shear stress, the tangent stiffness became very high for a large stress range with a substantial change in the non-linearity of stress-strain relation.

Key words: ageing effect, cement-mixed gravelly soil, non-linearity, small strain stiffness, triaxial compression test, viscous property (IGC: D6/D7)

INTRODUCTION

The first new type bridge abutment comprising the backfill of well-compacted cement-mixed well-graded gravelly soil was successfully constructed for a new bullet train line (Shinkansen) in 2003 in Kyushu, Japan (Aoki et al., 2003; Watanabe et al., 2003b; Tatsuoka, 2004). This case typically shows that such permanent civil engineering structures as the bridge abutment described above that need a high stability while allowing a limited amount of instantaneous and residual deformation can be constructed by using cement-mixed backfill. In this project, a series of geotechnical investigation were performed including those on the stress-strain characteristics of compacted cement-mixed gravelly soil. Watanabe et al. (2003a), Kongsukprasert and Tatsuoka (2003), Lohani et al. (2004) and Kongsukprasert et al. (2005) reported the effects of influencing factors (i.e., compaction conditions, cement content, curing conditions, strain rate and so on). Kongsukprasert and Tatsuoka (2005) reported significant effects of ageing with a shear stress and loading history on the stress-strain behaviour and peak strength as well as the viscous effects on these properties. Those papers reported mainly the peak strength and global stress-strain behaviour. On the other hand, accurate evaluation of small strain stiffness and pre-peak non-linearity of stress-strain relation of this material becomes necessary to reliably analyse the deformation when subjected to static and dynamic loading of structures comprising this material. Systematic studies on this issue cannot be found in the literature when compared with those on other artificial bound particulate or granular materials, such as compacted dam concrete (e.g., Hansen and Reinhardt, 1990; Schrader, 1996), cement-mixed sand (e.g., Barbosa-Cruz and Tatsuoka, 1999, 2000; Kongsukprasert et al., 2001) and cement-mixed soft clay (e.g., Sugai et al., 2000; Sugai and Tatsuoka, 2003). Similarities and differences in the strength and deformation properties among these bound materials having different stiffness and strength values are analysed in this paper and in Kongsukprasert et al. (2007).

This paper reports first the small strain stiffness and pre-peak non-linearity of stress-strain relation of compacted cement-mixed well-graded gravelly soil obtained from a comprehensive series of consolidated drained (CD) triaxial compression (TC) tests. In particular, the effects of strain rate and curing period on the small strain stiffness at small strains and non-linear stress-strain relation of compacted cement-mixed well-graded gravelly soil as well as the ageing effects were evaluated by drained triaxial compression tests on compacted moist specimens cured for different periods at isotropic and different anisotropic stress states. In all the tests, the initial stress-strain relation at small strains less than about 0.001 \( \varepsilon \) was essentially elastic and the initial Young’s modulus, \( E_0 \), was essentially the same as the \( E_{eq} \) value evaluated by applying unload/reload cycles under otherwise the same conditions. The \( E_{eq} \) values were rather independent of strain rate. The \( E_{eq} \) value from the first unload/reload cycle applied during otherwise continuous ML became noticeably lower than the elastic modulus evaluated at the same stress state, more as approaching the peak stress state. After a number of small unload/reload cycles and long sustained loading, the \( E_{eq} \) value became closer to the elastic modulus due to a decrease in the viscous effects. The ratio of \( E_0 \) to the compressive strength (\( q_{\text{max}} \)) was similar to that of concrete but noticeably larger than those of uncompacted cement-mixed soil, sedimentary softrock and unbound gravelly soil. Both \( E_0 \) and \( q_{\text{max}} \) increased with time by ageing, while the \( E_0/q_{\text{max}} \) ratio decreased with time. When ML was restarted at a constant strain rate after ageing with a shear stress, the tangent stiffness became very high for a large stress range with a substantial change in the non-linearity of stress-strain relation.

Key words: ageing effect, cement-mixed gravelly soil, non-linearity, small strain stiffness, triaxial compression test, viscous property (IGC: D6/D7)
stiffness are reported. The ratio of small strain stiffness to compressive strength of this material is compared with those of concretes, other types of cement-mixed soil, sedimentary soft rocks and unbound granular materials. The effects of ageing with a shear stress on the small strain stiffness and non-linearity of stress-strain relation are then reported.

**TEST METHOD**

The test material and test method employed in the present study as well as setting of local deformation transducers (LDTs) are described below only briefly as Kongsukprasert et al. (2005) and Kongsukprasert and Tatsuoka (2005) described them in detail.

**Test Material**

A well-graded quarry gravelly soil of crushed sandstone with a specific gravity, \( G_s \), equal to 2.71 so-called ‘Chiba gravel’, was sieved to remove particles larger than 10 mm to obtain the test material (called ‘model Chiba gravel’ with \( G_s = 2.74 \)). Materials of five different batches, approximated 200 kg each, were used (Fig. 1). The batch number of the material used to prepare the respective specimen is shown in the respective table describing the test conditions and/or other relevant places. Effects of using different batches on the test results were insignificant (Kongsukprasert, 2003; Kongsukprasert and Tatsuoka, 2005). Ordinary Portland cement with \( G_s = 3.16 \) obtained from a single bag of 20 kg kept in an airtight container was used throughout the present study.

**Specimen Preparation**

Compaction tests were performed on batch No. 1 material using a mould with an inner diameter of 10 cm and a volume of 1,000 cm\(^3\) at an energy level \( E_0 = 550 \) kJ/m\(^2\) (the standard Proctor). The optimum water contents, \( w_{opt} \), for the original material (without cement) and the cement-mixed material (with a cement-to-gravel ratio by dry weight, \( c/g \), equal to 2.5%) were essentially the same (about 8.75%; Fig. 2).

Cement and gravel prepared according to the respective pre-scribed \( c/g \) value were mixed thoroughly before adding a relevant amount of water so that the initial water content, \( w_i \), in the ratio to the dry weight of soil plus cement became \( w_{opt} \) for \( E_0 \) (= 8.75%). The mixture was kept in a closed container during the whole subsequent preparation process, approximately 25–35 minutes, to keep the water content constant. The specimens were compacted manually in five even layers in a rectangular prismatic mould (95 mm × 95 mm × 190 mm) to the respective target compacted dry density of solid (\( \rho_d \)). The compacted specimens were cured being sealed inside the compaction mould under the atmospheric pressure at constant water content in a temperature-controlled room (25°C) for five days. When the initial curing period (defined zero at the start of compaction), \( t_{ini} \), entered the fifth day, the specimens were removed from the mould and wrapped with a piece of kitchen wrapping plastic sheet for further curing under the atmospheric pressure at constant water content. The specimens were subjected to drained TC loading at the water content when prepared and cured, without water-saturation.

**Triaxial Compression Test Procedures**

An automated displacement-controlled triaxial apparatus consisting of a precision gear system that responds sharply to control signals from a computer without any noticeable backlash at load reversal was used (Tatsuoka et al., 1994, 1999a; Santucci de Magistris et al., 1999). Axial strains were sensitively and accurately measured by using a pair of 160 mm-long LDTs (Goto et al., 1991) set on a pair of the opposite side faces of the specimen. To alleviate the effects of membrane penetration and bedding error, lateral strains were measured locally in the horizontal direction parallel to the side face of the rectangular prismatic specimen by using three pairs of 70 mm-long lateral LDTs set at 5/6, 3/6 and 1/6 of the specimen height on a pair of the opposite specimen side faces. Isotropic deformation in the horizontal planes was assumed to obtain average local lateral strains from the readings of the lateral LDTs. The detailed setup of LDTs...
is illustrated in Kongsukprasert et al. (2005) and Kongsukprasert and Tatsuoka (2005).

The specimen was isotropically consolidated to a confining pressure ($\sigma_c$) equal to 19.8 kPa by means of partial vacuum and cured for one hour, unless otherwise specified, before the start of ML drained TC at constant $\sigma_c$ (= 19.8 kPa). The axial strain rates reported in this paper, unless otherwise indicated, are those obtained from the displacement rates of the loading piston, which differs to varying extents from those measured locally due to effects of system compliance and bedding error. Locally measured axial strain rates were used when evaluating the effects of strain rate on measured quantities. At a number of arbitrary stress states during otherwise continuous monotonic loading (ML), one or five unload/reload cycles with a single axial strain amplitude of about 0.005% were applied to evaluate the small strain stiffness at each stress state. In some TC tests, sustained loading stage was applied for some period before applying small unload/reload cycles.

### EVALUATION OF SMALL STRAIN STIFFNESS

#### Background

The small strain stiffness (or the elastic modulus) of geomaterial is usually evaluated by either dynamic tests (i.e., resonant-column tests and wave propagation tests) or static tests (i.e., monotonic or cyclic loading tests measuring stresses and strains) or both. When the wave length is too short compared to the scale of in-homogeneity of a test specimen or mass, the small strain stiffness measured by wave propagation tests may represent the stiffness of relatively stiff part that is much larger than the average stiffness as measured statically. Otherwise, when measured at the same stress conditions with the same small strain amplitude, corresponding dynamically and statically measured small strain stiffness values should be essentially the same, in particular when accounting for effects of strain rate (e.g., Tatsuoka and Kohata, 1995; Tatsuoka et al., 1995, 1999a, 1999b). In the present study, by taking into account the fact that the specimens of compacted well-graded gravely soil are not highly homogeneous, the static tests are relevant to evaluate the average stiffness at small strains.

To evaluate the small strain stiffness, a single or multiple unload/reload cycle(s) with a single amplitude axial strain of the order of 0.001% is usually applied at different isotropic stress states and also at different anisotropic stress states during otherwise ML of TC, for example. The elastic Young’s modulus, $E_e$, is ideally defined as the slope of a reversible stress-strain relation of an unload-reload cycle while the initial value at small strains is specifically defined as the initial Young’s modulus, $E_0$ (Fig. 3). However, an actual hysteretic stress-strain relation of an unload-reload cycle cannot be perfectly reversible due to visco-plastic deformation while the overall stress-strain relation is shifted toward larger strains due to the viscous property of the test material. The value of peak-to-peak secant modulus from a unload/reload cycle is defined as the equivalent Young’s modulus, $E_{eq}$, as shown in Fig. 3, to be distinguished from the elastic Young’s modulus, $E_e$. The $E_{eq}$ value may be essentially the same as, or lower than, the $E_e$ value at the same stress state. This difference is one of the topics discussed in this paper.

The following remarks are relevant to the small strain stiffness obtained as above of unbound geomaterial (i.e., clay, sand and gravel):

1. Even at very small strains less than 0.001%, the stress-strain behaviour cannot be perfectly elastic but rate-dependent at different degrees, as illustrated in Fig. 4. In a ML test, an elastic zone may appear at the initial stage and it becomes larger with an increase in the strain rate, while it may disappear at very low strain rates. Therefore, in triaxial tests, as the strain rate (or the loading frequency for cyclic loading) becomes smaller, the values of $E_0$ and $E_{eq}$ defined at an axial strain (or axial strain amplitude) of the order of 0.001% become smaller to a larger extent than the elastic Young’s modulus $E_e$ obtained for the stress-strain behaviour that is essentially elastic (i.e., independent of strain rate) and linear.

2. The $E_e$ value defined as $(d\sigma_e/d\epsilon_e)_{\epsilon_e=0}$ of geomaterial are essentially a function of the instantaneous effective axial stress, $\sigma'_e$ (i.e., the hypo-elastic elasticity; e.g., Hoque and Tatsuoka, 1998). When approaching the failure state, the micro-structure is
inevitably permanently changed by irreversible shear
strain, which may reduce the elastic modulus, $E^e$, 
compared to the value predicted based on such an 
empirical relation as above, established based on the 
data from tests without this damaging effect (e.g., 
Hoque and Tatsuoka, 2004). Moreover, loading 
history, including cyclic loading with different
numbers of cycles, may affect noticeably the $E^e$ value 
(e.g., Hoque and Tatsuoka, 2004).

3) The values of $E_{eq}$ evaluated by applying small unload/
reload cycle(s) at arbitrary stress states during other-
wise ML TC at a constant strain rate are inevitably 
subjected to effects of visco-plastic strains and 
become a function of closeness to the peak stress 
state as well as strain amplitude and strain rate 
applied during unload/reload cycles. That is, the 
stress-strain behaviour during respective unload/
reload cycle may not be perfectly elastic but may 
include a large inelastic (or irreversible) strain com-
ponent, which may make difficult the reliable evalua-
tion of elastic deformation characteristics by this 
method.

These trends of behaviour are also relevant to cement-
mixed sand (Kohata et al., 1997). Moreover, not only 
the peak strength and pre-peak stress-strain behaviour 
but also the small-strain stiffness of cement-mixed 
soil largely changes with time by ageing effects. With cement-mixed 
well-graded gravelly soil, these issues listed above 
have not been systematically studied and therefore are not well 
understood. These issues are analysed based on triaxial 
test results in the following.

The Poisson ratio, the other important elastic defor-
mation property, is not discussed here owing to the paper 
page limit. This will be a discussion topic in a further 
paper by the authors.

Effects of Strain Rate on Small Strain Stiffness

Figures 5(a) and 5(b) show results from three CD TC 
tests performed at relatively high axial strain rates, $\dot{\varepsilon}_a$, 
so that the ageing effects developing during ML became 
significant (Table 1; Kongsukprasert and Tatsuoka, 
2005). Figure 5(c) shows the initial stress-strain relations 
at small strains from these tests. A relatively large scatter 
seen in the data in Fig. 5(c) is probably due to a limited 
accuracy of the load cell and a limited resolution of the 
LDTs. In the first two tests, ML was continued at $\dot{\varepsilon}_a = 
0.01\%$/min (test JS001) or 0.03 $\%$/min (test J023). In 
another test (test JS002), the $\dot{\varepsilon}_a$ value was changed step-
wise many times during otherwise ML at a constant $\dot{\varepsilon}_a$, 
either 0.01$/min or 0.03$/min. In these three tests, one 
small unload/reload cycle was applied many times during 
otherwise ML at a constant axial strain rate. It may be 
seen that, despite that the effect of strain rate is noticeable 
in the overall pre-peak behaviour (Figs. 5(a) and 5(b)), it 
is insignificant at small strains (Fig. 5(c)). At these small 
strains (Fig. 5(c)), the initial stiñness $E_0$ deñined at strains 
less than about 0.001$\%$ from the ML stress-strain curves 
is essentially the same as the $E_{eq}$ values from unload/
reload cycles applied when $q$ was still small. To exam-
in these issues more in detail, a more comprehensive 
series of tests was performed as shown below.

In another test (SP012; Table 2), unload-reload cycles 
were applied at different axial strain rates for a deviator 
stress range between 0 and 180 kPa. The nominal axial 
strain rates were 0.0012, 0.03, and 0.15$/min, while 
those based on locally measured axial strains were smaller

![Fig. 5. Results from three CD TC tests to evaluate the effects of strain rate (Kongsukprasert and Tatsuoka, 2005). (Arrows in Figs. a and b show the moments when ML at respectively indicated strain rates started)](image-url)
by about one order of magnitude, 0.000086, 0.00193, and 0.01/\% min. Figures 6(a) and 6(b) show two typical stress-strain curves of the reloading part of the unload/reload cycles. These curves were fitted by:

$$q = a + b(e_c) + c(e_c)^2$$  \hspace{1cm} (1)

where \(a, b, c\) and \(e\) are constants. The values of \(E_{\text{eq}} = (q/e_{\text{eq}})\) listed in Table 2 were obtained from the \(q\) value obtained by substituting \(e_{\text{eq}} = 0.001\%\) into Eq. (1). These \(E_{\text{eq}}\) values are plotted in Fig. 7.

Figure 7 summarises the strain rate-dependency of the initial stiffness values \(E_0\) (defined at \(e_{\text{eq}} = 0.001\%\)) and the values of \(E_{\text{eq}}\) (defined for a single amplitude at \(\Delta e_{\text{eq}} = 0.001\%\)) of various types of geomaterial obtained as follows:

1. Cyclic triaxial tests (U = undrained; D = drained) performed on specimens of:
   - Sagamihara soft mudstone \(\sigma'_c = 470.4\text{ kPa}; Tatsuoka et al., 1995\);
   - OAP clay \(\sigma'_c = 676.2\text{ kPa and } \sigma'_k = 333.2\text{ kPa}; Tatsuoka et al., 1995\);
   - Chiba gravel \(\sigma'_c = 0.247, w = 3.7\%\) and \(\sigma'_c = 19.8\text{ kPa}; Jiang et al., 1997\);
   - air-pluviated Toyoura sand \(\sigma'_c = 0.658\) and \(\sigma'_k = 98\text{ kPa}; Tatsuoka et al., 1995\);
   - air-pluviated Hostun sand \(\sigma'_c = 78.4 - 245\text{ kPa and } \sigma'_k = 78.4\text{ kPa}; Hoque, 1996\);
   - and compacted Metramo silty sand \(\sigma'_c = 392\text{ kPa}; Santucci de Magistris et al., 1999\);
   - compacted cemented soil and gravel (CSG) \(\sigma'_c = 200\text{ kPa}; Omae et al., 2003\).

2. Consolidated undrained (CU) TC tests on normal consolidated (NC) kaolin \(\rho'_c = 29.4\text{ kPa and } K_c = 0.6 - 1.0; Tatsuoka et al., 1995\).

3. Unconfined cyclic tests and ultrasonic tests on hard rock cores, concrete and mortar; and resonant-column tests on concrete and mortar (Sato et al., 1997a and b). The strain rates in the ultrasonic tests were evaluated from the wave frequency and particle velocity.

It may be seen from Fig. 7 that the Young’s modulus defined at a strain level or for a strain amplitude as small as 0.0001\% is generally insensitive to the strain rate. With hard rock cores, unbound dense sands and gravels as well as compacted cement-mixed well-graded gravelly soil from the study by Omae et al. (2003) and the present study, the stress-strain behaviour at small strain of

<table>
<thead>
<tr>
<th>Test</th>
<th>Batch no.</th>
<th>(w_i) (%)</th>
<th>(\rho_i) (g/cm(^3))</th>
<th>(c/g) (%)</th>
<th>Initial curing period, (t_{\text{ini}}^c) (day)</th>
<th>Total curing, (t_{\text{total}}^c) (day)</th>
<th>TC loading mode</th>
<th>Axial strain rate (%/min)</th>
<th>(E_0) (GPa)</th>
<th>(q_{\text{max}}) (kPa)</th>
<th>(e_{\text{at}}) at (q_{\text{max}}) (%)</th>
<th>(E_0/q_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>J023</td>
<td>1</td>
<td>8.746</td>
<td>2.000</td>
<td>2.500</td>
<td>7.04</td>
<td>7.07</td>
<td>ML</td>
<td>(x^b)</td>
<td>5.33</td>
<td>1.988</td>
<td>0.312</td>
<td>2.682</td>
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<tr>
<td>JS001</td>
<td></td>
<td>8.752</td>
<td>2.000</td>
<td>2.498</td>
<td>7.14</td>
<td>7.23</td>
<td>ML</td>
<td>(x/3)</td>
<td>6.05</td>
<td>1.745</td>
<td>0.325</td>
<td>3.466</td>
</tr>
<tr>
<td>JS002</td>
<td></td>
<td>8.750</td>
<td>2.000</td>
<td>2.499</td>
<td>7.16</td>
<td>7.18</td>
<td>Stepwise</td>
<td>(x/20, x/3, x_{3.34x})</td>
<td>5.69</td>
<td>1.912</td>
<td>0.296</td>
<td>2.973</td>
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</tbody>
</table>

\(^a\)Time elapsed from start of compaction until end of 1 hour consolidation.

\(^b\)Time elapsed from start of compaction until peak state.

\(^c\)Nominal axial strain rate, \(x = 0.03\%/min\).

Table 2. Results from three ML TC tests to investigate the loading rate effects

<table>
<thead>
<tr>
<th>Cycle no.</th>
<th>(\epsilon_{\text{eq}}^1) (%)</th>
<th>(\epsilon_{\text{eq}}^2) (%)</th>
<th>(\Delta t) (sec)</th>
<th>(\Delta e) (%)</th>
<th>Local axial strain rate (%/min)</th>
<th>Average local axial strain rate (%/min)</th>
<th>(E_{\text{eq}}) (GPa)</th>
<th>Average (E_{\text{eq}}) (GPa)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000</td>
<td>0.00390</td>
<td>119</td>
<td>0.00390</td>
<td>0.00197</td>
<td>0.001933</td>
<td>4.405</td>
<td>4.593</td>
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<td>2</td>
<td>0.000325</td>
<td>0.00455</td>
<td>117</td>
<td>0.00422</td>
<td>0.00217</td>
<td>0.001933</td>
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<tr>
<td>3</td>
<td>0.001300</td>
<td>0.00487</td>
<td>119</td>
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<td>0.00180</td>
<td>0.001933</td>
<td>4.512</td>
<td>4.593</td>
</tr>
<tr>
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<td>0.001300</td>
<td>0.00487</td>
<td>119</td>
<td>0.00357</td>
<td>0.00180</td>
<td>0.001933</td>
<td>4.623</td>
<td>4.643</td>
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<tr>
<td>5</td>
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<td>0.00455</td>
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<td>0.00009</td>
<td>0.001933</td>
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<tr>
<td>6</td>
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<td>0.00422</td>
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<td>0.00389</td>
<td>0.00008</td>
<td>0.001933</td>
<td>4.712</td>
<td>4.721</td>
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<tr>
<td>7</td>
<td>0.001300</td>
<td>0.00520</td>
<td>23</td>
<td>0.00390</td>
<td>0.0017</td>
<td>0.001933</td>
<td>4.619</td>
<td>4.721</td>
</tr>
<tr>
<td>8</td>
<td>0.000000</td>
<td>0.00390</td>
<td>2,810</td>
<td>0.00389</td>
<td>0.00008</td>
<td>0.001933</td>
<td>4.761</td>
<td>4.792</td>
</tr>
</tbody>
</table>

\(^d\)Average axial strain at the start of reloading.
0.001% exhibits a particularly small strain rate-dependency. In this case, the stiffness value is close to the truly elastic property. However, it is not the case with silty sands, sedimentary soft rocks and stiff and soft clays. In view of the above, the stiffness obtained at a strain of 0.001% should be called the quasi-elastic property (Tatsuoka et al., 1999). In the present study, the $E_{eq}$ value from the initial stress—strain relation until the axial strain $e_a$ becomes about 0.001%, where the stress state is close to the isotropic stress state, is defined as the initial (quasi-elastic) stiffness, $E_0$.

**Effects of Shear Stress Level on Small Strain Stiffness**

Figures 8(a) and 8(b) show the stress-strain relations from ML tests JA012 and JA016 at a nominal axial strain rate, $e_a$, of 0.03%/min. Either a single small unload/reload cycle or five cycles was (were) applied many times during otherwise ML. Figures 8(c) and 8(d) show typical local stress-strain relations including a unload/reload cycle(s). It may be seen that the unload/reload stress-strain curves become more open when evaluated at shear stress levels closer to the peak stress state. Correspondingly, the slope of the reloading stress-strain relation becomes lower than the elastic value, $E_e$, to a more extent with an increase in the shear stress level. This under-estimation of $E_e$ becomes more serious with a decrease in the strain rate applied. In an attempt to alleviate the problem described above, as illustrated in Fig. 9, the equivalent Young’s modulus at a given stress state, $E_{eq}$, was evaluated based on the average axial strain increment for unload and reload branches.

On the other hand, due to an inherent drawback of the axial loading apparatus, the axial strain rate for a very small axial strain range (less than about 0.001%) immediately after the start of unloading became very high (e.g., part $a$ to $b$ in Fig. 8(c)-3), followed by much slower unloading part $b$ to $c$. This uncontrolled high axial strain rate was on average about 0.06%/min based on the time history of locally measured axial strain, which is faster by a factor of about 30 than the one applied for the subsequent part of unload/reload cycle. Taking advantage of this behaviour, the slope of this fast unloading part was defined as another type of small strain modulus, $E_d$. As shown below, it is likely that each $E_d$ value is closer to its corresponding $E_e$ value than the $E_{eq}$ value.

Figures 10(a) and 10(b) show the Young’s moduli, $E_d$ and $E_{eq}$, calculated for each unload/reload cycle, $n$, at each stage during otherwise ML from tests JA016 and JA012, respectively, plotted against the total (or cumulative) number of loading cycles, $N$. The solid squares in Fig. 10(b) represent the average, $\bar{E}_d$, of the $E_d$ values for the five consecutive unload/reload cycles ($E_{d1}$ through $E_{d5}$) at the same stress state in test JA012 obtained as:

$$E_d = \frac{\Delta q}{\text{average of } \Delta e_a \text{ for five cycles}}$$
Figure 11 compares the values of $E_d$ and $E_{eq}$ presented in Figs. 10(a) and 10(b) by replotting them against each shear stress level, $q_{eq}/q_{max}$, where these values were evaluated. Figure 12 shows results from another test (SP011), similar to those from test JA012. In this test, the specimen was initially cured unconstrained for 14 days and then was sheared at a constant nominal axial strain rate, equal to 0.03%/min, except for a stress range $q =$ 0.3–0.9 MPa, where the strain rate was increased to five times. It may be seen from Figs. 10, 11 and 12 that, in tests JA012 and SP011, the $E_{eq}$ value at each stress state is rather independent of the number of loading cycles when the deviator stress, $q_{eq}$, is lower than about 0.8 MPa (i.e., $q_{eq}/q_{max} = 0.4$). However, the trend that the $E_{eq}$ value increases with cyclic loading at the respective stress state, associated with a decrease in the creep strain, becomes more obvious at higher deviator stresses. This result indicates that the $E_{eq}$ values evaluated from the first single small unload/reload cycle underestimate the elastic modulus more at higher shear stress levels.

The $E_d$ values scatter to some extent in Figs. 10 and 11. This trend can be attributed to a limited accuracy of stress and strain measurements. Despite the above, it may be seen that the $E_d$ value is rather constant in the nearly full pre-peak regime. This result shows that viscous effects on the $E_d$ values are insignificant even when measured at high shear stresses. This could be attributed to the following two factors; $E_d$ was defined a) at relatively high strain

$\frac{5}{E_{d1} + E_{d2} + E_{d3} + E_{d4} + E_{d5}}$ (2)
Fig. 9. Definitions of $E_0$, $E_{\text{tan}}$ and $E_{\text{sec}}$ for primary stress-strain curve and $E_d$ and $E_{\text{eq}}$ for unclosed unload/reload stress-strain curves in a single unload/reload cycle; $q_{\text{av}}$ – the average $q$ during an unload/reload cycle for which $E_{\text{eq}}$ was defined.

Fig. 10. $E_d$ and $E_{\text{eq}}$ values for: a) a single unload/reload cycle at each stage in test JA016 and b) five cycles at each stage in test JA012.

Fig. 11. $E_d$ and $E_{\text{eq}}$ values obtained at consecutive unload/reload cycle(s) at each stress state during otherwise drained ML TC, plotted against $q_{\text{eq}}/q_{\text{max}}$, tests JA016 and JA012.

Fig. 12. Relationships between $E_{\text{eq}}$ values during five unload/reload cycles and: a) number of cycle and b) $q_{\text{eq}}/q_{\text{max}}$; test SP011 (batch No. 3; $w_i = 8.75\%$, $r_d = 2.0\%$, $c = 2.5\%$; and b) for a relatively small strain amplitudes. From the above, it is understood that the $E_d$ is more representative of the elastic modulus than the $E_{\text{eq}}$ value.

AGEING EFFECTS ON SMALL STRAIN STIFFNESS

Four series of CD TC tests ($\sigma_0 = 19.8\%$) were performed at $\dot{e}_a = 0.03\%$/min on specimens with different values of $c/g$ and $r_d$, compacted at $w_i = w_{\text{opt}} = 8.75\%$ and aged for different initial curing periods, $t_{\text{ini}}$, under the atmospheric pressure (Table 3). Kongsukprasert and Tatsuoka (2005) reported part of the test results. Figures 13 and 14 show the overall stress-strain relations and their close-ups at small strains for $r_d = 2.0 g/cm^3$ and $c/g = 2.5\%$, while Fig. 15 for $r_d = 2.0 g/cm^3$ and $c/g = 4.0\%$; and Fig. 16 for $r_d = 2.1 g/cm^3$ and $c/g = 2.5\%$. In the three tests presented in Fig. 14(a), in the post-peak regime, the deviator stress, $q$, jumped upon a step increase in the axial strain rate from 0.03%/min to 0.15%/min that was made to evaluate the viscous effects on the stress-strain behaviour. This issue is discussed in detail in Kongsukprasert and Tatsuoka (2005).

Figure 17 shows the relationships between the initial Young’s modulus defined at $e_a = 0.001\%$, $E_0$, and the initial curing period, $t_{\text{ini}}$, for these test results (listed in Table 3). It may be seen that the $E_0$ value increases with...
Table 3. Test conditions and part of results from CD TC tests to evaluate the effects of initial curing period

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>Batch no.</th>
<th>$w_i$ (%)</th>
<th>$\rho_i$ (g/cm³)</th>
<th>$c/g$ (%)</th>
<th>Initial curing period, $t_{ini}$ (day)</th>
<th>Total curing period, $t_c$ (day)</th>
<th>$E_{0,50}$ (GPa)</th>
<th>$E_0$ (GPa)</th>
<th>$q_{max}$ (kPa)</th>
<th>$\varepsilon_d$ at $q_{max}$ (%)</th>
<th>$E_0/q_{max}$</th>
<th>$(E_0)/q_{max}$ (GPa)</th>
<th>$(E_{0,50})/q_{max}$ (GPa)</th>
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<td>2.500</td>
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<td>7.07</td>
<td>3.19</td>
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*The compressive strength was inferred from the stress-strain curve not reaching the peak estimated, as the ultimate strength of the specimen exceeded the maximum capacity of load (50 kN) of the apparatus.

Fig. 13. Effects of initial unconfinement curing on the stress-strain relation, series 1 (batch No. 1 and 2; $w_i=8.75\%$, $\rho_i=2.0$ g/cm³ and $c/g=2.5\%$)

Fig. 14. Effects of initial unconfinement curing on the stress-strain relation, series 2 (batch No. 3; $w_i=8.75\%$, $\rho_i=2.0$ g/cm³ and $c/g=2.5\%$)
and increase in \( t_{\text{ini}} \). Figure 18(a) shows the relationship between the \( E_0 \) value and the value of \( E_{\text{eq}} \) of the first single unload/reload cycle at each stage, averaged for a stress range of \( q_{\text{eq}}/q_{\text{max}} = 0 \sim 0.3 \), \( (E_{\text{eq}})_{\text{ave0.3}} \), at different curing periods, while Fig. 18(b) for \( q_{\text{eq}}/q_{\text{max}} = 0.3 \sim 0.7 \), \( (E_{\text{eq}})_{\text{ave0.7}} \). These boundary values of \( q_{\text{eq}}/q_{\text{max}} \) were chosen after several times of trial and error to properly illustrate the effects of shear stress level on the \( E_{\text{eq}} \) value obtained from the first single unload/reload cycle applied during otherwise ML. It may be seen that the \( E_0 \) value and the \( (E_{\text{eq}})_{\text{ave0.3}} \) value are essentially the same at all the curing periods examined. For this reason, the respective \( (E_{\text{eq}})_{\text{ave0.3}} \) values are used to obtain normalised non-linear relations later in this paper. These results show that the initial ML stress-strain relation at axial strains less than 0.001% is essentially reversible, as seen from the fact that the unload/reload stress-strain relations at axial strains less than about 0.002% are nearly perfectly overlapping the corresponding initial ML stress-strain curve (Figs. 13(b), 14(b), 15(b) and 16(b)). On the other hand, the \( (E_{\text{eq}})_{\text{ave0.7}} \) value is noticeably smaller than the \( E_0 \) value (Fig. 18(b)), which indicates that, already for \( q_{\text{eq}}/q_{\text{max}} = 0.3 \sim 0.7 \), the value of \( E_{\text{eq}} \) of the first unload/reload cycle at each stage is noticeably affected by the viscous property of the test material. Figures 19(a) and 19(b) show the values of \( (E_{\text{eq}})_{\text{ave0.3}} \) and \( (E_{\text{eq}})_{\text{ave0.7}} \) plotted against the total curing time when the respective \( E_{\text{eq}} \) was measured, \( t = t_{\text{ini}} + \Delta t \), where \( \Delta t \) is the additional time since the start of ML TC, which was 30 ~ 35 minutes at the peak stress state. The trend of behaviour seen in Fig. 19 is very similar to that seen with \( E_0 \) shown in Fig. 17.

It may be seen from Figs. 17 and 19 that the increasing rate of the values of \( E_0 \) and \( E_{\text{eq}} \) with \( t_{\text{ini}} \) when \( t_{\text{ini}} \) becomes larger than 14 days is noticeably smaller in series 1 and 2 than in series 3 and 4. This may be due to combined negative effects of smaller \( c/g \) (2.5%) and \( p_t \) (2.0 g/cm³) in series 1 and 2 (Kongsukprasert and Tatsuoka, 2005).

**RELATION BETWEEN \( E_0 \) AND \( q_{\text{max}} \)**

It is convenient if the \( E_0 \) value can be estimated from the compressive strength, \( q_{\text{max}} \), because the accurate evaluation of \( E_0 \) is usually much more difficult than that of \( q_{\text{max}} \). Figure 20 shows the relationships between \( E_0 \) and \( "q_{\text{max}} \) at \( \dot{e}_0 = 0.03\%/\text{min}" \) obtained from the data presented above (see also Table 3). It may be seen that the
range of the ratio $E_0/q_{\text{max}}$ of these data is narrow. However, with the test results from series 1 and 2, the ratio $E_0/q_{\text{max}}$ slightly decreases with $t_c$ for $t_c$ longer than 14 days. As the strain at the peak stress tends to decrease with time (Figs. 13 and 14), a decrease in the ratio $E_0/q_{\text{max}}$ means that the pre-peak stress-strain relation becomes more linear. Combining the test results presented in Figs. 17 and 20, it is likely that the rate at which the stress-strain curve becomes more linear with $t_c$ increases with a decrease in $c/g$ or $p_0$ or in both. This issue is discussed more in detail later in this paper.

Figure 21(a) shows the relationships between the initial Young’s modulus, $E_0$, and the compressive strength, $q_{\text{max}}$, from the CD TC tests on compacted cement-mixed well-graded gravelly soil from the present study as well as those reported by Kongskuprasert and Tatsuoka (2005). The data from CD or CU TC tests on various types of cement-mixed sandy and gravelly soils (except for one case, DMM-Kaw/u, in which the base material is clay) (Tatsuoka et al., 1997, 2002; Tatsuoka, 2004) are also presented in Fig. 21(a). In this figure, the range of the data from CD or CU TC tests on various types of sedimentary soft rocks (Tatsuoka and Shibuya, 1991; Tatsuoka and Kohata, 1995; Tatsuoka et al., 2002) is also presented, of which the data points are presented in Fig. 21(b). In Fig. 21(c), the data from CD TC tests on unbound natural gravelly soils (Tatsuoka and Shibuya, 1991; Tatsuoka and Kohata, 1995) and unconstrained compression tests on various types of concrete (Yamaguchi et al., 1988; Withey, 1961) are presented. Figure 21(d) summarises all these data. The test conditions for these data are listed in Table 4. The following trends of behaviour may be seen:

1. The $E_0$ values of unbound gravelly soils are around 100–800 MPa, which is smallest among the data presented in this figure. The ratio $E_0/q_{\text{max}}$ largely scatters, which is due mainly to the fact that the values of $E_0$ and $q_{\text{max}}$ are different functions of the
same variables, including confining pressure, dry density and soil type (Tatsuoka and Shibuya, 1991).

2. The values of $E_0$ of concretes are around 20–50 GPa, which is largest among these data. The range of the $E_0$ and $q_{\text{max}}$ relations of various types of concretes with different types of cement, mixing designs and so on is relatively narrow. However, as the increasing rate of $E_0$ with an increase in the curing period, $t_c$, is generally smaller than $q_{\text{max}}$, the ratio $E_0/q_{\text{max}}$ decreases with $t_c$, resulting into a wide range of $E_0/q_{\text{max}}$.

3. The $E_0$ values of sedimentary soft rocks are in between those of unbound gravelly soils and concretes. The ratio, $E_0/q_{\text{max}}$, largely scatters as unbound gravelly soils.

4. The ranges of the $q_{\text{max}}$ and $E_0$ values of cement-mixed soils are comparable with those of natural sedimentary soft rocks, which are often employed as the foundation ground to support heavy permanent structure allowing very limited displacements (e.g., the Akashi Strait Bridge; Tatsuoka and Kohata, 1995). However, the followings different trends can be noted:

   a) The ratio, $E_0/q_{\text{max}}$, of all the cement-mixed soils is greater than about 500 with an exception (explained below) and generally larger than...
<table>
<thead>
<tr>
<th>Series name</th>
<th>Material</th>
<th>Test</th>
<th>$t_c$ (day)</th>
<th>Site remark</th>
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<tr>
<td>Y-Napoli/d, u</td>
<td>Yellow tuff</td>
<td>CD, CU</td>
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<td>CD</td>
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<td>Isomi gravel</td>
<td>CD</td>
<td>—</td>
<td>Isomi, Japan</td>
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<td>CD</td>
<td>—</td>
<td>Kobe, Japan</td>
</tr>
<tr>
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<td></td>
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<td>—</td>
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<tr>
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<td>CD</td>
<td>—</td>
<td>Chiba gravel, Japan</td>
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<tr>
<td>Chiba2</td>
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<td>CD</td>
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<tr>
<td>A-1, A-2, A-3, A-4</td>
<td>Dam concrete with fly ash</td>
<td>CD</td>
<td>7–365</td>
<td>Japan(^6)</td>
</tr>
<tr>
<td>W124–30</td>
<td>Ordinary concrete</td>
<td>UV(^3))</td>
<td>30 years</td>
<td>USA(^3)</td>
</tr>
<tr>
<td>W136–30</td>
<td></td>
<td>UV(^3))</td>
<td>50 years</td>
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</table>

\(^1\)“no” means no compaction in production.
\(^2\)The base material is clay only in this case.
\(^3\)Unconfined compression tests only in this case (the others are all CD or CU TC tests).
\(^4\)Yamaguchi et al., 1988; \(^5\)Withey, 1961.
those of sedimentary soft rocks and unbound gravelly soils. The exception deviating from the major part of cement-mixed soils is exceptionally weak and soft uncompacted cement-mixed sand from the field (i.e., data termed “Dry-Kis/d” in Table 4).

b) The scatter of the $E_0$ and $q_{\text{max}}$ relations of a large number of different types of uncompacted cement-mixed granular soils (except for CM-Haw) and those of compacted cement-mixed well-graded gravelly soil (i.e., model Chiba gravel) is smaller than those of natural sedimentary soft rocks and unbound gravelly soils. The exception is the data termed “Dry-Kis/d”, explained above. A general trend that the ratio $E_0/q_{\text{max}}$ decreases as $q_{\text{max}}$ increases (like unbound gravelly soils) and as $t_1$ increases (like concretes) may be seen.

These results listed above indicate that there is no unique $E_0$ and $q_{\text{max}}$ relation for different types of material. Therefore, when needed, the correlation specific to each material of concern should be evaluated experimentally. Yet, the ratio $E_0/q_{\text{max}}$ of compacted well-graded gravelly soil exhibits the smallest scatter and is of the order of 3,000. By taking advantage of this ratio, the approximated value of $E_0$ can be estimated from the value of $q_{\text{max}}$ obtained by relevant laboratory stress-strain tests, while the approximated value of $q_{\text{max}}$ can be estimated from the value of $E_0$ obtained by relevant field shear wave velocity measurements. Furthermore, this ratio is noticeably higher than the other types of cement-mixed soil that were prepared without compaction (except for CM-Haw).

**AGEING EFFECTS ON THE NON-LINEARITY OF STRESS-STRAIN RELATION**

The effects of ageing (i.e., drained sustained loading) at an isotropic stress state and with a shear stress (i.e., at an anisotropic stress state during ML TC) on the non-linearity of stress-strain relation were examined while referring to the stress state-dependency of small strain stiffness, $E_{\text{eq}}$. As shown below, the effects of the shear stress during ageing on the non-linearity are significant.

**Ageing at Isotropic Stress States**

Figure 22(a) shows the relationships between the $E_{\text{eq}}$ value of the first and single unload/reload cycle with a single amplitude axial strain of about 0.005% applied at different stress states during otherwise ML, and the deviator stress at which $E_{\text{eq}}$ was evaluated, $q_{\text{eq}}$, obtained from the data presented in Figs. 13–16. The specimens were cured for different periods without a shear stress. Figure 22(b) shows the normalised relationships between $(E_{\text{eq}})_{h0.03}$ and $q_{\text{eq}}/q_{\text{max}}$. As the $E_{\text{eq}}$ value evaluated when $q_{\text{eq}}$ is low is nearly the same as the initial Young’s modulus, $E_0$, the relations presented in Fig. 22(b) are essentially the same as those between $E_{\text{eq}}/E_0$ and $q_{\text{eq}}/q_{\text{max}}$. The following trends of behaviour may be seen:

1) All the $E_{\text{eq}}$ values evaluated at different deviator stresses increase with an increase in the period of ageing without a shear stress, $t_1$.

2) At any $t_1$, the $E_{\text{eq}}$ value decreases noticeably with an increase in the shear stress level, $q_{\text{eq}}/q_{\text{max}}$. This trend of behaviour is due largely to that the unload/reload hysteresis loop becomes more open due to viscous effects that increase with an increase in $q_{\text{eq}}/q_{\text{max}}$.

3) The normalised relationship between $E_{\text{eq}}/(E_{\text{eq}})_{h0.03}$ and $q_{\text{eq}}/q_{\text{max}}$ is rather independent of $t_1$ within the limit of $t_1$ examined in the present study.

Figures 23(a) and 23(b) show the $E_{\text{eq}}$ values evaluated at the fifth small unload/reload cycles applied at the respective stress state, $E_{\text{eq},5}$, plotted similarly in Figs. 22(a) and 22(b), obtained from only tests SP009–SP011 of series 2 and all tests of series 3. It may be seen by comparing Figs. 22(a) and 23(a) that the $E_{\text{eq},5}$ value becomes larger than the value at the first cycle to a greater extent as the stress level increases. This confirms the trend of behaviour seen from Fig. 8. The normalised relationship between $E_{\text{eq},5}/(E_{\text{eq}})_{h0.03}$ and $q_{\text{eq}}/q_{\text{max}}$ is rather independent of ageing time (Fig. 23(b)). This trend is the same as the $E_{\text{eq},5}/(E_{\text{eq}})_{h0.03}$ and $q_{\text{eq}}/q_{\text{max}}$ relation (Fig.
Effects of Ageing with a Shear Stress

Figure 25 shows loading histories with ageing stages for various periods applied at anisotropic stress states during otherwise ML in the CD TC tests (Table 5). Figure 26 shows the test results. It may be seen that the stiffness is very high in a large stress range immediately after ML has restarted following the respective ageing stages. The effects of ageing with a shear stress on the creep deformation property and the peak strength are discussed in Kongsukprasert and Tatsuoka (2005). The effects on the small strain stiffness and the non-linearity of the subsequent stress-strain relation are analysed below.

Figure 27 shows the $E_{eq}$ values obtained from a single small unload/reload cycle applied before and after the respective ageing stages at different deviator stresses during otherwise continuous ML TC. Note that the $E_{eq}$ value evaluated when $q_{eq}/q_{max}$ is less than about 0.3 is nearly the same as the initial Young’s modulus, $E_0$. 

Fig. 23. a) Effects of $t_{ini}$ on the relationship between $E_{eq,3}$ at the fifth unload/reloading cycle and $q_{eq}$ and b) $E_{eq}/(E_{eq})_{ave0.3} - q_{eq}/q_{max}$ relations

Fig. 24. Normalised relations between $E_{tan}/E_0$ and $q_{eq}/q_{max}$; a) series 1, b) series 2, c) series 3 and d) series 4

22(b)). Figure 24 shows the relationships between the normalized tangent modulus, $E_{tan}/(E_{eq})_{ave0.3}$ and the normalized deviator stress, $q/q_{max}$, which are essentially the same as those between $E_{tan}/E_0$ and $q_{eq}/q_{max}$. The definition of the tangent modulus, $E_{tan}$, is given in Fig. 9. These $E_{tan}$ values were obtained from the respective $q - e_a$ relations. To this end, Eq. (1) was fitted to the local $q - e_a$ relation for a small $e_a$ range with the centre at the $e_a$ value where the $E_{tan}$ value is to be obtained ($e_a^*$), and then deviated to obtain $E_{tan} = b + 2ce_a^*$. This procedure was repeated in the full pre-peak regime. As explained earlier, $E_{tan}/(E_{eq})_{ave0.3}$ is equivalent to $E_{tan}/E_0$. These relations represent the non-linearity of pre-peak stress-strain relations. It may be seen that the stress-strain relation becomes less non-linear with an increase in the period of ageing without a shear stress, $t_{ini}$, unlike negligible effects of $t_{ini}$ on the relationships between $E_{eq}/(E_{eq})_{ave0.3}$ and $q_{eq}/q_{max}$ and between $E_{eq}/(E_{eq})_{ave0.3}$ and $q_{eq}/q_{max}$ (Figs. 22(a) and 23(a)).
Table 5. Test conditions and part of results from CD TC tests to evaluate the effects of sustained loading

<table>
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<tr>
<th>Series</th>
<th>Test</th>
<th>Batch no.</th>
<th>w (%)</th>
<th>ρ₀ (g/cm³)</th>
<th>c'/g (%)</th>
<th>Initial curing period, tₐ₀ (day)</th>
<th>Sustained loading</th>
<th>Axial strain rate (%/min)</th>
<th>Eₐ₀ (GPa)</th>
<th>E₀ (GPa)</th>
<th>(Eₐ₀/E₀)₀.5 (GPa)</th>
<th>qₐₚ (kPa)</th>
<th>qₐₚ/qₚ (kPa)</th>
<th>Eₐ₀/qₚ (GPa)</th>
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<td>2</td>
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<td>—</td>
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<td>7.05</td>
<td>1,000</td>
<td>234,000</td>
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Fig. 25. Loading histories employed to evaluate effects of ageing with a shear stress, series JAS.

(Fig. 18(a)). The Eₐ₀ values are plotted against the deviator stress, qₘₚ, where the respective Eₐ₀ value was measured. Figure 28 shows their normalised relations. Figure 29 shows the corresponding relationships between the normalised tangent Young's modulus, Eₗₐ₀/(Eₐ₀)₀.5, and the normalised shear stress level, qₐₚ/qₚ (see Fig. 9 for the definitions of Eₗₐ₀ and qₚ). The following trends of behaviour may be seen from these figures:

1) The Eₐ₀ value increases markedly with ageing to a value even greater than the respective Eₐ₀ (Fig. 27). This increase in Eₐ₀ is due to: both i) a decrease in the viscous strain component during a small unload/reload cycle by ageing with a shear stress; and ii) an increase in the elastic modulus by ageing. It seems that the effects of these two factors are not fully persistent during the subsequent loading stage, as seen from the fact that the difference between the values of Eₐ₀/(Eₐ₀)₀.5 after and before the respective ageing stage gradually decreases with an increase in qₐₚ/qₚ (Fig. 28).

2) When ML is restarted following the respective ageing stages with a shear stress, the stress-strain behaviour becomes nearly elastic for a large stress range (Fig. 26). This trend can also be seen from a sudden large change in the relationship between Eₗₐ₀/(Eₐ₀)₀.5 and qₗₐ₀/qₚ (Fig. 29). This result indicates a paramount importance of taking into account the effects of ageing with a shear stress when evaluating the small strain deformation of cement-mixed soil that has been subjected to ageing with a shear stress.

To further investigate into the effects of recent loading history on the small strain stiffness, another pair of specimens were prepared by initially ageing under unconfining conditions for tₐ₀ = 7 days (test SP004) and 14 days (test SP003) and then subjected to CD TC employing the following specific loading histories (Fig. 30(a)).

1) In test SP004 (tₐ₀ = 7 days), the specimen was subjected to ML at a nominal axial strain rate of x = 0.03%/min followed by drained sustained loading (i.e., ageing) at q = 1.0 MPa for 24 hours. At this ageing stage, two consecutive sets of ten and two unload/reload cycles with a single amplitude of deviator stress equal to 150 kPa (the neutral deviator stress = 1.0 MPa) were applied at very low nominal axial strain rates, equal to 5X* and 40X*, where X* = x/625. (Fig. 30(d)). In this figure, the result from test SP009, in which the specimen was not subjected to small unload/reload cycles during the ageing for
other the same loading histories as test SP004 is also presented.

2) In test SP003 ($t_{\text{ini}} = 14$ days), after ML at a nominal axial strain rate of $x = 0.03\%$/min, the specimen was subjected to the following loading histories at $q = 1.0$ MPa (see Fig. 30(c)); i) ageing for 24 hours; ii) 40 unload/reload cycles with a single amplitude of $q$ of 150 kPa (the neutral deviator stress = 1.0 MPa) at a axial strain rate of $x = 0.03\%$/min; iii) another ageing from 24 hours; iv) another cyclic loading as iii); v) another ageing for 2.5 hours; and vi) restart of ML. In this figure, the result from test SP011, in which the specimen was not subjected to small unload/reload cycles during the ageing for the same other loading histories as test SP003 is also presented.

As shown in Fig. 30(b), the specimens were also subjected to several step changes in the axial strain rate during otherwise ML at a constant strain rate. Figure 30(c) compares the overall stress-strain relations from tests SP004 and SP003 with those from their reference tests, SP009 and SP011 (presented in Fig. 14).

In test SP003, small unload/reload cycles were applied after creep loading for 24 hours (Fig. 30(a)). Figure 30(f) shows the relationship between $E_{\text{eq}}$ and the number of loading cycles, $N$. It may be seen that the $E_{\text{eq}}$ value did not increase noticeably by cyclic loading in the respective sets of cyclic loading. Larger values of $E_{\text{eq}}$ in the second set of cyclic loading are due to ageing effects that developed during the ageing stage for 24 hours between the first and second sets of cyclic loading. In test SP004, the stiffness value largely increases upon a change in strain rate. This increase in the stiffness may not be due to strain rate effects, but more possibly due to ageing effects that developed during the preceding very slow 10 unload/reload cycles, taking in total 2 days. This inference is based on the fact there is no significant effects of strain rate on $E_{\text{eq}}$ values in the data of compacted cement-mixed well-graded gravelly soil presented in Figs. 7 and 12.

CONCLUSIONS

The following conclusions with respect to the deformation characteristics of compacted cement-mixed well-graded gravelly soil can be derived from the test results presented in this paper:

1. The initial stress-strain relation at small strains less than about 0.001% in monotonic loading tests was essentially reversible. The initial Young's modulus from the initial stress-strain relation is essentially the same from cyclic loading tests performed under otherwise the same conditions.

2. The effects of strain rate on the small strain stiffness evaluated by applying small unload/reload cycles were very small.

3. When multiple unload/reload cycles with small
strain amplitude were applied without pre-sustained and pre-cycling loading at an anisotropic stress state, the $E_{eq}$ values increased with cyclic loading due to a gradual decrease in the creep strain developing during cyclic loading. Therefore, the $E_{eq}$ value from the first and single unload/reload cycle applied during otherwise continuous ML could be significantly lower than the elastic modulus. This trend was not important when the stress state was closer to an isotropic one, but the importance increased with an increase in the ratio of the shear stress level to the peak stress state, $q_{eq}/q_{\text{max}}$.

4. By applying a long ageing period or a large number of small unload/reload cycles at the same anisotropic stress stage, the peak-to-peak secant modulus, $E_{eq}$, of a given small unload/reload cycle became closer to the elastic modulus.

5. The ratio of initial quasi-elastic Young’s modulus to compressive strength, $E_0/q_{\text{max}}$, of compacted cement-mixed gravelly soil was generally similar to that of concrete, but noticeably larger than those of uncompacted cement-mixed soils, unbound gravelly soil and sedimentary soft rock.

6. The elastic modulus, pre-peak average stiffness, linearity of pre-peak stress-strain relation and peak strength increased with an increase in the period of drained sustained loading (i.e., curing or ageing) under unconfined or isotropically confined conditions. By ageing with a shear stress, the small strain
stiffness increased due to not only the effects of ageing (i.e., cement-hydration) on the elastic modulus but also a decrease in the viscous deformation. By the same mechanism, the stress-strain behaviour exhibited highly elastic and stiff behaviour for a large stress range when ML was restarted following a relatively long ageing stage with a shear stress.

ACKNOWLEDGEMENTS

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