UPPER BOUND PLASTICITY ANALYSIS OF A PARTIALLY-EMBEDDED PIPE UNDER COMBINED VERTICAL AND HORIZONTAL LOADING

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ABSTRACT

Seabed pipelines undergo temperature cycles that create axial load which can be relieved through controlled lateral buckling. The prediction of lateral buckling in design requires accurate assessment of the lateral breakout resistance. This Technical Note describes upper bound plasticity analysis of a partially-embedded pipe on undrained soil. The purpose is to generate failure envelopes for vertical and horizontal loading to provide a theoretical basis for estimating breakout resistance. The following cases have been considered: smooth and rough pipes, with and without separation at the rear face of the pipe. The envelopes are similar to those developed previously for surface foundations, but capture additional effects that are due to the curved geometry of the pipe surface. The breakout resistance and the movement of the pipe at failure are strongly influenced by the separation condition. Pipe roughness and soil self-weight have a relatively minor effect on breakout resistance. Existing empirical expressions usually assume a linear variation in breakout resistance with embedment and vertical load. This theoretical analysis demonstrates that these relationships are non-linear. The resulting envelopes provide a more rigorous basis for predicting the breakout resistance of partially-embedded pipelines.

Key words: plasticity theory, soft clay, undrained shear strength (IGC: E3)

INTRODUCTION

Subsea pipelines for oil or gas transportation are required to operate at high temperatures and pressures. The axial stress induced by the change in temperature from the as-laid condition causes a tendency for the hot pipeline to buckle. Such buckling action, when not correctly controlled, has led to at least two pipeline failures (Pasqualino et al., 2001; McKinnon et al., 2001). The conventional technique to avoid buckling has been to retrain the pipeline by trenching and burial. Alternatively, the stress in the pipeline can be relieved with the use of in-line expansion spools. These methods are, however, becoming less cost-effective as operating temperatures and pressures increase and as developments move into deeper water where trenching is impractical. In these new frontiers of hydrocarbon extraction, the mitigation of thermal buckling represents a critical new challenge facing pipeline designers (Perinet and Frazer, 2006).

An alternative design solution, which can be highly economical, is to control the formation of lateral buckles along the unburied pipeline, which is laid in a snaked pattern. The regular formation of lateral buckles relieves the axial compressive stress in the pipe and represents an elegant solution to the problem of thermal buckling. The key uncertainty in lateral buckling design is the prediction of the soil resistance encountered by the partially-embedded pipe during lateral movement (Bruton et al., 2006). To ensure that the lateral buckles form as planned, the force required to move the pipe sideways—known as the breakout resistance—must be accurately predicted.

The purpose of this Technical Note is to present a simple upper bound plasticity solution for the allowable combinations of vertical and horizontal load on a partially-embedded pipeline. This solution provides a theoretical basis for the assessment of breakout resistance. The nomenclature for this scenario is shown in Fig. 1. The soil is characterised by an undrained strength, $s_u$, as is appropriate for the soft clays typically encountered in deep water offshore. The shear stress mobilisable at the pipe-soil interface is $\alpha s_u$. The limits of $\alpha = 0$ and $\alpha = 1$ are...
denoted ‘smooth’ and ‘rough’ respectively.

BACKGROUND

Vertical Load-embedment Response

The lateral soil resistance exerted on a partially embedded pipe is strongly affected by the pipe embedment, w, defined as the penetration of the pipe invert below the level of the undisturbed seabed. In order to assess breakout resistance it is necessary to first predict the embedment of the pipe based on the best assessment of the load imposed during laying.

Upper and lower bound plasticity solutions for a partially-embedded pipe under vertical load were presented by Murff et al. (1989), modified from the mechanisms presented by Randolph and Houlsby (1984) (Fig. 2). An improved upper bound solution has been found, based on rigid circular blocks of soil rotating about two points symmetrically on either side of the pipe centre for smooth pipes and embedments in the range 0.15 < w/D < 0.5. This refinement is a modification of the Martin mechanism (Martin and Randolph, 2006) which is doubly symmetric about the pipe centre. The modified Martin mechanism for pure vertical loading is shown in Fig. 4(a). Approximate expressions that predict the penetration load to within 10% of the upper bound solutions up to an embedment of 0.5D are given by Eqs. (1) and (2).

For rough pipes:

\[ V_{DSu} = 8.1 \left( \frac{w}{D} \right)^{0.43} \]  

(1)

For smooth pipes:

\[ V_{DSu} = 5.85 \left( \frac{w}{D} \right)^{0.32} \]  

(2)

Field observations show that the as-laid pipe embedment usually exceeds the penetration that would occur due to the self-weight of the pipe by a significant margin (Lund, 2000). This additional embedment occurs due to (i) the concentration of stresses at the touchdown point due to the catenary shape as the pipe is laid, and (ii) additional loading associated with movement of the lay vessel and hydrodynamic loading of the hanging length of pipe.

The as-laid pipe embedment is typically in the range 0.2 < w/D < 0.5. When the pipeline is in operation, with thermal loading driving lateral movement, the normalised pipe weight is typically in the range of 0.5 < V/suD < 2, depending on the pipeline dimensions, construction and contents. These conditions correspond to the shaded region in Fig. 2 which indicates that the operating vertical load is a small fraction of the static vertical load that would be required to achieve the as-laid embedment – a condition referred to as ‘over-penetration’. Zhang et al. (2002) quantified this condition by defining an over-penetration ratio, \( R_{pen} = V_{max}/V \), which is analogous to the over-consolidation ratio of soil. For prototype pipes, \( R_{pen} \) typically lies between 3 and 20.

Empirical Assessment of Horizontal Breakout Resistance

Current design practice is to assess the breakout resistance using empirical expressions calibrated from model tests, with the pipe embedment, pipe weight and the soil strength as input parameters (e.g., Brenodden et al., 1989; Wagner et al., 1989; Verley and Lund, 1995; Bruton et al., 2006).

These expressions usually comprise of two terms: (i) a ‘frictional’ resistance component, proportional to the current vertical load, \( V \), (which is the operating pipe weight), and (ii) a passive resistance component, linked to the embedded depth of the pipe, \( w \), and the soil undrained strength, \( s_u \):

\[ H = \mu V + \lambda s_u w \]  

(3)

Recommended values of the friction coefficient, \( \mu \), and the embedment coefficient, \( \lambda \), vary with soil type, but are typically in the range 0.2 < \( \mu < 1 \) and 0.5 < \( \lambda < 2 \). Verley and Lund (1995) and Bruton et al. (2006) link \( \lambda \) to the ratio of soil strength to unit weight \( (s_{u}/\gamma) \).

Equation (3) predicts a linear increase in the breakout resistance with both embedment depth and pipe weight, matching the general trends observed in the underlying model test database. However, these expressions are wholly empirical and subjected to the uncertainties within the underlying data. It is desirable to establish a sound theoretical basis for predicting breakout resistance.

Failure Envelopes for Combined Loading

An alternative approach to describe the variation in pipe breakout resistance is to express the allowable combinations of vertical load, \( V \), and horizontal load, \( H \), as a failure envelope in \( V-H \) space. This approach is well-established for describing the behaviour of surface foundations under combined loading (Butterfield and Gottardi, 1994). Prior to breakout, the pipe load state, solely comprising of \( V \) due to the self-weight, will lie within the failure envelope, due to the over-penetration during laying. The thermal loading creates an increasing horizontal load. When this load path of increasing \( H \) intersects with the failure envelope, breakout will occur.
This Technical Note presents an upper bound solution for a partially embedded pipe under combined vertical and horizontal loading. For a given pipe embedment, the geometry of a 2-parameter upper bound mechanism based on a circular slip surface has been optimised for all ratios of horizontal to vertical loading, in order to derive a failure envelope. The soil has been idealised as a Tresca material representative of undrained clay, with uniform strength.

The following variables have been considered: (i) the roughness of the soil-pipe interface, (ii) the ratio of soil strength to weight, and (iii) the possibility of separation at the rear of the pipe. During fast loading, tensile resistance may be sustained by suction at the rear of the pipe, so it may be assumed that the pipe is bonded to the soil. During slower rates of loading, although undrained conditions may prevail within the soil mass, a crack may open at the rear of the pipe. This condition can be modelled by assuming separation between the rear face of the pipe and the soil.

The results focus on the range of over-penetration ratios appropriate for typical on-bottom pipelines ($R_{pen} = 3–20$).

**UPPER BOUND MECHANISM**

A simple upper bound mechanism for $V$-$H$ loading of a pipe based on a single slip circle is shown in Fig. 3. The geometry of this mechanism depends on two parameters, which can either be considered as the centre of the slip circle, $x_0$, $y_0$, or the radius of the slip circle and the inclination at the tangent to the pipe surface. This tangent corresponds to the direction of pipe movement.

In order to retain two-dimensional loading and geometry, it is assumed that the horizontal force acts through the centre of the pipe and the pipe does not rotate. For a smooth pipe ($\alpha = 0$), the work equation is identical to an expression of moment equilibrium about the centre of the slip circle. For a rough pipe, additional dissipation due to relative movement along the perimeter of the pipe in contact with the soil (of length $P$) is included. The resulting work equation can be expressed as:

$$V \cdot v + H \cdot u = s_u L \cdot \Omega R_{slip} + \alpha s_u P \cdot \Omega \frac{D}{2}$$  \hspace{1cm} (4)

where $u =$ horizontal pipe velocity, $y_0 \Omega$;
$v =$ vertical pipe velocity, $x_0 \Omega$;
$L =$ length of the slip surface;
$R_{slip} =$ radius of the slip surface, and
$\Omega =$ angular velocity of the slip circle.

To optimise the mechanism for each combination of $V$-$H$ loading, a search algorithm was used to locate the centre of the slip circle corresponding to the mechanism offering the lowest collapse load. The search process starts with a coarse grid such that a large search zone can be covered within a reasonable computational time. For a given value of $V$, the horizontal forces calculated from Eq. (5) using each of the grid points as the centre of the slip circle are compared to locate the local minimum which gives the lowest horizontal force. A refined search is then carried out around the local minimum using a finer mesh to enhance the accuracy of the solutions.

$$H = \frac{s_u L R_{slip} + \alpha s_u P \frac{D}{2} - V x_0}{y_0}$$ \hspace{1cm} (5)

To model separation behind the pipe, the rear portion of the slip circle is omitted from the calculation, leading to reduced values of $L$ and $P$.

The slip circle upper bound is significantly non-optimal for purely vertical loading, although this region of the failure envelope is not relevant for most pipeline applications due to the high over-penetration ratio applicable in practice. The values of $V$ for $H = 0$ deduced from the single slip circle mechanism exceed the solution of Murff et al. (1989) by more than $35\%$ for $w/D < 0.2$. To refine the failure envelope within this region of predominantly vertical loading, an alternative upper bound mechanism has been used – a modification of the Martin circular rigid block mechanism (Martin and Randolph, 2006). To
generalise this mechanism to $V$-$H$ loading, the inclination of the pipe movement is allowed to vary (Fig. 4(b)). The rigid circular blocks become centred on the diametric line across the pipe that is perpendicular to the direction of movement. By varying the direction of movement, the optimal mechanism is deduced by minimising $H$ for a given value of $V$. This mechanism provides a significant improvement over the slip circle solution for low values of $H/V$.

RESULTS

**Case 1: Without Pipe-soil Separation**

The failure envelope for a rough pipe with no separation is shown in Fig. 5 for increments of normalised embedment in the range $0.05 < w/D < 0.5$. Normality (or associated flow) is observed from the directions of the calculated pipe movement: the predicted pipe displacement vector is always perpendicular to the failure envelope. The transition points from the slip circle to the Martin mechanism are indicated. Examples of the optimal mechanisms for two combinations of vertical load and embedment are shown in Figs. 6(a) and 6(b). At very low vertical load, the pipe movement is horizontal, as required by symmetry.

Yield envelopes have been previously established for the response of surface strip foundations under combined loading (Green, 1954; Murff et al., 1994; Bransby and Randolph, 1998), which are analogous to a pipeline at infinitesimal embedment ($w \to 0$). The solution by Green (1954) applies for undrained loading, and is applicable regardless of whether separation is permitted (for $V < 0$) (Houlsby and Puzrin, 1999). The maximum value of horizontal capacity given by Green's solution is $H/D' \leq 1$ (where $D'$ is the effective contact width as shown in Fig. 1; $D'/D = 0.436$ for $w/D = 0.05$) and applies for $V/V_{\text{max}} < 0.5$. (Fig. 5). The shape of the yield envelope derived from Green's solution is similar in general shape to the yield envelopes found in the present analysis for the no-separation case, but the curved surface of the pipe and the resulting embedment leads to additional passive and active resistance so that the pipe envelope for $w/D = 0.05$ (marked A on Fig. 5) is significantly larger than Green's solution for a surface footing with the same contact width (marked B on Fig. 5).

At very shallow embedment, the failure envelope is similar in shape to the Green (1954) solution for a surface foundation. It is interesting to note that, as for Green's solution, the optimal mechanism under purely vertical load involve inclined movement of the pipe. As the embedment increases, the envelope becomes more circular, reflecting the transition towards fully-buried behaviour, when the resistance is independent of the direction of loading. For a deeply buried pipe, when the behaviour is independent of the free surface, the failure envelope is a circle with a radius of 9.14–11.92, depending on the pipe roughness, based on the plasticity solution for flow

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**Fig. 5.** Failure envelope for rough pipe without separation

**Fig. 6.** Example mechanisms for rough pipe
around a cylinder (Martin and Randolph, 2006).

For smooth pipes, the failure envelope is of similar shape, but the uniaxial horizontal capacity (i.e., $H$ for $V = 0$) reduces by approximately 13% irrespective of the pipe embedment (Fig. 7).

Figure 8 plots the location of the centre of the most optimum slip circle for both rough and smooth pipes at different embedments and vertical loads. It can be seen that, at high vertical load, the centre of the slip circle lies in front of the pipe in the direction of the horizontal pipe movement, implying a “deep-seated” type of failure. As the vertical load decreases, the slip circle centre moves towards the vertical centreline of the pipe. This indicates that the most kinematically favourable mechanism becomes a slip centre symmetrical about the pipe vertical centreline, implying pure horizontal pipe movement at low vertical load when separation is not permitted.

### Case 2: With Pipe-soil Separation

An analysis with separation at the rear of the pipe is applicable to the prediction of breakout resistance if suction and tensile resistance at the rear of the pipe is lost. For separation to occur, the soil behind the pipe has to be stable after the support from the pipe is removed. The solution of the maximum height of a vertical cut is $\sim 4s_u/g'$.

Typical pipelines have an outer diameter in the range of 4–30 inches (100–750 mm). If a pipe with $D = 0.75$ m is embedded at $D/2$, the soil at the rear of the pipe would not collapse if $s_u > 0.5$ kPa, assuming $g' \sim 5$ kN/m$^3$. This low required value of $s_u$ suggests that soil-pipe separation can occur in normal seabed soil conditions.

Figure 9 shows the failure envelopes for a rough pipe over an embedment range of $0.05 < w/D < 0.5$. The single slip mechanism is non-optimal at larger values of $V/V_{\text{max}}$, and the transition to the modified Martin mechanism is identified. The maximum predicted breakout resistance for any given embedment occurs at roughly $V/V_{\text{max}} \sim 0.5$. For the case with separation, the yield envelope for the pipe is fundamentally different to Green’s solution (which is also applicable to conditions of separation). The pipe yield envelopes pass through the load origin ($V = 0$, $H = 0$) for $w/D < 0.5$, indicating zero breakout resistance in the absence of vertical load. This contrast shows that for undrained conditions in which separation is permitted, the curved shape of the pipe surface leads to a fundamental difference in the shape of the $V$-$H$ yield envelope compared to a surface foundation. For a surface foundation under a vertical load $V/V_{\text{max}} < 0.5$, the optimal failure mechanism is always horizontal sliding at the foundation base, and $H/D/s_u = 1$ (so the slope of the yield envelope, $dH/dV = 0$). In contrast, for a pipe under a comparable vertical load level, the failure mechanism involving horizontal translation includes a significant component of resistance due to passive failure of the soil in front of the pipe. This resistance can be reduced by a failure mechanism involving upwards pipe movement, which
creates a region of yield surface for which \( dH/dV > 0 \)
(due to normality). The corresponding mechanism involves a slip circle extending from the front face of the pipe towards the soil surface. At very low values of \( V/V_{\text{max}} \), the failure envelopes feature a ‘frictional’ cut-off of constant \( H/V \) (Fig. 9). Within this region the optimal failure mechanism involves upwards translation of the pipe at a gradient parallel to a tangent to the pipe wall at the soil surface and the slip circle radius tends to infinity (so the slip surface becomes planar). This direction of movement removes all contact between the pipe and the soil so there is no energy dissipation. Instead, the horizontal load creates sufficient work input to balance the energy required to lift the vertical load. From the geometry of the pipe wall at \( \omega \) (Fig. 1), it can be shown that the frictional cut-off is:

\[
H/V = \sqrt{\frac{1 - \left(1 - \frac{2w}{D}\right)^2}{1 - \frac{2w}{D}}} \quad (6)
\]

As the embedment approaches \( w/D = 0.5 \), Eq. (6) tends to infinity. The failure envelope for \( w/D = 0.5 \) intersects with horizontal load axis at \( H/s_D = 0.5 \).

For surface foundations, these two distinct shapes of failure envelope, either passing through the load origin, or reaching maximum \( H \) at \( V = 0 \), are associated with drained and undrained conditions respectively. See, for example, Bransby and Randolph (1998) or Murff et al. (1994) for undrained examples, and Butterfield and Gottardi (1994) and Nova and Montrasio (1991) for drained examples. In contrast, due to the curved geometry of the pipe surface, the failure envelope for undrained soil conditions (albeit with separation at the rear face) passes through the load origin.

The failure envelopes for smooth pipes are plotted in Fig. 10. At very low values of \( V/V_{\text{max}} \), the solutions are identical to those for rough pipes since no energy is dissipated at soil-pipe interface. The maximum breakout resistance, which occurs at \( V/V_{\text{max}} \approx 0.5 \), is \( \approx 15\% \) lower than the rough case for all values of \( w/D \). Compared to the case without separation, the reduction in breakout resistance from rough to smooth is minimal for \( V < V_{\text{max}} \) due to the shorter region of slip at the pipe surface.

The location of the optimum slip circle centre for the case with soil-pipe separation is plotted in Fig. 11. At high vertical load, the location of the slip circle centre lies in the front of the pipe in the direction of the horizontal pipe movement, and therefore predicts a downward pipe movement. As the vertical load decreases, the centre of the slip circle moves away from the vertical centreline of the pipe, indicating an asymmetric failure mechanism, which is in contrast to the case with no-separation (Fig. 8). In the present analysis the slip circle centre was limited to remain within \( 30D \) of the pipe centre. However, the optimal centre point tends towards infinity as the solution approaches the frictional cut-off defined by Eq. (6), which involves rigid body pipe movement with no rotation.
Comparison with Empirical Expressions for Breakout Resistance

Figure 12 compares the theoretical breakout resistance with predictions from Eq. (3) using empirical parameters $\lambda = 2$ and $\mu = 0.3$, over the ranges of embedment and normalised load that are practically relevant. The comparison suggests that the empirical equation (which was calibrated from model test data) is linked to the situation where pipe-soil separation is possible. The empirical predictions are within 30% of the theoretical values for the separation case, but are at least 2 times smaller than the theoretical solutions for the case without separation. The theoretical solutions indicate that the linear empirical expressions are a simplification of the underlying behaviour, which involves a reduction in apparent ‘friction factor’ $H/V$ with increasing vertical load.

Effect of Soil Weight

Figure 13 shows the upper bound solutions including the effects of soil weight for typical values of $V/Ds_u$. In the analyses, it has been assumed that water could fill the void created by the rotating block, and therefore the submerged unit weight of the material is used. For the no-separation case, soil weight has a minimal influence on the failure envelope since the net change in the potential energy of the soil within the two-sided slip circle mechanism is minimal. The effects become slightly larger but still of little engineering significance as vertical load and embedment increase. For the range of values shown on Fig. 13, the maximum difference in breakout resistance occurs at $w/D = 0.5$ and $V/Ds_u = 2$, and is only ~3% and ~5% for rough and smooth pipes respectively, as the $s_u/\gamma D$ ratio reduces from infinity to 0.5.

With pipe-soil separation, the increase in potential energy of the soil lifted ahead of the pipe has an influence on the breakout resistance. For values of pipe weight in the range $0.3 < V/Ds_u < 2$, the maximum change in breakout resistance due to soil weight is 12% for $s_u/\gamma D = 0.5$. This relatively small influence is due to the small size of the region of soil that is lifted ahead of the pipe (Fig. 6(c)).

It should be noted that these theoretical failure envelopes ignore the effect of soil heave during embedment. The soil displaced by the pipe during embedment will offer additional resistance during breakout, including a surcharge load that will increase the influence of soil weight.

DISCUSSION

This upper bound plasticity analysis provides insights into both the breakout loads of partially-embedded pipes, and also the pipe movements at failure. Breakout resistance is strongly influenced by the separation condition at the rear of the pipe. If pipe-soil separation is prevented by negative excess pore pressure and the tensile strength of the soil, the breakout resistance is typically 2–5 times higher than if separation occurs (for the typical embedments and over-penetration ratios). The associated failure mechanism is approximately symmetrical about the vertical pipe axis implying horizontal or slightly downward pipe movement. In contrast, separation at the rear of the pipe changes the most kinematically favourable mechanism to a passive “wedge” mechanism which causes the pipe to rise upwards through the soil ahead of it.

For a typical pipe weight of $V/Ds_u = 1.5$, over-penetrated to an embedment of $0.5D$, the breakout load with separation is 25% of the value for the case without separation. The difference reduces for heavier pipes and lower embedment (i.e., a lower over-penetration ratio).
In field conditions, the likelihood of separation at the soil-pipe interface is affected by the pipe coating material, the soil type and the rate at which the loading is applied. The broad agreement between typical empirical expressions for breakout resistance and the theoretical values for the separation case suggests that this case prevails in practice.

CONCLUSIONS

This Technical Note describes upper bound plasticity analysis of a partially-embedded pipe. The purpose is to establish failure envelopes under combined vertical and horizontal loading in order to provide a theoretical basis for estimating breakout resistance. Failure envelopes have been generated for smooth and rough pipes, with and without separation between the pipe and the soil at the rear face.

These failure envelopes resemble those developed previously for surface foundations, but capture additional effects that arise from the curved geometry of the pipe surface. The breakout resistance is strongly affected by the separation condition, being reduced typically by a factor 4 if separation is permitted. In addition, the displacement of the pipe at failure is altered, with separation the pipe typically rises, whereas without separation the movement is close to horizontal. The influences of pipe roughness and soil weight are found to be relatively minor.

Empirical expressions used in practice tend to produce predictions closer to the separation case. However, these expressions assume a linear variation in breakout resistance with embedment and vertical load. This theoretical analysis demonstrates that these relationships are non-linear. The resulting envelopes provide a more rigorous basis for predicting the breakout resistance of partially-embedded pipelines.

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