DISCRETE ELEMENT ANALYSIS OF THE RESPONSE OF GRANULAR MATERIALS DURING CYCLIC LOADING

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ABSTRACT

Soil can experience cyclic or repeated loadings in a range of situations and prediction of cyclic soil response is obviously important to geotechnical engineers. The particulate nature of soil results in highly complex, non-linear response characteristics, and developing models that can capture soil response under cyclic loading is non-trivial. The distinct element method (DEM) can be used to study the fundamental, particle-scale mechanics of granular materials, and offers much promise as a tool to advance understanding of soil response. The first stage in adopting DEM to model cyclic soil response is to quantitatively demonstrate that a DEM model can replicate physical test data, as analytical validation of DEM models for random assemblies of particles under repeated loading is not viable. This paper describes a series of strain-controlled cyclic triaxial tests on an ideal granular material (steel spheres) that were used to validate the capability of an axi-symmetric DEM model to analyse cyclic loading. The DEM model was then used in a parametric study to examine the particle-scale mechanics of the response of specimens of uniform spheres to 50 cycles of loading with various strain amplitudes. The distribution of contact force orientations and magnitudes during testing was examined. The simulations indicate that both the fabric anisotropy and coordination number continued to evolve over the 50 cycles considered. While the variation in the macro-scale response was less marked, there is a clear relation between the micro-scale parameters and the overall specimen response.

Key words: cyclic triaxial test, discrete element method, fabric, validation (IGC: E8/E13/E14)

INTRODUCTION

The response of soil under cyclic loading is of great interest to geotechnical engineers. Cyclic loads of relatively high frequency are felt during earthquakes, however lower frequency repeated loadings are also important. Such loading scenarios include the foundations to road pavements (e.g., Lekarp et al., 2000), wind turbine foundations, foundations to reciprocating machines, soil adjacent to integral bridge abutments (e.g., Clayton et al., 2006), and soil in dams where the reservoir level fluctuates. Janbu and Seneset (1981) proposed that observed time-dependent settlements of structures on high permeability soils may be due to combined effects of cumulative displacements under cyclic loading and creep. Reflecting the importance of understanding cyclic soil response, much research has been undertaken in this area, and constitutive models for analysis of field scale boundary value problems have been proposed. The complexities of cyclic soil response arise largely due to the particulate nature of soil. Due to the limitations of conventional laboratory testing, much of the experimental research has considered the overall material response by making external measurements on representative samples. While this macro scale approach has helped greatly in developing our understanding of soil response, there is merit in understanding the fundamental particle-scale interactions that underlie the observed macro-scale complexity.

Some researchers have inferred the micro-mechanics of soil response based upon (macro-scale) observations in typical element tests. For example, Mitchell (1993) stated that cyclic straining causes a loose or a medium dense sand to densify by disturbing the initial soil fabric and repositioning the soil grains into more efficient packings. The effect of initial fabric on soil liquefaction susceptibility is well known. Youd (1977) provided a schematic illustration of packing changes within sands subject to cyclic loading, and also provided a detailed micro-scale discussion on the variation of the void ratio during cyclic loading. England et al. (1997) noted the need for constitutive models that can capture the evolution of fabric under cyclic loading. Papadimitriou and Bouckovalas (2002) recognised the importance of fabric evolution during cy-
cyclic loading of soil and developed an analytical approach to account for such fabric changes in a continuum elastoplastic constitutive model. Wichitman and Triantafyllidis (2004) proposed hypotheses on the way soil microstructure changes during cyclic loading to explain the observed increase in soil stiffness with cyclic loading. One recent study that assessed the effects of fabric on drained sand response is that of Chaudhury et al. (2002) who, in a series of Hollow Cylinder Apparatus tests, demonstrated that the shape of the hysteresis loops depended on the orientation of loading relative to the initial fabric and that the volumetric strain response also was affected by fabric orientation. However, in the absence of micro-mechanical measurements detailed, quantitative conclusions regarding the particle-scale mechanics cannot be achieved.

Discrete element modelling (DEM) is a numerical tool that can greatly enhance our understanding of fundamental soil mechanics (e.g., Thornton, 2000). DEM is particularly useful for developing an understanding of the response of cohesionless (granular) soils. Using DEM we can simulate conventional soil mechanics element tests and gain detailed information about the soil fabric and its evolution during testing (e.g., Cui and O’Sullivan, 2006). DEM has been used in a limited number of studies to examine the micro mechanics of cyclic soil response. An early study by Ng and Dobry (1994) considered the response of systems of disks (2D) and spheres (3D) in a periodic cell in undrained (constant volume) cyclic simple shear loading conditions. The results of these simulations qualitatively agreed with physical tests on sand. The potential to use DEM to understand the micro-mechanics of soil response was demonstrated by examining the influence of particle rotation on the macro-scale response in the 2D simulations. More recently, in a two dimensional study, Sitharam (2003) simulated undrained and drained cyclic biaxial tests on assemblies of disks. David et al. (2005) carried out a series of 3D DEM simulations on spherical particles enclosed within rigid walls. In these simulations the spheres were subject to stress controlled cyclic loading and the influence of friction on the macro-scale response was explored. From the authors’ perspective, the current status of the use of DEM to analyse cyclic soil response can be summarized as follows: the applicability of DEM to develop insight into cyclic soil response has been demonstrated, but there has been limited use of DEM to develop an understanding of the micro-mechanics of cyclic soil response.

The research described here coupled a series of relatively simple strain-controlled cyclic triaxial tests with discrete element method (DEM) simulations to quantitatively explore the ability of a DEM model to capture the cyclic response of a granular material. Having validated the DEM model, a parametric study was carried out to examine the influence of the amplitude of cyclic loading on the specimen response, considering both the macro-scale response and the particle-scale interactions.

**TEST AND SIMULATION DESCRIPTIONS**

The approach to DEM code validation taken by Cui and O’Sullivan (2006) and Cui et al. (2007) was also adopted here. The philosophy of model validation in these studies is to perform geotechnical laboratory tests on specimens of Grade 25 steel spheres. As these spheres are fabricated with tight tolerances (the sphere diameter and sphericity is controlled to within $7.5 \times 10^{-4}$ mm during fabrication), the particle geometry can be accurately replicated in the numerical model. Furthermore the inter-particle friction coefficient between these particles has been accurately measured. These assemblies of steel spheres are an ideal material and are considered here to be an analogue soil. For example, this material has a geometry that is substantially simpler than the geometry of real sands (the importance of geometry on cyclic soil response is considered by Clayton et al., 2006), and this material will not exhibit the degree of particle abrasion under cyclic loading observed by Festag and Katzenbach (2001). The use of such an ideal material is in this study is motivated by a desire to directly compare simulations performed using the DEM model with physical test data for validation purposes. Once the ability of the DEM model to capture the response of this relatively simple material is qualitatively demonstrated, the model can then be extended with some confidence to analyse more realistic particle assemblies. We note that many researchers have gained insight into soil response by considering such “simple” granular materials (e.g., Thornton, 2000; Rowe, 1962; Kuwano, 1999). Cui et al. (2007) described both the physical test configuration (for monotonic triaxial tests) and the DEM simulation approach in some detail. For completeness, however, a brief overview of the test configuration and DEM model is included here.

**Physical Tests**

The laboratory tests used assemblies of dry Grade 25 chrome steel balls under vacuum confinement of 80 kPa. As measured by the manufacturer, Thomson Precision Ball, the sphere material density is $7.8 \times 10^3$ kg/m$^3$, the shear modulus is $7.9 \times 10^{10}$ Pa, the Poisson’s ratio is 0.28. The inter-particle friction coefficient was measured by O’Sullivan et al. (2004) to be 0.096 for equivalent spheres, while the sphere-boundary coefficient was measured by Cui (2006) in a series of tilt tests to be 0.228.

Two specimen types were considered, the uniform specimens contained spheres of radii of 2.5 mm, while the non-uniform specimens contained a mixture of spheres with radii of 2 mm, 2.5 mm and 3 mm in a 1:1:1 mix. The specimens were 101 mm in diameter and 203 mm high. The samples were prepared by sealing the latex membrane against the inside of a cylindrical mould using a vacuum. The spheres were then placed using a funnel with a long shaft, the height of the shaft was increased 5 times during the specimen preparation process. The uniform specimens had a void ratio of 0.612, while the non-uniform specimens had a void ratio of 0.603. A representative
Fig. 1. Illustration of specimen configuration in laboratory and simulations

(a) Representative laboratory test specimen

(b) Representative “virtual” specimen for DEM analyses (boundary conditions indicated)

(c) Subplot of the Voronoi diagram used to simulate membrane in DEM analyses

The samples were tested under cyclic triaxial test conditions. As observed by Kramer (1996) the cyclic triaxial test has been the most commonly used test for measurement of dynamic soil properties at high strain levels. The principal limitations of cyclic triaxial testing include the reversal of the principal stresses, the orientation of the applied shear stresses, and the direct measurement of $E$ rather than $G$ (Riemer, 2004). Despite these limitations, the cyclic triaxial test is suited to fulfill the objectives of the current study, namely to provide a database of physical test results for DEM model validation and to subsequently examine the evolution of fabric during cyclic loading using the DEM model. A number of research studies on sand response have used the cyclic triaxial apparatus, however the loading approach differs in these studies, for example Prahan et al. (1989) kept $p'$ constant, while Clayton et al. (2006) cycled the radial stress. The tests described here were strain controlled (similar to the tests described by Cosgrove et al. (2001)), with the axial strain (in compression) being increased to 1% and then reduced to 0%. Due to equipment restrictions, the number of strain cycles in each test was limited to 15 cycles, and these cycles were executed at a rate of 0.0333 mm/s by raising and lowering the bottom boundary. The amplitude of the axial strain was 1% in all cases (giving a frequency of cyclic loading of 0.5 cycles per minute). The vertical force applied to the specimen was measured on the stationary top boundary. Full details of these tests can be found in O’Neill (2005).

DEM Simulations

The DEM simulations were performed using a mixed boundary test environment as proposed by Cui et al. (2007). Three different types of boundary condition are used in the “mixed boundary” simulations (refer to Fig. 1(b)); rigid, planar surfaces are used to simulate the top and bottom platens, a “stress-controlled membrane” is used to simulate the latex membrane that enclosed the specimen laterally in the laboratory, and a set of two, orthogonal vertical periodic boundaries are used so that only one segment of the axi-symmetric specimen need be considered in the simulation. This approach therefore uses the axi-symmetrical property of the triaxial cell to reduce the computational costs of DEM simulations. To achieve an axi-symmetric analysis while maintaining a continuous contact network in the circumferential direction two vertical, circumferential periodic boundaries are introduced in the model (Fig. 1(b)). For ease of implementation a 90° segment is considered in the current study. Contact is detected between particles close to one periodic boundary and particles along the other periodic boundary by multiplying the particle coordinates by an orthogonal rotation tensor. The rotation tensor is also used to rotate the contact force vector for application to the particles, where appropriate. The authors are confident that their approach, including an assumption of geometric axi-symmetry is valid for this system in general, and for the study presented here in particular. The specimen itself is cylindrical and in the physical test the particles were deposited under gravity, therefore it can reasonably be expected that the material fabric and properties are isotropic along planes orthogonal to the central, vertical, axis passing through the specimen. The only significant disadvantage of this approach is its inability to
capture a single shear band passing through the sample as observed, post peak, in triaxial tests on dense or cemented sands. This limitation is not relevant in the context of the current study, the material is relatively loose and the stress levels considered are lower than the peak stresses mobilized in monotonic tests on this material.

To model the flexible latex membrane enclosing the specimen, the confining pressure was applied using a “stress-controlled membrane”. The numerical “membrane” is formed by identifying the “membrane spheres”, along the outer surface of the sample. A Voronoi diagram of the sphere centroids is used to calculate the force that should be applied to each membrane sphere to achieve the required confining pressure measured in the laboratory, as illustrated in Fig. 1(c). The force to be applied to each “membrane sphere” is calculated by multiplying the confining pressure and the area of the Voronoi polygon surrounding the sphere centroid. In this test environment, the vertical (deviatoric) load is applied by cycling the rigid top boundaries.

To prepare the “virtual” specimens for the DEM simulations, a loose specimen of uniform spheres was firstly generated using a code developed by Jodrey and Tory (1985). The specimen density was increased by expanding the radii of the spheres and varying the confining pressure. It was found that using a confining pressure of 3000 kPa ensured that the void ratios of the numerical specimens ($e=0.615$) were close to the laboratory specimens ($e=0.612$). Considering the discrepancy between the confining pressure used in the physical tests and the confining pressure used in the DEM simulations, for the specimen generation approach used, the authors could control either the void ratio alone or the average stress alone, with the other parameter being a resultant value. In the simulations presented here, we chose to control the void ratio as the specimen response is quite sensitive to the void ratio, and the applied stresses are the stresses under which the specimen came into equilibrium for the specified void ratio.

The “virtual” one-quarter cylindrical specimen generated for the DEM simulations of the uniform specimens contained 3852 spheres with radii of 2.47 mm, with a radius of 50 mm and a height of 200 mm, and a void ratio of $e=0.615$. Using a similar approach to the uniform case, a non-uniform specimen containing the same mixture of spheres as in the non-uniform physical tests was developed. This non-uniform specimen contains 3464 non-uniform spheres and had a void ratio of 0.604 (physical test $e=0.603$).

Considering the input parameters for the DEM simulations, contact was modelled using the elastic Hertz-Mindlin contact model (implementation detailed by Lin and Ng, 1997) with the no-slip tangential contact proposed by Mindlin (1949) being used to calculate shear contact forces. The input parameters for this model are the shear modulus ($7.9 \times 10^{10}$ Pa), and the Poisson’s ratio (0.28). The average inter-sphere friction coefficient of 0.096 measured by O’Sullivan et al. (2004) for equivalent spheres was directly input, having established that the results of simulations on randomly packed spheres are insensitive to the small distribution of friction values obtained in these earlier tests. No additional (numerical) damping was applied to the system during the test simulations. As the central difference time-integration system adopted is non-dissipative, (refer to O’Sullivan and Bray (2001) for example), energy dissipation in the system occurs via frictional sliding and loss of contact between particles. A scaled density value was used ($7.8 \times 10^{12}$ kg/m$^3$). Density scaling was adopted to increase the critical time-step and reduce the computational cost of the DEM simulations (see also Thornton, 2000; O’Sullivan et al., 2004).

The option to use density scaling to control the maximum time step for stable DEM simulations is also available within the commercial DEM code, PFC (Itasca, 2004) and, as outlined by Itasca, controlling the time-increment in this way is valid as long as the inertia forces remain negligible in comparison with the contact forces. The simulations as completed were highly computationally expensive, a simulation of 50 cycles taking on average one week to complete.

An additional input parameter in DEM simulations is the time step for the analysis. Recognising that DEM simulations using the central-difference time integration approach are conditionally stable (O’Sullivan and Bray, 2003), the simulation time step ($\Delta t$) was calculated as:

$$\Delta t = \beta \sqrt{\frac{I_{\text{min}}}{k_{\text{eq}}}}$$

where $I_{\text{min}}$ is the moment of inertia of the smallest particle in the system, and $k_{\text{eq}}$ is an equivalent linear contact spring stiffness. Referring to O’Sullivan and Bray (2003) and Itasca (2002) the time-step for stable analyses in DEM is proportional to $\sqrt{I_{\text{min}}/k_{\text{eq}}}$, the parameter ($\beta < 1$) is then used to select a precise value for the time increment. As the Herzian contact spring is non-linear (elastic), the following expression is used to estimate the equivalent linear spring stiffness:

$$k_{\text{eq}} = \frac{2}{3} \sqrt{2Gr_{\text{max}}/\alpha}$$

where $r_{\text{max}}$ is the maximum sphere radius and $\alpha r_{\text{min}}$ is the maximum allowable overlap of the spheres in the system (the program exits with an error message if the overlap at any contact point in the system exceeds $\alpha r_{\text{min}}$). O’Sullivan and Bray (2003) demonstrated that $\beta$ should not exceed 0.221 for stable three-dimensional analyses with multiple particle contacts. Throughout the current study, to be conservative and ensure the risk of instability was minimized, a $\beta$ value less than 0.1 was adopted. Furthermore, this approach ensured that all the non-linearity associated with changing contact conditions was captured in the simulations.

An initial simulation using the loading rate of 0.0333 mm/s was rejected as the specimen response was not “quasi-static”, i.e., there was a significant difference between the force measured on the top and bottom boundaries in the simulation. Therefore the loading velocity and the density scaling values used were adjusted to achieve a
quasi-static response for all the cyclic strain amplitudes considered in the parametric study. Table 1 lists the value of loading velocity and density values used to ensure quasi-static conditions during the simulations (i.e., a negligible difference between the measured forces along the top and bottom boundaries).

A parametric study considering a single, monotonic triaxial test on a specimen of uniform spheres was used to validate the suitability of the DEM input parameters. Earlier experience in direct shear test simulations (Cui and O’Sullivan, 2006) informed this study. Once the model parameters had been determined for this single test, their validity was confirmed in a further monotonic DEM-laboratory test comparisons (Cui et al., 2007), as well as the cyclic tests described here.

**COMPARISON OF THE LABORATORY CYCLIC TESTS AND THE DEM SIMULATIONS**

Understanding the material response in the earlier monotonic triaxial tests is important to appreciate the findings presented here. Consequently representative data from the study described by Cui et al. (2007) is presented in Fig. 2. Figure 2 considers the variation in normalized deviator stress \((\sigma_z - \sigma_r)/\sigma_r\) as a function of axial strain. To calculate this normalised deviator stress, \(\sigma_z\) is taken from measuring the stresses along the top platen, while \(\sigma_r\) is taken to be the confining pressure (cell pressure). (Note the laboratory test conditions and DEM input parameters were consistent for both the monotonic and cyclic studies.) During the vacuum confined physical tests, no measure of the volumetric strains could be made due to the absence of a fluid filled cell used in typical triaxial tests. Consequently no area correction could be applied and the physical tests and DEM simulations are compared without application of an area correction. Cui et al. (2007) repeated their tests and simulations on a number of similar specimens, equivalent to the specimens considered in the current study. In these monotonic tests the uniform specimens attained peak stress ratios \(((\sigma_z - \sigma_r)/\sigma_r)\) in the range of 0.90 to 1.03 at axial strain levels between 5.3% and 9.2%. For the non-uniform specimens the monotonic peak stress ratios were in the range of 0.94 to 1.03 and they were attained at axial strain levels 6.4% and 8.0%.

**Table 1. DEM simulation parameters**

<table>
<thead>
<tr>
<th>Specimen type</th>
<th>Strain amplitude</th>
<th>Loading velocity (mm/s)</th>
<th>Density of material (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform spheres</td>
<td>1%</td>
<td>0.00333</td>
<td>(7.8 \times 10^{12})</td>
</tr>
<tr>
<td>Non-uniform spheres</td>
<td>1%</td>
<td>0.00333</td>
<td>(7.8 \times 10^{12})</td>
</tr>
<tr>
<td>Uniform spheres</td>
<td>0.5%</td>
<td>0.00333</td>
<td>(7.8 \times 10^{12})</td>
</tr>
<tr>
<td>Uniform spheres</td>
<td>0.1%</td>
<td>0.000333</td>
<td>(7.8 \times 10^{12})</td>
</tr>
</tbody>
</table>

Fig. 2. Response of uniform and non-uniform specimens in monotonic triaxial tests (laboratory tests and DEM simulations) (Cui et al., 2007)

Fig. 3. Comparison of macro-scale response observed in the laboratory tests and the DEM simulations (without area correction)
For the current cyclic triaxial study, a comparison of the macro-scale response observed in a laboratory test and the DEM simulation for the uniform specimen without area correction is illustrated in Fig. 3(a), while the non-uniform specimen is considered in Fig. 3(b). As with the monotonic tests and simulations, the deviator stresses shown are normalized by the confining pressures ($\sigma_v$), (i.e., the plots indicate the variation in $(\sigma_d - \sigma_v)/\sigma_v$). Furthermore, the comparison is made without application of an area correction, as before. In the initial, specimen preparation stage, of the DEM simulations a system of rigid walls enclosed the specimen and the position of these walls was adjusted to control the stresses within the specimen. At the start of the test, upon removal of these walls there was adjustment of the particle positions within the specimen, with a consequent reduction in the vertical stress measured along the top and bottom boundaries. Consequently at the beginning of the first cycle $\sigma_v$ was slightly less than $\sigma_v$, however as the magnitude of this difference was small it cannot be expected to have a significant influence on the overall material response. For clarity, only the 1st, 2nd, and 10th load cycles are illustrated in both cases. Using the DEM simulation results, additional plots applying a standard area correction were developed (for both specimen types) and the observed response did not noticeably change. This insensitivity can be understood by recognising that the strain levels considered here are relatively small, recalling that the peak stress ratios in the monotonic tests were attained at axial strain levels between 5.3% and 9.2%. The peak stress ratio attained in the first cycle of the physical test on the uniform specimens was 0.23, while the peak stress ratio attained in cycle 1 in the tests on the non-uniform specimens was 0.25.

During cyclic loading hysteresis loops were observed, as is typical for cyclic loading of granular materials, indicating dissipation in energy. Considering both specimens in the first loading cycle the response was somewhat bi-linear with a relatively stiff response (with decreasing stiffness) being observed up until $\varepsilon_a = 0.25\%$ and a significantly less stiff response being observed between $\varepsilon_a = 0.25\%$ and $\varepsilon_a = 1.00\%$. Such a marked change in stiffness was not observed during the subsequent loading cycles as shape of the specimen response during loading evolved with each cycle. For the second loading cycle, the initial stress ratio was about $-0.33$ for both specimens, but the stress ratio at $\varepsilon_a = 1.00\%$ in this cycle was similar to the stress ratio at $\varepsilon_a = 1.00\%$ in the first cycle, i.e., the overall average stiffness of each specimen in the second cycle was higher than in the first cycle, (the differences in stiffness are related to changes in the specimen fabric below).

During the unloading stages, the stress ratio decreased quickly at the outset of unloading to reach a value of 0 at $\varepsilon_a$ values between 0.8% and 0.9%. For all three load cycles considered, the decrease in stiffness during unloading was somewhat bi-linear, with a relatively stiff response observed until $\varepsilon_a = 0.9\%$ and a less stiff response being observed between $\varepsilon_a = 0.9\%$ and $\varepsilon_a = 0\%$. The stress ratio at the end of unloading ($\varepsilon_a = 0\%$) appeared to decrease slightly with increased cyclic loading, with this decrease being less marked for the non-uniform specimen. Small changes in the peak stresses during strain controlled cyclic loading on an ideal material were also observed by Clayton et al. (2006). In their study Clayton et al. compared the response of Leighton Buzzard Sand and glass ballotini under strain controlled cyclic loading. A significant decrease in minimum deviatoric stress was observed for the Leighton Buzzard Sand samples, and not for the glass ballotini samples. Clayton et al attributed this observed difference in response to particle geometry effects.

All of the trends in material response noted here were observed in the both the physical tests and the DEM simulations (cycles 1, 2 and 10).
simulations. A good quantitative agreement was attained between the physical test and the simulations with the stress ratio at $e_a = 1\%$ being very similar. For the uniform specimens the average stress ratio at 1\% was 0.635 for the physical tests and 0.666 for the numerical simulations. For the non-uniform specimens of the average stress ratio at 1\% was 0.676 for the physical test and 0.679 for the DEM simulation. However, the observed decrease in the stress ratio at the end of unloading with increasing cyclic loading was greater in the laboratory tests than in the DEM simulations.

Comparisons of the secant stiffnesses for the laboratory tests and the simulations for both the uniform specimen and the non-uniform specimen are shown in Fig. 4. In both cases the secant stiffness ($E_{sec}$) calculated was normalized by the confining pressure and $E_{sec}$ was calculated using the stress and strain conditions at the start of the current loading cycle as the origin. The simulation captured the decrease in stiffness with increasing strain relatively accurately for the loading cycles considered here. Considering the loading cycles the numerical simulations indicated a decrease in the rate at which stiffness decreases with increasing axial strain with increasing number of cycles and this was not observed in the physical tests. The shape of stiffness-strain plot in unloading in the first cycle differs significantly from subsequent cycles and this trend is observed in the laboratory tests and the DEM simulations.

The observed slight differences between the physical tests and the numerical simulations can possibly be explained by the simple rheological model used to model contact between the particles, or possibly damage to the surface of the spheres in the physical tests during loading. However despite these (relatively small) differences the authors are satisfied with the agreement attained between the physical tests and the numerical simulations and confident that the particle-scale interactions that can be analysed using the DEM simulation data give an accurate representation of the physical reality. The subsequent sections of this paper describe the findings of a parametric study involving a series of DEM simulations where the amplitude of cyclic loading was systematically reduced and the particle-scale interactions in the DEM specimens were monitored.

**SENSITIVITY OF MACRO-SCALE RESPONSE TO THE AMPLITUDE OF CYCLIC STRAINING**

The parametric study considered uniform DEM specimens subjected to strain-controlled cyclic triaxial tests with cyclic strain amplitudes of $\varepsilon_a^{max} = 1\%$, $\varepsilon_a = 0.5\%$, and $\varepsilon_a^{max} = 0.1\%$. Note that all the analyses were carried out on the same specimen. The macro-scale test results are illustrated in Fig. 5 to 9. The term “macro-scale” is used here to clearly indicate that these stresses are calculated by considering the boundary forces and the applied cell pressure, as would be the case in a physical test. Figure 5 illustrates the sensitivity of the shape of the stress-strain plot to the cyclic strain amplitude (for clarity, for each simulation only the 1st, 2nd, 10th and 50th cycles are shown). When $\varepsilon_a^{max} = 0.5\%$ the bi-linear response in cycle 1 noted above for the case where $\varepsilon_a^{max} = 1\%$ is again evident, while at the lower cyclic strain amplitude of $\varepsilon_a^{max} = 0.1\%$ the decrease in stiffness is smoother and the shape of the response in subsequent cycles is similar to the response observed in the first cycle. The peak deviatoric stress decreased as the maximum strain amplitude decreased, as would be expected.

In the DEM simulations, the variation of a number of parameters during cycling was monitored by considering the situation at the start of each cycle ($e_a = 0\%$), mid-way through each cycle ($e_a = e_a^{max}$), at the mid-point of the loading phase and the mid-point of the unloading phase. The resulting data is plotted as a function of the cycle number, $n$, (starting from $n = 1$) using the following convention; for a given cycle, $n$, the $e_a = 0\%$ data are plotted at point $n-1$, the $e_a = 0.5\varepsilon_a^{max}$ (loading) data are plotted at point $n-0.75$, the $e_a = \varepsilon_a^{max}$ data for unloading are plotted at point $n-0.5$, and the $e_a = 0.5\varepsilon_a^{max}$ (unloading) data are plotted at point $n-0.25$. As illustrated in Fig. 6 the deviator stresses at axial strain values of $e_a = \varepsilon_a^{max}$, $e_a = 0.5\varepsilon_a^{max}$, and $e_a = 0$ all tended to decrease as the cyclic loading con-

![Fig. 5. Sensitivity of the macro-scale specimen response of uniform sphere specimens in DEM simulations to the cyclic strain amplitude (considering cycles 1, 2, 10, and 50)](image-url)
continued. In the simulations with $e_{\text{max}}^{a}=1\%$ and $e_{\text{max}}^{a}=0.5\%$, there appears to be a consistent gradual decrease in the deviator stress at $e_{a}=e_{\text{max}}^{a}$. In contrast for these simulations when $e_{a}=0$, and for $e_{a}=0.5e_{\text{max}}^{a}$ (both loading and unloading) there was a notable decrease in the deviator stress over the first 3 cycles, followed by a more gradual decrease as cyclic loading progressed. For the simulation with $e_{a}=0.1e_{\text{max}}^{a}$ there was a significant decrease in the deviator stress at $e_{a}=e_{\text{max}}^{a}$ over the initial 10 cycles, in comparison with a more gradual decrease observed in the subsequent 40 cycles. For this simulation, as for the other two simulations, the decrease in deviator stress at $e_{a}=0.5e_{\text{max}}^{a}$ was more noticeable in comparison with the stress decrease at $e_{a}=e_{\text{max}}^{a}$. Referring to Fig. 4, this decrease in the deviatoric stress values during loading results in a reduction in the area of the hysteresis loop and reduction in the amount of energy dissipated in each cycle as loading continues.

Figure 7 illustrates the variation in volumetric strain during cycling at $e_{a}=e_{\text{max}}^{a}$ and $e_{a}=0$. At all cyclic strain amplitudes there is a tendency for the volumetric strain to increase slightly at $e_{a}=0$, with the increase in volume being greatest for the case of $e_{\text{max}}^{a}=1\%$. Considering the volumetric strains at $e_{a}=e_{\text{max}}^{a}$, when the strain amplitude was $1\%$ a slight increase in volume was observed, while a slight decrease in volume was observed for the other two simulations. Overall the most significant volumetric strains were observed over the first cycle of loading, however the straining continued throughout the simulations and did not appear to have ceased after 50 cycles.

The variation in normalized secant stiffness ($E_{\text{sec}}/\sigma_{t}$) during the 1st, 2nd and 10th cycles for each simulation considered is illustrated in Fig. 8, while the variation of $E_{\text{sec}}/\sigma_{t}$ at $e_{a}=0$, $e_{a}=0.5e_{\text{max}}^{a}$, and $e_{a}=e_{\text{max}}^{a}$ as a function of the number of load cycles is illustrated in Fig. 9.

Considering the data presented in Fig. 8, there is no noticeable differences in the variation in stiffness as a function of strain during subsequent cycles (i.e., the response observed in the 10th cycle is indistinguishable from the response observed in the 50th cycle). Referring to Fig. 9, for all the simulations there is a significant increase in $E_{\text{sec}}/\sigma_{t}$ over the first cycle, this can be explained by reference to Fig. 5. The $E_{\text{sec}}$ values for each cycle are measured relative to the stress conditions at $e_{a}=0$ for that cycle. Referring to Fig. 5 there is a significant decrease in the deviator stress at $e_{a}=0$ over the first cycle, while the deviator stress at $e_{a}=e_{\text{max}}^{a}$ does not change significantly. Following the initial increase, the $E_{\text{sec}}/\sigma_{t}$ values at $e_{a}=$.
$e_a^{\max}$ and $e_a = 0.5e_a^{\max}$ (unloading) do not noticeably change as cyclic loading continues. However for all three simulations, following the initial increase, there is a decrease in the $E_{sec}/\sigma_3$ values at $e_a = 0.5e_a^{\max}$ (loading) over the first ten cycles. This decrease in stiffness corresponds with a decrease in the deviatoric stresses at $e_a = 0.5e_a^{\max}$ (loading) (Fig. 6).

The fluctuations observed in the response at low levels of straining (Fig. 8, $e_a^{\max} = 0.1\%$) are a consequence of the elastic nature of the contact springs used in the rheological model used to represent contact in the DEM model. These changes in stiffness correspond to the small fluctuations observed in the stress–strain plots (Fig. 5). These small fluctuations will have had little influence on the overall response, we chose not to remove them by filtering. Such a response would not be observed in a physical granular material and highlights the need to implement a dissipative contact model to study granular material response at very small strains.

**EVOLUTION OF MICRO-SCALE PARAMETERS DURING CYCLIC LOADING**

Using the DEM simulation results, the particle scale interactions during cyclic loading can be analysed. The analyses presented here considered the evolution of the contact force network, the fabric, and the coordination number.

**Evolution of Contact Force Network**

Diagrams of the contact force network and plots indicating the magnitudes and orientations of contact forces at the middle and the end of the 50th cycle are illustrated in Fig. 10 for the simulations with $e_a^{\max} = 1\%$ and $e_a^{\max} = 0.1\%$. For these diagrams, only the strong contact forces, i.e., those contact force exceeding the average contact force plus one standard deviation are considered. For all three simulations these strong contact forces make up 15% of the total number of contacts (and this proportion does not vary significantly during the simulations). In Fig. 10 lines are drawn between the centres of contacting spheres, and the line thickness is proportional to the magnitude of the force, the entire specimen volume is considered here. For the case where $e_a^{\max} = 1\%$, most of the large contact forces were oriented in the vertical direction at the maximum axial strain value (Fig. 10(a)), while the largest contact forces were oriented in the horizontal direction at axial strain of 0% (Fig. 10(c)). At $e_a^{\max} = 0.1\%$, while the contact forces are also clearly aligning themselves with the orientation of the maximum prin-
Fig. 10. Contact force network at maximum and minimum strain levels, 50th cycle, uniform specimen: \( \epsilon_a = \epsilon_{a,\text{max}} = 1\% \) and \( \epsilon_a = \epsilon_{a,\text{max}} = 0.1\% \) (considering only forces > average force + 1 std. dev.)

Principal stress (Fig. 10(b)), for this smaller strain amplitude the anisotropy in the contact force network is less marked at \( \epsilon_a = 0\% \) (Fig. 10(d)).

Plots of the contact force network such as those provided in Fig. 10 can give only a qualitative assessment of the arrangement of the network of contacts transmitting stress through the material given the complexity of the network and its three-dimensional geometry. A quantitative assessment of the contact forces may be made by reference to the polar histogram plots provided as Figs. 11 to 14. All non-zero contact forces were considered in the development of these plots. As the specimen is axi-symmetric we need consider only one quadrant of the system to plot the histograms. As would be expected for this axi-symmetric system, the distribution of contact forces orientations in the horizontal plane is approximately uniform, therefore only the vertical projections are considered here. Each 10° bin in the histogram has been shaded and the degree of shading indicates the average contact force magnitude in that bin, normalized by the overall average contact force for the strain level considered. Therefore these plots give an indication of both the orientation of the contact forces in the system, as well as the relative magnitudes of the forces transmitted in each direction.

Figures 11 and 12 consider all the contacts in the specimen for the simulations with \( \epsilon_a = \epsilon_{a,\text{max}} = 0.1\% \) and \( \epsilon_a = \epsilon_{a,\text{max}} = 1\% \) respectively. In both figures the distribution of the contact forces at the beginning and end of the first and last load cycles simulated is considered. Comparing firstly the distribution of contact force orientations, initially, as a consequence of the specimen generation approach there are more contact normals orientated vertically (i.e., with an inclination to the horizontal exceeding 45°) than horizontally. This anisotropy in the contact orientation is more pronounced when \( \epsilon_a = \epsilon_{a,\text{max}} \) in both simulations, and there is a slight difference in the distribution with the simulation where \( \epsilon_a = \epsilon_{a,\text{max}} = 0.1\% \) having more horizontally orientated contacts in comparison with the simulation where \( \epsilon_a = \epsilon_{a,\text{max}} = 1\% \). A more notable difference between the two simulations is the distribution in the magnitude of contact forces as a function of contact orientation. In the simulation with \( \epsilon_a = \epsilon_{a,\text{max}} = 0.1\% \), the forces orientated in the vertical direction (i.e., \( > 80^\circ \) to the horizontal) tend to be about 1.15 times the average force, however where \( \epsilon_a = \epsilon_{a,\text{max}} = 1\% \), the forces tend to be closer to 1.2 times the average force and the normalized horizontal force magnitudes are smaller. Looking at the difference between the contact force distributions at the beginning and end of the last load cycle it is clear that as the orientation of the major principal stress rotates, and the deviator stress moves from a negative to a positive value, (refer to Figs. 4 and
5), there is a significant difference in the contact force orientations as well as the relative magnitudes of the forces transmitted in the horizontal and vertical directions. Considering Figs. 11(b) and 12(b) we can see that when the orientation of the major principal stress is horizontal (negative deviator stress), while the distribution of contact orientations is almost uniform there are still, on average, more forces orientated vertically. However the horizontally orientated contacts transmit more force than the vertically orientated contacts. When the major principal stress is orientated vertically the distribution of forces is considerably more anisotropic, with about 70% of the contacts having an orientation exceeding 50° to the horizontal in both cases.

These shaded contour plots can also be used to look at the inhomogeneities in the contact network structure the within the specimen. Three zones are considered, Zone 1 considers all the contacts with elevations less than $H/6$ (where $H$ is the total specimen height), Zone 2 considers contacts with elevations between $H/6$ and $H/3$, and Zone 3 considers contacts with elevations between $H/3$ and $H/2$. Shaded histograms illustrating the distribution of the magnitude and orientation of the contact forces are given in Figs. 13 and 14. There a greater number of horizontally orientated contacts in the section close to the boundary in comparison with the central portions of the specimen. Furthermore, at $\varepsilon_a = \varepsilon_{\text{max}}^a$, the trend for the vertically orientated contacts to transmit the largest forces is more marked close to the centre of the specimen (Zone 3).

**Fabric Tensor Analysis**

For spherical particles, the fabric tensor is given by

$$F_{ij} = \frac{1}{2N_c} \sum_{k=1}^{N_c} n_i^{(k)} n_j^{(k)}$$

(1)

where $N_c$ is the number of contacts, $n_i$ is the component of the unit branch vector in the $i$ direction, and the branch vector is the vector joining the centroids of the two contacting particles. The principal values, $F_1$, $F_2$ and $F_3$, and the principal directions of the fabric tensor can be calculated by considering the eigenvalues and eigenvectors of the fabric tensor. The deviator fabric ($F_1-F_3$) quantifies the anisotropy of the microstructure (see also Thornton, 2000; Cui and O’Sullivan, 2006). For the three simulations considered the fabric tensor was calculated firstly by considering all the contacts in the specimen and then by considering only the strong contacts (i.e., contacts where the contact force exceeded the average force + 1 standard deviation). In both cases the deviator fabric and the orientation of the principal fabric to the vertical ($\beta$) was considered.

The evolution of the overall anisotropy (i.e., $\Phi_1-\Phi_3$ considering all the contacts) for the duration of each of the three simulations considered is illustrated in Fig. 15. A comparison of Figs. 15 and 6 reveals a clear link between the variation in deviator stress and the evolution of
fabric anisotropy. Considering firstly the deviator fabric at 0% strain, initially $\Phi_1-\Phi_2$ is almost zero, indicating an isotropic fabric, corresponding with the initial isotropic stress condition. Comparing the simulations with $\varepsilon_a^{\max} = 1\%$ and $\varepsilon_a^{\max} = 0.5\%$, there is a more noticeable decrease in deviator stress at $\varepsilon_a = \varepsilon_a^{\max}$ for the $\varepsilon_a^{\max} = 0.5\%$ simulation (Fig. 6) and this corresponds with a more noticeable decrease in deviator fabric (Fig. 15). Considering the response at $\varepsilon_a = 0\%$, for all three simulations a significant decrease in deviator stress over the initial cycles of loading (i.e., magnitude increased) is observed to correspond with a significant increase in anisotropy. Note also that as cyclic loading continued the deviator fabric at $\varepsilon_a = 0\%$ exceeded the deviator fabric at $\varepsilon_a = \varepsilon_a^{\max}$ for all three simulations, even though the deviator stress at $\varepsilon_a = \varepsilon_a^{\max}$ exceeded the deviator stress at $\varepsilon_a = 0\%$. (This is explored further below). The variation in fabric with increased cyclic loading appears more marked than the variation in the deviator stress. This suggests that the fabric does not depend only upon the deviator stress but also on the previous loading of the specimen. The deviator fabric is clearly continuing to evolve as cycling progresses and has not reached a stable state after 50 load cycles, even when $\varepsilon_a^{\max} = 0.1\%$.

Recognizing that there is significant heterogeneity in the contact force network, and considering the strong force chains that are typically observed in two dimensional DEM analyses, the fabric tensor was re-evaluated, considering only the contacts transmitting the largest contact forces (i.e., forces exceeding the average contact force plus one standard deviation). The deviator fabric in this case, i.e., the “strong force” deviator fabric, is illustrated in Fig. 16. Note that following the definition of mechanical coordination number proposed by Thornton (2000), the fabric tensor was also calculated considering only contacts where each sphere meeting at that contact point had two or more contacts, however the deviator fabric did not noticeably change for any of the simulations, in comparison with the values presented in Fig. 15. However, the strong force deviator fabric, illustrated in Fig. 16, clearly correlates more strongly with the stress data in Fig. 5, in comparison with the overall deviator fabric illustrated in Fig. 15. The deviator fabric at $\varepsilon_a = 0\%$ never exceeds the strong force deviator fabric at $\varepsilon_a = \varepsilon_a^{\max}$, and the variation in strong force deviator fabric with increased cyclic loading more closely resembles the variation in the deviator stress with increased cyclic loading.

The relationship between the two calculated fabric tensors and the stresses can be better appreciated when the extent of the contribution of the strong contact forces to the stress transmission within the specimen is quantified. Figure 17 is a plot of the ratio of the principal stress difference calculated considering all the strong contacts in the specimen to the overall principal stress difference (as considered in Fig. 13). It can clearly be seen that while only 15% of the contacts can be classified as “strong”
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(i.e., exceeding the average contact force plus one standard deviation), these contacts transmit the majority of the deviator stress. The stress transmitted by the strong contacts is about 80% of the total stress at $\varepsilon_a = \varepsilon_{a,\max}$, and a smaller proportion (about 60%) is transmitted at $\varepsilon_a = 0\%$. The proportion of stress carried by the strong contacts does not vary significantly during the simulations.

The eigenvectors of the fabric tensor will give information regarding the orientation of the principal fabric. Figures 18 and 19 illustrate the orientation of the major principal fabric to the positive z (vertical) axis during the simulation ($\beta$), considering both the overall fabric and the strong force fabric. At $\varepsilon_a = 0\%$ the major fabric orientation is close to the horizontal, and the principal strong-force fabric is more consistently horizontal in comparison with the overall principal fabric, especially where $\varepsilon_a = \varepsilon_{a,\max} = 0.1\%$. At $\varepsilon_a = \varepsilon_{a,\max}$, the difference in the orientations of the principal fabrics is more marked. In this case the strong force principal fabric is almost vertical, with $\beta$ being small, while the overall principal fabric tends deviate more from the vertical. Considering the case where $\varepsilon_{a,\max} = 0.1\%$, the orientation of the overall major fabric deviates significantly from the vertical, suggesting an isotropic fabric, while the strong force fabric is clearly

Fig. 13. Polar histograms of contact force orientation in Zones 1, 2, and 3 for cycle 50; $\varepsilon_{a,\max} = 0.1\%$, shading illustrates normalized contact force magnitudes
vertically orientated. There was less variation in the strong force fabric with increased cycling in comparison with the overall fabric (considering both the deviator fabric and the principal fabric orientation). This reflects the distribution of the stresses; there are a large number of contacts transmitting relatively small forces and their orientation is random in comparison with the strong contacts, whose orientation coincides closely with the macroscale applied stress state.

The simulations of the physical tests can be used to compare the fabric for the uniform and non-uniform specimens. As illustrated in Fig. 20, the trends observed for the uniform specimens also are apparent in the non-uniform specimen over the first 16 load cycles.

**Coordination Number Evolution**

The evolution of the coordination number \( N \) during the three simulations is illustrated in Figs. 21 and 22. The coordination number was calculated as

\[
N = 2N_c/N_p
\]

where \( N_c \) is the number of contacts and \( N_p \) is the number of particles. Figure 21 illustrates the variation in \( N \) as a function of axial strain for selected loading cycles, while
Fig. 22 illustrates the variation of $N$ as a function of the number of cycles at selected strain levels. Referring to Fig. 21, for each simulation the $N$ value decreased significantly in the first cycle, this is consistent with the significant decrease in coordination number at small strains observed in the monotonic triaxial simulations of Cui et al. (2007). The magnitude of the reduction in the coordination number decreased with decreasing values of $\varepsilon_{a}^{\text{max}}$. The variations in coordination number in subsequent cycles were noticeably smaller. The coordination number data clearly then indicates that the biggest change in the specimen fabric took place in the first cycle, correlating with the significant changes in the macro scale response in the first cycle (Figs. 6 and 9).

It is interesting to observe that the maximum coordination number occurred when $\varepsilon_{a}$ was close to $0.5\varepsilon_{a}^{\text{max}}$ during
Fig. 18. Orientation of principal fabric at 0% axial strain

Fig. 19. Orientation of principal fabric at maximum axial strain

Fig. 20. Comparison of specimen anisotropy for uniform and non-uniform specimens over 16 cycles of loading
Fig. 21 Variation in coordination number with strain for various amplitudes of cyclic loading (cycles 1, 2, 10, and 50)

Fig. 22 Evolution of coordination number with number of load cycles

the loading stage, and not at the maximum $e_a$ value. Referring to Fig. 22, after the initial load cycles (approximately 5) the coordination numbers tended to increase as cyclic loading continued. Values of $N$ can be related to the observed macro-scale response, the significant decrease in $N$ at $e_a = 0\%$ in the first cycles of loading corresponded with a significant decrease in deviator stress. As noted above the area of the hysteresis loops decreases slightly as cyclic loading continues, energy will be dissipated in friction and also as particles loose contact. Comparing the variation in coordination number with axial strain for cycles 2 and 50 for all three simulations, there is no apparent reduction in the number of contacts that are broken and reformed in a given load cycle as the number of cycles of loading increases.

In comparison with the macro-scale stress measurements, a clear correlation between the secant stiffness (Fig. 9) and the micro scale parameters of $\Phi_1 - \Phi_3$ and $N$, is less evident. For all three simulations, the secant stiffness at $e_a = e_{ax}^{max}$ does not vary significantly after the initial loading cycle, (reflecting the fact that the shape of the hysteresis loop does not evolve significantly), even though there are noticeable variations in $\Phi_1 - \Phi_3$ and $N$.

For the three simulations described here, the computational cost of the simulations restricted the number of load cycles to 50. Recognizing that, even after 50 cycles both the macro-scale response and the specimen fabric are continuing to evolve, a single simulation with 200 cycles of loading and with $e_{ax}^{max} = 0.5\%$ was completed. The results of this simulation, as illustrated in Fig. 23, indicate that the fabric continues to evolve as cyclic loading continues. Comparing Figs. 23 and 22, the definite increase in coordination number that can be observed in Fig. 23(d) as cycling continues is not evident if only 50 cycles are considered.

CONCLUSION

This paper has described two series of DEM simulations of cyclic triaxial tests that used a mixed boundary test simulation environment, including both cylindrical periodic boundaries and a three-dimensional stress controlled membrane. The first series of simulations were carried out to validate the DEM model by comparing the macro-scale simulation results with the data from physical tests on equivalent specimens. The physical tests were carried out under a vacuum confinement of 80 kPa, and the amplitude of cyclic loading was 1%. The granular
material considered was made up of Grade 25 steel spheres so that the particle geometry could be accurately replicated in the physical tests. A good agreement was attained between the physical test data and the DEM simulation results, with the stress ratios at axial strains of 1% being similar. The numerical model also captured the trend of variation in stiffness with strain. Given the success with which the numerical model could capture the response observed in the physical tests, the authors are confident that the numerical model can be used to examine the micro-scale response of this granular material under quasi-static cyclic loading conditions. Extension of this research to include, in the first instance, stress-controlled (rather than strain controlled) cyclic loading and consideration of non-spherical particles will yield valuable insight into the fundamental mechanics of cyclic soil response.

The parametric study described here explored the sensitivity of both the macro- and micro-scale responses to the amplitude of cyclic loading. Three simulations, each with 50 cycles of loading, with cyclic strain amplitudes of 1%, 0.5%, and 0.1% were analysed. The principal conclusions of this parametric study are as follows:

1. There is a decrease in the deviator stress as calculated using macro-scale parameters as cyclic loading progressed for all simulations, and this decrease is most noticeable over the initial load cycles.
2. Following an initial increase in the calculated secant stiffness values, the secant stiffness at the maximum strain values remained approximately constant, while there was an apparent decrease in the secant stiffness at $e_a = 0.5 e_{a max}$.
3. The distribution of contact force orientations and magnitudes reflects the macro-scale applied stress state. There is a clear redistribution in the magnitude of contact force during cyclic loading, with the
contacts carrying the largest forces tending to be orientated in the direction of the major principal stress. The variation in the number of contacts in orientated in the major principal stress is direction is less marked. A comparison of three horizontal slices through the specimen indicates that there is a variation in the contact force network within the specimen as a consequence of boundary effects.

4. Comparing the principal stress differences and the deviator fabric, the stress measurements correlated better with the strong fabric tensor (i.e., the fabric tensor calculated using only the contacts transmitting the largest contact forces). At $e_a = e_a^{\text{max}}$ most of the stress is transmitted through the system by only 15% of the total number of contacts. The strong fabric tensor varied less with increased cyclic loading, in comparison with the overall fabric tensor.

5. The coordination number at $e_a = e_a^{\text{max}}$ tends to increase slightly as cyclic loading continues, the variation in coordination number at $e_a = 0$ is less obvious. While a stiffer material response tends to coincide with a higher coordination number, the deviator fabric relates more strongly with the stress strain response observed at the macro scale, than does the coordination number.

6. The parametric study has illustrated that even under relatively small amplitude cyclic loading ($e_a^{\text{max}} = 0.1\%$), the specimen fabric continues to evolve as cyclic loading continues, a steady state is not achieved after 50 cycles of loading. A single simulation with $e_a^{\text{max}} = 0.5\%$ indicated that a steady state is not reached even after 200 cycles.

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