EFFECTS OF PARTICLE CHARACTERISTICS ON THE VISCOUS PROPERTIES OF GRANULAR MATERIALS IN SHEAR

TADAO ENOMOTOi, SHOHEI KAWABEii, FUMIO TATSUOKAIii, HERVE DI BENEDETTOiv) and ANTOINE DUTTINEvi)

ABSTRACT

The viscous properties of a wide variety of unbound granular materials (GMs) were evaluated by drained shear tests. The specimens were reconstituted ones that were loose or dense and air-dried or moist or saturated, of mostly natural sands and gravels, having different mean particle diameters, uniformity coefficients, fines contents, degrees of particle angularity and particle crushabilities. The tests were mostly triaxial compression (TC) tests and partly plane strain compression tests, both at fixed confining pressure, and direct shear tests at fixed normal pressure. The viscous properties of GMs were evaluated by stepwise changing the loading rate and performing sustained loading (SL) tests during otherwise monotonic loading (ML) at a constant loading rate. The viscous properties are characterised in terms of the rate-sensitivity coefficient (β), the viscosity type parameter (θ = βr/b) and the decay parameter (r). Correlations among these parameters and effects of particle characteristics on these parameters are analysed. Creep strains are compared with residual strains by cyclic loading under otherwise the same TC conditions. As the particles become less angular, as the grading becomes more uniform and as the particles become less crushable, the viscosity type deviates more from the Isotach type (i.e., θ = 1.0) changing toward the P & N type (i.e., θ < 0) associated with a decrease in β and r, while creep strains by SL decreases and residual strains by many unload/reload cycles increases. It is shown that the loading rate effects observed in the experiments can be simulated well by the three-component model taking into account the effects of particle characteristics on the viscous property parameters.

Key words: creep deformation, cyclic loading, granular material, particle properties, triaxial compression, viscous properties (IGC: D6/D7)

INTRODUCTION

Unbound granular materials (unbound GMs) (e.g., unbound sands and gravels) may exhibit significant rate-dependent behaviour (including significant creep deformation) in field full-scale cases (e.g., Tatsuoka et al., 2000, 2001; Jardine et al., 2005, 2006; Oldecop and Alonso, 2007). A number of laboratory stress-strain tests showed that unbound GMs have significant viscous properties and their trend is similar to that of saturated clays and bound materials (i.e., sedimentary soft rocks and cement-mixed soils) (e.g., Yamamuro and Lade, 1993; Matsushita et al., 1999; Hayano et al., 2001; Di Benedetto et al., 2002; Tatsuoka et al., 1999, 2002, 2004, 2006a, 2008a, b; Tatsuoka, 2007; Komoto et al., 2003; Nawir et al., 2003; Aqil et al., 2005; Kongsukprasert and Tatsuoka, 2005; Anh Dan et al., 2006; Kiyota and Tatsuoka, 2006; Enomoto et al., 2007a, b; Sorensen et al., 2007; Duttine et al., 2008a; Kongkitkul et al., 2008; among many others). The major findings obtained by these previous studies can be summarised as follows.

The viscosity type of geomaterial is not unique but different depending on the geomaterial type. Classical elasto-plastic models cannot describe these different viscosity types of geomaterial. A number of different elasto-viscoplastic models for geomaterials have been proposed: e.g., the overstress model (Perzyna, 1963). As a more general and flexible elasto-viscoplastic model framework that can define and classify these different viscosity types, the non-linear three-component model (Fig. 1) is employed in the present study. According to this model, the stress (i.e., the measured effective stress), σ, is obtained by adding the viscous component, σv, to the inviscid (or rate-independent) component, σi, at the same εv value. The strain rate, ē, is obtained by adding the irreversible (or inelastic or visco-plastic) component, ēir, to the hypo-elastic component, ēe. A combination of E and P bodies in series represents the classical elasto-plastic models.

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Four basic viscosity types illustrated in Fig. 2 were found by recent laboratory shear tests of a number of different geomaterial types. The Isotach type is the most classical one. In continuous monotonic loading (ML), the increment of $\sigma^\prime$ ($\Delta \sigma^\prime$) that develops at any moment by $\Delta \varepsilon^\prime$ or $\Delta \varepsilon^\prime_{ir}$ or both does not decay with an increase in $\varepsilon^\prime$ during subsequent loading. So, the current $\sigma^\prime$ is a unique function of instantaneous $\varepsilon^\prime$ and its rate, $\dot{\varepsilon}^\prime$, and the strength during ML at a constant $\dot{\varepsilon}$ increases with $\dot{\varepsilon}$. With the other three types, $\Delta \sigma^\prime$ decays with $\varepsilon^\prime$ towards different residual values. With the TESRA type, $\Delta \sigma^\prime$ decays eventually toward zero, so the strength during ML at a constant $\dot{\varepsilon}$ is essentially independent of $\dot{\varepsilon}$. ‘TESRA’ stands for ‘temporary effects of strain rate and strain acceleration.’ The Combined type is a combination of the Isotach and TESRA types. With the Positive and Negative (P\&N) type, a positive $\Delta \sigma^\prime$ value decays towards a negative value, so the strength during ML at a constant $\dot{\varepsilon}$ decreases with an increase in $\dot{\varepsilon}$. Table 1 summarises the effects of geomaterial type and strain level on the viscosity type. The arrows indicate that the viscosity type changes with $\varepsilon^\prime_{ir}$ in a single ML test. Moreover, for the description by the model (Fig. 1) of the stress-strain behaviour of unbound geomaterials (including GMs) in drained triaxial compression (TC), triaxial extension (TE) and plane strain compression (PSC) tests, the effective principal stress ratio, $R = \sigma_1/\sigma_3$, and the shear strain, $\gamma = \varepsilon_1 - \varepsilon_3$, are the relevant stress and strain parameters ($\sigma$ and $\varepsilon$). Then, by referring to Fig. 3, the viscous properties of unbound geomaterials can be characterised firstly as follows:

$$\frac{\Delta R}{R} = \beta \cdot \log_{10} \left( \frac{\gamma_{after}^{ir}}{\gamma_{before}^{ir}} \right)$$

where $\beta$ is the rate-sensitivity coefficient; and $\Delta R$ is the jump in $R$ that takes place upon a step change in the irreversible shear strain rate ($\gamma^{ir} = \gamma^{after} - \gamma^{before}$) from $\gamma_{before}^{ir}$ to $\gamma_{after}^{ir}$ when $R = R$. In Fig. 3, $\gamma_{after}^{ir}/\gamma_{before}^{ir} = 10$, therefore, $\beta = \Delta R/R$. Furthermore, different viscosity types exhibit different values of the residual rate-sensitivity coefficient, $\beta_{residual}$, defined as:

$$\frac{\Delta R_{ir}}{R} = \beta_{residual} \cdot \log_{10} \left( \frac{\gamma_{after}^{ir}}{\gamma_{before}^{ir}} \right)$$

where $\Delta R_{ir}$ is the residual value of $\Delta R$ that has taken place upon a step change in $\gamma^{ir}$ from $\gamma_{before}^{ir}$ to $\gamma_{after}^{ir}$ and then has fully decayed during subsequent ML at a constant $\dot{\gamma}$; and $R_{ir}$ is the $R$ value (when $\Delta R_{ir}$ is defined) given along the stress-strain curve obtained if ML had continued at the constant $\dot{\gamma}$ kept to the original value (i.e., $\gamma_0$ in Fig. 3). In Fig. 3, $\gamma_{after}^{ir}/\gamma_{before}^{ir} = 10$, therefore, $\Delta R_{ir}/R = \beta_{residual}$. As both $\beta$ and $\beta_{residual}$ values can be different among different geomaterial types that exhibit the same viscosity type and they may change with strain in a single test (Tatsuoka et al., 2008a; Kongkitkul et al., 2008; Dutine et al., 2008a), it is relevant to define the current viscosity type by the viscosity type parameter, $\theta$, defined as:
\[ \theta = \frac{\beta_{\text{residual}}}{\beta} \]  

where \( \beta_{\text{residual}} \) and \( \beta \) are the instantaneous values measured at the same \( \gamma' \). Then, different viscosity types can be represented by different \( \theta \) values: i.e., Isotach (\( \theta = 1.0 \)); Combined (\( 0.0 < \theta < 1.0 \)); TESRA (\( \theta = 0.0 \)); and P\&N (\( \theta < 0.0 \)). Tatsuoka (2007) and Tatsuoka et al. (2008a) proposed a mathematical expression incorporating the viscosity type parameter, \( \theta \) that can describe these four viscosity types and transitions among them. Note that \( \theta \) is a function of \( \gamma' \).

Lastly, the \( \beta \) value of drained unbound GMs having mean particle diameters larger than certain values (i.e., sands and gravels) is basically independent of particle size (Tatsuoka et al., 2006b). With Toyoura sand in drained TC tests (Nawir et al., 2003) and drained PSC tests (Kongkitkul et al., 2007), the effects of wet conditions (air-dried or saturated), confining pressure and dry density on the \( \beta \) value are insignificant, if any. The \( \beta \) value of Toyoura sand is essentially the same in drained TC, TE and PSC tests, as well as drained DS tests (Duttine et al., 2008a), and the effects of over-consolidation are insignificant (Kiyota and Tatsuoka, 2006). Kawabe et al. (2008) also showed that the effects of confining pressure and over-consolidation on the viscous property of drained saturated Albany sand, which exhibits the P\&N viscosity in drained TC, are negligible.

Despite these many findings described above, the effects of particle characteristics (i.e., grading, particle shape, particle crushability and so on) on the viscous properties of a wide variety of unbound GMs as encountered in the field are still poorly understood. In particular, most of the GMs used in the previous studies are poorly graded having relatively stiff (i.e., less crushable) particles. Furthermore, similarities or differences between the effects of particle characteristics on the residual strains developed by cyclic loading (CL) with unbound GMs and those on the creep strains are poorly understood. In view of the above, by using much wider GM types than in the previous studies, a series of drained TC tests were performed to evaluate the effects of these particle characteristics on the viscous properties of unbound GMs. The results from these TC tests were analysed referring to those from drained PSC and DS tests performed in the previous and present studies. Moreover, a number of cyclic triaxial tests were performed on air-dried specimens for these research objectives. The ageing effect is beyond the scope of this study. The rate-dependency under undrained conditions is also beyond the scope of the present study.

**TEST METHOD AND MATERIALS**

**Test Materials**

Figure 4 shows the particle pictures of GMs representative of those tested in the present study. Figure 5 shows the grading curves of these GMs and others referred to in this paper. The values of mean diameter (\( D_{50} \)), uniformity

![Fig. 4. Some representative granular materials tested in the present study (*: one unit length = 0.5 mm)](image)
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Fig. 5. Grading curves of granular materials referred to in this paper

![Grading curves of granular materials](image)

Names of granular materials ($D_{50}$, $U_c$ & FC)

- Toyoura sand* ($0.188$ mm; 1.63; 0%)
- Hostun sand* ($0.322$ mm; 2.01; 0%)
- Silica No.3 sand* ($1.511$ mm; 1.69; 0%)
- Silica No.4 sand* ($1.400$ mm; 1.66; 0%)
- Silica No.5 sand* ($0.555$ mm; 2.24; 1.8%)
- Silica No.6 sand* ($0.299$ mm; 2.43; 3.1%)
- Silica No.8 sand* ($0.099$ mm; 2.24; 23.4%)
- Mixed silica sand* ($0.811$ mm; 13.1; 7.6%)
- Coral sand A* ($0.171$ mm; 2.07; 3.7%)
- Coral sand B* ($0.377$ mm; 2.15; 0.8%)
- S.L.B. sand* ($0.688$ mm; 1.43; 0%)
- Ticino sand* ($0.533$ mm; 1.52; 0%)
- Nanta sand* ($0.171$ mm; 16.6; 19.1%)
- Fujinomori clay* ($0.017$ mm; 10; 98.1%)
- Model Chiba gravel H ($2.00$ mm; 2.28; 0%)

- Albany silica sand* ($0.303$ mm; 2.22; 0.1%)
- Inagi sand* ($0.188$ mm; 20.8; 17.2%)
- Ishihama beach sand* ($0.344$ mm; 2.12; 0.1%)
- Omigawa sand* ($0.177$ mm; 4.62; 13.1%)
- Hime gravel* ($1.54; 3.55; 0%$
- Shinanogawa Riverbed gravel* ($11.25; 73.7; 4.2%$
- Corundum A* ($1.4m; 1.62; 0%$
- MACH* ($1.099; 6.34; 0.1%$
- Jamuna river sand ($0.169; 1.99; 7%$
- Original Chiba gravel ($7.9m; 11.2; 0%$
- Ottawa sand* ($0.174m; 1.76; 0%$
- Crushed concrete aggregate ($5.5; 6.5mm; 18.8; 1.2-2.1%$
- Model Chiba gravel A ($0.5m; 2.1; 0%$
- Glass beads A* ($0.44m; 1.21; 0%$
- Glass beads B* ($0.22m; 1.19; 0%$
- Glass beads C* ($0.11m; 1.09; 0%$
- Monterey sand* ($0.464m; 14; 0%$

*: tested in the present study

Coefficient ($U_c$) and fines content (FC) are listed next to the respective material names in the bottom of Fig. 5 and these values together with the values of specific gravity ($G_s$), maximum and minimum void ratios ($e_{max}$ and $e_{min}$) and the $\beta$ values of these GMs, as well as test methods, are listed in Table A1 of APPENDIX A1. The hydraulic conductivities of these GMs are not listed, as they have no direct link to the rate-dependent behaviour discussed in this paper. This is because it is certain that the drained tests performed on saturated specimens referred to in this paper were essentially under fully drained conditions.

Among those tested, Silica sands Nos. 3, 4, 5, 6 and 8 are poorly graded consisting of relatively angular and stiff particles having different $D_{50}$ values. Mixed silica sand was produced by mixing these silica sands to have a larger $U_c$ value. Coral sand A is a fine poorly graded sand consisting of relatively angular and stiff particles. Corundum A (granular aluminium oxide), Hime gravel and Albany silica, Ottawa, Ticino, Monterey and SLB (Silver Leighton Buzzard) sands are poorly graded consisting of relatively round and stiff particles. MACH is a well-graded GM with $U_c=6.3$ produced by mixing Albany sand, Corundum A and Hime gravel. Corundum B is a very fine round powder of granular aluminium oxide. Glass beads A, B and C consist of spherical stiff particles, having different $D_{50}$ values. Ishihama Beach sand is poorly graded consisting of sub-angular and stiff particles. Shinanogawa riverbed gravel is rather well-graded consisting of round gravel particles and relatively angular sand particles (Fig. 4). Model Chiba gravel H ($D_{50}=2.0$ mm and $U_c=2.28$) is relatively poorly-graded consisting of relatively angular and stiff sandstone particles, produced by removing particles larger than 4 mm from original Chiba gravel. Air-dried specimens were used in stress-controlled TC tests ($\sigma_1=40$ kPa) to evaluate residual deformation due to sustained and cyclic loading.

In comparison with those described above, coral sand B is poorly graded including relatively crushable shell fractions, while Tanno, Inagi and Omigawa sands are inland weathered sands having relatively angular and relatively crushable particles, as noted by an increase in the fines content by compaction tests (Kawaharazono, 2007; Seida et al., 2008). Study on the quantification of particle crushability is underway and the results will be reported in the near future.

Only saturated loose specimens were produced with Ishihama sand. Only saturated dense specimens were produced with Omigawa, Narita, Tan-no and mixed silica sands. Only dense air-dried specimens were produced with Shinanogawa riverbed gravel (a large specimen) and
The specimens were axially compressed in an automated way using a high precision gear-type axial loading system at constant cell pressure controlled by using an E/P transducer. Isotropic compression was performed at an axial strain rate of 0.00625/min towards an effective mean stress \( p' = (\sigma' + 2\sigma_0)/3 \approx 400 \text{ kPa} \), where \( \sigma' \) and \( \sigma_0 \) are the effective vertical and horizontal principal stresses. At \( p' = 50, 100, 200 \) and 300 kPa in the course of isotropic compression, eight cycles with an axial strain (double amplitude) of the order of 0.002 were applied to evaluate the elastic property (Fig. 7). The quasi-elastic vertical local deformation transducer arranged outside the triaxial cell are reported in this paper unless otherwise noted. Elastic properties were evaluated based on axial strains locally measured along the specimen’s lateral side with a pair of local deformation transducer (LDT; Goto et al., 1991). Although the effects of bedding error on the externally measured axial strains generally cannot be ignored, their effects on the parameters describing the viscous properties are negligible (Enomoto et al., 2007a; Tatsuoka et al., 2008a; as also shown below). The volume change of saturated specimens was obtained by measuring the water height in a burette connected to a specimen by using a low-capacity differential pressure transducer. The volume change of air-dried and moist specimens in strain-controlled TC tests was estimated by substituting respective measured values of \( R (= \sigma'/\sigma) \) and axial strain increments into the modified Rowe’s stress-dilatancy relation calibrated by the corresponding drained TC tests on saturated specimens (see APPENDIX B of Tatsuoka et al., 2008a).

The specimens were saturated with saturated specimens of Inagi and Narita sands, which are well-graded relatively angular inland sands obtained from Pleistocene Era deposits having essentially the same grading while including slightly weathered therefore crushable particles (more with Inagi sand) (Kawaharazono, 2007; Hirakawa et al., 2008); and air-dried specimens of Jamuna River sand, which is a poorly graded sand from Bangladesh, consisting of angular particles with some fraction of mica (Yasin et al., 2003).

### Triaxial Apparatuses and Test Procedures

Strain-controlled drained TC tests on small specimens (the present study): The major part of the experiments performed in the present study used an automated strain-controlled triaxial apparatus (e.g., Santucci de Magistris et al., 1999; Kyota and Tatsuoka, 2006). The top and bottom ends of saturated or air-dried specimens (7 cm in diameter and 15 – 15.5 cm in height) were well-lubricated by using a 0.3 mm-thick latex rubber smeared with a 0.05 mm-thick silicone grease layer (Tatsuoka et al., 1984). External axial strains that were obtained from axial displacements of the loading piston measured with a deformation transducer arranged outside the triaxial cell are reported in this paper unless otherwise noted. Elastic properties were evaluated based on axial strains locally measured along the specimen’s lateral side with a pair of local deformation transducer (LDT; Goto et al., 1991). Although the effects of bedding error on the externally measured axial strains generally cannot be ignored, their effects on the parameters describing the viscous properties are negligible (Enomoto et al., 2007a; Tatsuoka et al., 2008a; as also shown below). The volume change of saturated specimens was obtained by measuring the water height in a burette connected to a specimen by using a low-capacity differential pressure transducer. The volume change of air-dried and moist specimens in strain-controlled TC tests was estimated by substituting respective measured values of \( R (= \sigma'/\sigma) \) and axial strain increments into the modified Rowe’s stress-dilatancy relation calibrated by the corresponding drained TC tests on saturated specimens (see APPENDIX B of Tatsuoka et al., 2008a).

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### Table 2. Degrees of angularity of representative GMs

<table>
<thead>
<tr>
<th>Materials</th>
<th>( A^* )</th>
<th>Materials</th>
<th>( A^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chiba gravel</td>
<td>1967</td>
<td>Inagi sand</td>
<td>772</td>
</tr>
<tr>
<td>Ishihama beach sand</td>
<td>1705</td>
<td>Omigawa sand</td>
<td>768</td>
</tr>
<tr>
<td>Silica No. 3 sand</td>
<td>1512</td>
<td>Ticino sand</td>
<td>449</td>
</tr>
<tr>
<td>Silica No. 4 sand</td>
<td>1619</td>
<td>Ottawa sand</td>
<td>434</td>
</tr>
<tr>
<td>Silica No. 5 sand</td>
<td>1600</td>
<td>Monterey sand</td>
<td>297</td>
</tr>
<tr>
<td>Tanno sand</td>
<td>1469</td>
<td>Albany silica sand</td>
<td>209</td>
</tr>
<tr>
<td>Hostun sand</td>
<td>1435</td>
<td>S.L.B. sand</td>
<td>163</td>
</tr>
<tr>
<td>Coral sand B</td>
<td>1214</td>
<td>Himie gravel</td>
<td>139</td>
</tr>
<tr>
<td>Silica No. 6 sand</td>
<td>1070</td>
<td>Glass beads A, B &amp; C</td>
<td>0</td>
</tr>
<tr>
<td>Coral sand A</td>
<td>1013</td>
<td>Corundum A</td>
<td>0</td>
</tr>
<tr>
<td>Toyoura sand</td>
<td>896</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The specimens a 6 cm-diameter hammer (Li et al., 2004; Deng and compacting air-dried clay powder inside a split mould in 15 cm high) of Fujinomori clay and kaolin produced by formed on air-dried specimens (7.5 cm in diameter and 15 to the rigid flat faces of stainless steel properties (Aqil et al., 2005; Tatsuoka et al., 2006a); and b) compacted air-dried model Chiba gravel A (Hirakawa et al., 2008). The top and bottom ends of the specimens were in contact with the rigid flat faces of stainless steel top cap and pedestal. Several drained TC tests were performed on air-dried specimens (7.5 cm in diameter and 15 cm high) of Fujinomori clay and kaolin produced by compacting air-dried clay powder inside a split mould in ten layers with six blows of about 20 kgf per layer using a 6 cm-diameter hammer (Li et al., 2004; Deng and Tatsuoka, 2007; Tatsuoka et al., 2006a). The specimens were isotropically consolidated to \( \sigma'_{90} = 77 \) kPa and 80 kPa (Fujinomori) and 100 kPa (kaolin).

Strain-controlled drained TC tests on large specimens (the present study): A large specimen (30 cm in diameter and 60 cm in height) of Shinanogawa riverbed gravelly soil was produced by compacting air-dried material inside a split mould in six layers using an electric motor-driven vibrator to an initial dry density equal to 2.05 g/cm\(^3\). The specimen was isotropically compressed to \( \sigma'_{90} = 40 \) kPa. Anh Dan et al. (2004) performed drained TC tests on large moist specimens of the same dimensions as above of original Chiba gravel isotropically compressed to \( \sigma'_{90} = 490 \) kPa. In all these TC tests, the axial strains were obtained both externally from axial displacements of the loading piston using a LVDT and locally by using a pair of LDTs (Tatsuoka et al., 1994).

Stress-controlled drained triaxial tests on small specimens (the present study): To evaluate both creep strains by sustained loading (SL) and residual strains by cyclic loading (CL), a series of stress-controlled triaxial tests were performed at \( \sigma'_{90} = 40 \) kPa on dense \( (D_r = 90\%) \) air-dried specimens \((d = 75 \) mm and \( h = 150 \) mm\) of selected GMs consisting of relatively angular stiff particles (i.e., Toyoura sand and model Chiba gravel H) and round stiff particles (Hime gravel and Corundum A). The specimens were produced by air-pluviation. The top and bottom ends of the specimens were lubricated by the same method as described above. The axial and horizontal deformations were measured locally by a pair of LDTs and a set of three clip gauges. The specimens were axially compressed at a constant stress rate of 12 kPa/min.

Strain-controlled drained PSC tests (the previous studies): A series of drained PSC tests were performed on rectangular prismatic specimens (20 cm high, 16 cm long and 8 cm wide in the \( \sigma_x \) direction) of 1) air-dried air-pluviated Jamuna River sand (Yasin et al., 2003); and 2) moist and saturated Inagi and Narita sands moist-tamped at the respective optimum water contents (Kawaharazono, 2007; Hirakawa et al., 2008). The top and bottom ends of the specimens were lubricated. The specimens were anisotropically compressed at \( R = 3.0 \) towards \( \sigma_x = 400 \) kPa (Jamuna River sand) and at \( R = 2.0 \) towards \( \sigma_x = 50 \) kPa (Inagi and Narita sands). Axial strains were measured both externally and locally (with a pair of LDTs), while lateral strains were measured locally by using two sets of four proximity transducers, each set measuring lateral displacements at each \( \sigma_9 \) surface (Yasin and Tatsuoka, 2000).

Drained direct shear (DS) tests (the present study): Displacement-controlled DS tests were performed at a constant normal pressure \( \sigma_n (100 \) kPa) on cubic air-dried specimens \((12 \text{ cm} \times 12 \text{ cm} \times 12 \text{ cm}\) of poorly graded sands consisting of relatively angular stiff particles (Hostun and Silica No. 6a sands) and relatively round stiff particles (Albany, Monterey and Ottawa sands). The grading of Silica No. 6a sand is slightly different from that of Silica No. 6 sand. The specimens were prepared by multi-layer volume-controlled tamping. During otherwise ML at a constant shear displacement rate \((0.055 \text{ mm/min})\), several SL tests were performed for two hours per stage. More details are described by Duttine et al. (2008a).

**EVALUATION OF VISCOS PROPERTIES**

**Viscosity Type and \( \beta \) Value**

Figures 8(a) and (b) show the relationships among \( R = \sigma'_{90}/\sigma'_{90} \) and the total axial (or vertical) and volumetric

\[
\begin{align*}
E_v &= E_{\infty} \left( \frac{\sigma_{90}'}{98} \right)^m \\
\nu_{vh} &= -\frac{\partial \epsilon_{vh}}{\partial \epsilon_v} = \frac{E_{\infty}}{E_{\infty} + 1} \left[ \frac{\sigma_{90}'}{\sigma_{90}'} \right] = \frac{\partial \epsilon_v}{\partial \epsilon_v} \left[ \frac{\sigma_{90}'}{\sigma_{90}'} \right]^n (5a)
\end{align*}
\]

Equation (5b), which is part of the hypo-elastic model proposed by Tatsuoka and Kohata (1995) and Hoque and Tatsuoka (1998), was used to estimate elastic lateral strain increments, where \( \nu_{vh} \) is the elastic Poisson’s ratio; \( n \) is the parameter representing the inherent anisotropy of the stiffness, which is assumed to be equal to 1.0 in the present study; and \( \nu_0 \) is the Poisson’s ratio when the material exhibits isotropic elastic property, which is assumed to be equal to 0.168 (Hoque and Tatsuoka, 1998). After drained sustained loading for thirty minutes at \( p' = 400 \) kPa (isotropic), drained TC was started.

Strain-controlled drained TC tests on small specimens (the previous studies): Drained TC tests at \( \sigma'_{90} = 40 \) kPa were performed on specimens \((d = 100 \) mm and \( h = 200 \) mm) of a) moist crushed concrete aggregate compacted at the optimum water content to different initial dry densities (Aqil et al., 2005; Tatsuoka et al., 2006a); and b) compacted air-dried model Chiba gravel A (Hirakawa et al., 2008). The top and bottom ends of the specimens were in contact with the rigid flat faces of stainless steel top cap and pedestal. Several drained TC tests were performed on air-dried specimens (7.5 cm in diameter and 15 cm high) of Fujinomori clay and kaolin produced by compacting air-dried clay powder inside a split mould in ten layers with six blows of about 20 kgf per layer using a 6 cm-diameter hammer (Li et al., 2004; Deng and Tatsuoka, 2007; Tatsuoka et al., 2006a). The specimens were isotropically consolidated to \( \sigma'_{90} = 77 \) kPa and 80 kPa (Fujinomori) and 100 kPa (kaolin).

Strain-controlled drained TC tests on large specimens (the present study): A large specimen (30 cm in diameter and 60 cm in height) of Shinanogawa riverbed gravelly soil was produced by compacting air-dried material inside a split mould in six layers using an electric motor-driven vibrator to an initial dry density equal to 2.05 g/cm\(^3\). The specimen was isotropically compressed to \( \sigma'_{90} = 40 \) kPa.
Fig. 8. Two CD TC tests ($\sigma_1 = 400$ kPa) of saturated dense silica No. 6 sand: a) $R$-$e_v$-vol relations, b) $R$-$gir$-vol relations, c) zoom-up of test 25 and d) evaluation of rate-sensitivity coefficient, $\beta$

strains ($e_v$ and $e_{vol}$) and those among $R$ and the irreversible shear and volumetric strains, $\gamma^\prime$ ($= e^\prime_v - e_0^\prime_v$) and $e^\prime_{vol}$ ($= e^\prime_v + 2e^\prime_h$), from two typical drained TC tests of dense silica No. 6 sand, a poorly graded consisting of relatively angular and stiff particles (Fig. 4 and Table 2). Here, $e^\prime_v$ is the irreversible vertical strain evaluated based on Eq. (5a), and $e^\prime_h$ is the elastic horizontal strain evaluated based on Eq. (5b). In test 24, ML was continued at a constant $e_v$. In test 25, $e_v$ was changed stepwise by a factor of up to 200 and SL for two hours per stage was performed twice during ML at a constant $e_v$. It is seen that the peak strengths at different strain rates are essentially the same. Moreover, the stress increment that takes place upon a step change in $e_v$ tends to fully decay with strain during subsequent ML at a constant $e_v$ in test 25 (Fig. 8(c)). These trends indicate that the viscosity type is TESRA (Fig. 2). This point can be confirmed by a successful simulation of test 25 by assuming the TESRA viscosity (Fig. 8(c)); the simulation and the reference stress-strain relation are explained later. It may also be seen that the measured flow characteristics is not sensitive to the stress changes associated with step changes in $e_v$, in particular when the specimen is
dilating (Figs. 8(a) and (b)). A stronger trend of contraction rate seen at smaller strain rates in Figs. 8(a) and (b) is considered due to contractive volume changes caused by the volumetric creep mechanism (Kiyota and Tatsuoka, 2006).

In the results from test 25, any trend that the rate of volume increase becomes smaller at higher strain rates, which would have taken place if the specimen had not been fully drained, cannot be seen. This result also indicates that the specimen were essentially under fully drained conditions. The above is also the case with the test results presented in Fig. 9. Di Benedetto et al. (2002), Tatsuoka et al. (2002) and Nawir et al. (2003) also showed that the trend of rate-dependent stress-strain behaviour of drained saturated specimens of unbound sands and gravels is essentially the same as the one of air-dried specimens.

Figure 8(d) shows the relationship between $\Delta R/R$ and $\log(e_v)_{after}/e_v)_{before}$, where $e_v$ is the externally measured axial strain, and between $\Delta R/R$ and $\log(y_{gir} after)/y_{gir} before)$, where $y$ is the shear strain from locally measured axial strains and measured volumetric strains, from test 25. Nearly the same linear relation is obtained by using these two strain parameters. This result is consistent with those reported by Enomoto et al. (2007a) and Tatsuoka et al. (2008a). The slope of the linear relation is defined as the rate-sensitivity coefficient $\beta$ (Eq. (1)). The values of $\beta$ obtained by using $(e_v)_{after}/(e_v)_{before}$ from this and other similar tests are used in further analysis.

Figure 9 shows results from a drained TC test, similar as test 25 (Fig. 8), of a loose saturated specimen of another poorly graded angular sand, silica No. 4. The simulation shown in this figure assumed the TESRA viscosity as explained later. Many other similar test results exhibiting the TESRA viscosity are reported by Di Benedetto et al. (2002) and Tatsuoka et al. (2002, 2008a).

Figures 10 and 11 show drained TC test results of air-dried specimens of loose Monterey sand and dense Ottawa sand, both poorly graded sands consisting of relatively round and stiff particles. A trend of $P&N$ viscosity (Fig. 2) is obvious already in the pre-peak regime, and this trend is very obvious in the post-peak regime. The specimens are air-dried and drained in these tests, as well as those described in Figs. 12 and 13. Similar results of dense Monterey sand and other poorly graded sands consisting of round and stiff particles exhibiting the $P&N$ viscosity are reported in Tatsuoka et al. (2008a). The simulation shown in these figures are explained later.

Figure 12 shows results from a CD TC test of dense air-dried MACH, a well-graded GM consisting of relatively round and stiff particles. It appears that the viscosity type is TESRA initially in the pre-peak regime (Fig. 12(b)) and it changes to $P&N$ when approaching the peak stress state (Fig. 12(c)). Yet, this trend of $P&N$ viscosity is noticeably weaker than those with the original three poorly graded round GMs (reported by Tatsuoka et al., 2008a). This observation was confirmed by simulation of this test taking into account these trends, presented in Fig. 12, and also

![Fig. 9. TESRA viscosity in a CD TC test ($\sigma_y = 400$ kPa) on saturated loose Silica No. 4 sand and its simulation: a) whole strain range, b) zoom-up and c) and d) time histories of creep vertical strain](image-url)
Fig. 10. P&N viscosity in a CD TC test on air-dried loose Monterey sand and its numerical simulation: a) whole strain range and b) and c) zoom-ups

Fig. 11. P&N viscosity observed in a CD TC test on dense air-dried Ottawa sand and its numerical simulation: a) whole strain range and b) zoom-up

by the analysis of the viscosity parameters shown later. This test result indicates that the trend of P&N viscosity becomes weaker with an increase in $U_c$, as summarised in Table 1.

Figure 13 shows results from a CD TC test of air-dried reconstituted Shinanogawa riverbed gravelly soil, a well-graded consisting of relatively large round gravel particles and relatively angular sand particles (Fig. 4). It appears that the viscosity type is TESRA in the pre-peak regime and this trend can be confirmed by simulation assuming the TESRA viscosity (Fig. 13(b)). In the post-peak regime (Fig. 13(c)), when referring to a continuously smooth relation inferred for continuous ML at a constant strain rate (depicted by a broken curve), it is seen that the viscosity type is obviously TESRA. It seems therefore that, despite that this gravelly soil comprises rather round large particles, due to a better inter-particle interlocking achieved by including relatively angular small particles and a large coefficient of uniformity, the viscosity type does not become P&N even in the post-peak regime. This test result is consistent with the trends that the viscous type deviates more from the P&N type as the grading becomes less uniform and the particles become more angular, as summarised in Table 1.

Non-Linear Three-Component Model and Simulations
Di Benedetto et al. (2002) and Tatsuoka et al. (2002, 2008a) showed that the viscous properties of unbound geomaterials observed in the drained TC and PSC tests at fixed confining pressure can be modelled by the three-component model (Fig. 1) as follows:

$$ R = R^f + R^v \quad (6) $$

where $R = \sigma / \sigma_c = (\sigma_1 / \sigma_c) / (\sigma_1 / \sigma_c')$ ($= \sigma / \sigma_c'$ under the TC and PSC stress conditions); $R^f$ is the inviscid component of $R$ ($= \sigma / \sigma_c'$); and $R^v$ is the viscosity component obtained as $R-R^f$. Eq. (6) means that the current stress...
\[ \mathbf{R'}(\mathbf{\bar{\gamma}}', \mathbf{\bar{\dot{\gamma}}}') = \mathbf{R}(\mathbf{\bar{\gamma}}) \cdot \mathbf{g}(\mathbf{\bar{\gamma}}') \]  

(7)

where \( \mathbf{\bar{\gamma}}' = \mathbf{\bar{\gamma}} - \mathbf{\bar{\gamma}} \) (as \( \mathbf{\bar{\gamma}} \)); and \( \mathbf{R'}(\mathbf{\bar{\gamma}}', \mathbf{\bar{\dot{\gamma}}}') \) is the current value of \( \mathbf{R'} \), which is a unique function of instantaneous values of \( \mathbf{\bar{\gamma}}' \) and its rate \( \mathbf{\bar{\dot{\gamma}}}' \) under loading conditions (i.e., when \( \mathbf{\bar{\gamma}}' \) is kept always positive); and \( \mathbf{g}(\mathbf{\bar{\gamma}}') \) is the viscosity function \( (\geq 0) \) that is a highly non-linear function of \( \mathbf{\bar{\gamma}}' \), where \( \mathbf{g}(0) = 0; \) and \( \mathbf{g}(\infty) = \alpha \) (a finite positive value). The incremental form of Eq. (7) is:

\[ d\mathbf{R'}(\mathbf{\bar{\gamma}}', \mathbf{\bar{\dot{\gamma}}}') = d\mathbf{R}(\mathbf{\bar{\gamma}}') \cdot \mathbf{g}(\mathbf{\bar{\gamma}}') \]  

(8)

d\( \mathbf{R'} \) when \( \mathbf{\bar{\gamma}}' = 0 \) is represented by vector \( \Delta \mathbf{B} \) in Fig. 14(b).

With the other viscosity types (i.e., Combined, TESRA and P&N), the current \( \mathbf{R'} \) when \( \mathbf{\bar{\gamma}}' = \mathbf{\bar{\gamma}} \) (i.e., \( \mathbf{R'} \)\( \mathbf{\bar{\gamma}} \)) Eq.
The equations for history-dependent as follows (Tatsuoka et al., 2008a):

so it cannot be obtained by Eq. (7), but it becomes load-

decrease with a decrease in \( g \) (i.e., the

where \( g_{\text{decay}}(y^\tau, y^\mu - \tau) \) is the generalised decay function, which is a function of the current \( y^\mu \) and the difference between \( y^\mu \) and the irreversible strain, \( \tau \), at which the increment \([d(R(y^\mu) \cdot g(y^\mu))]\) developed. \( r(y^\mu) \) is the decay parameter, which is basically a function of \( y^\mu \) and it may decrease with \( y^\mu \) under the condition that \( 0 < r(y^\mu) \leq 1 \).

The parameter \( r_i \) denotes the decay rate of a viscous stress increment that developed when the irreversible strain was equal to \( \tau \) associated with an increase in the irreversible strain from \( \tau \) to the current value \( e^\tau \) (i.e., a decay by an amount of \( e^\tau - \tau \)). When \( r_i = 1.0 \), no decay takes place (i.e., the Isotach viscous property) and the decay rate increases with a decrease in \( r_i \). More details are explained in Tatsuoka et al. (2008a). In the simulations performed in the present study, for simplicity on one hand and due to a lack of reliable data on the other hand, \( r_i(y^\mu) \) is assumed to be a positive constant with respective GM types. \( \theta(y^\mu) = \beta_i/\beta \) is the viscous type parameter (Eq. (3)). Equation (7) (for Isotach viscosity) is obtained by substituting \( \theta = 1.0 \) into Eqs. (9a) and (b) and the equation for TESRA viscosity by substituting \( \theta = 0.0 \) into Eqs. (9a) and (b). In that case, Eq. (9b) becomes \( g_{\text{decay}}(y^\mu, y^\mu - \tau) = [r_i(y^\mu)]^{\tau - \tau} \). The equations for Combined and P&N types are obtained by substituting, respectively, positive values of \( \theta \) less than 1.0 and negative values into Eqs. (9a) and (b). Therefore, Eqs. (9a) and (b), as well as Eq. (8), are valid to all the viscosity types depicted in Fig. 2. Tatsuoka et al. (2008a) proposed the following equation for \( \theta(y^\mu) \) that may decrease with \( y^\mu \) under the condition that \( \theta(y^\mu) \leq 1 \) in a single test:

where \( \theta_{\text{ini}}, \theta_{\text{end}}, c \) and \( \gamma^\mu_0 \) are the parameters that control the manner of viscosity type transition with strain. Figure 15 summarises the \( \theta - \varepsilon \) relations used in the simulations described in this paper. As the residual condition could not be reached in the drained TC tests performed in the present study, the values of \( \theta_{\text{end}} \) in these drained TC tests are the \( \theta \) values at \( \varepsilon = 15\% \), which may be different from those at the residual state in the DS tests. In the DS tests at a fixed shear displacement rate, the residual state where the average stress ratio does not change with shear displacement can be easily reached. The \( \theta \) values at the residual state of poorly graded relatively angular GMs that exhibit the TESRA viscosity with \( \theta = 0.0 \) in the pre-peak regime (e.g., Toyoura, Hostun, Ticino and Silica No. 6a sands) are noticeably negative (see Figs. 12 and 13 of Duttine et al., 2008a).

The simulations presented in Figs. 8 through 13 were made based on Eqs. (6) through (10) after having replaced \( y^\mu \) with \( e^\mu \). This replacement was made because the major part of the drained TC tests referred to in this paper was performed on air-dried specimens without measuring horizontal strains, for which the analysis of rate effects on the relationship between \( R = \sigma^g/\sigma^s \) and the axial (vertical) strain, \( \varepsilon \), is straightforward. The reference curve (i.e., the \( R - \varepsilon \) curve, representing the elasto-plastic behaviour; i.e. the stress-strain behaviour of E and P components connected in series in Fig. 1) in the respective simulations was obtained by exploring the \( R - \varepsilon \) relations measured by continuous ML tests at different constant strain rates toward the one at zero strain rate. In
Table 3. Viscosity parameters of granular materials in drained TC tests simulated in this study

<table>
<thead>
<tr>
<th>Materials</th>
<th>Test number</th>
<th>Consolidated void ratio, $e_c$</th>
<th>Decay parameter*, $r_1$</th>
<th>Viscosity type parameter, $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\theta_{01}$</td>
</tr>
<tr>
<td>Toyoura sand</td>
<td></td>
<td>0.854</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hostun sand</td>
<td></td>
<td>0.864</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Silica No. 3 sand</td>
<td></td>
<td>0.824</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silica No. 4 sand</td>
<td></td>
<td>0.862</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silica No. 5 sand</td>
<td></td>
<td>0.857</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Silica No. 6 sand</td>
<td></td>
<td>0.980</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Silica No. 8 sand</td>
<td>1</td>
<td>1.173</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Mixed silica sand</td>
<td></td>
<td>0.582</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coral sand A</td>
<td></td>
<td>0.735</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Coral sand B</td>
<td></td>
<td>0.905</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tanno sand</td>
<td></td>
<td>1.122</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Ishihama beach sand</td>
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<td>0.848</td>
<td>0.1</td>
<td></td>
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<tr>
<td>Inagi sand</td>
<td></td>
<td>1.073</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Omigawa sand</td>
<td></td>
<td>0.874</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Shinanogawa riverbed gravel</td>
<td></td>
<td>2.069</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Albany silica sand</td>
<td>2</td>
<td>0.723</td>
<td>$10^{-7}$</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>3*</td>
<td>0.535</td>
<td>$10^{-7}$</td>
<td>-0.3</td>
</tr>
<tr>
<td>Hime gravel</td>
<td>4</td>
<td>0.639</td>
<td>$5 \times 10^{-8}$</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>5*</td>
<td>0.527</td>
<td>$10^{-7}$</td>
<td>-0.1</td>
</tr>
<tr>
<td>Corundum A</td>
<td>6</td>
<td>0.935</td>
<td>$10^{-7}$</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>7*</td>
<td>0.820</td>
<td>$10^{-3}$</td>
<td>-0.4</td>
</tr>
<tr>
<td>MACH</td>
<td>8</td>
<td>0.479</td>
<td>$10^{-6}$</td>
<td>0.0</td>
</tr>
<tr>
<td>Monterey sand</td>
<td>9</td>
<td>0.765</td>
<td>$10^{-6}$</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.587</td>
<td>-0.2</td>
<td>-0.9</td>
</tr>
<tr>
<td>Ottawa sand</td>
<td>11</td>
<td>0.755</td>
<td>$10^{-1}$</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.559</td>
<td>$10^{-1}$</td>
<td>-0.6</td>
</tr>
<tr>
<td>S.L.B. sand</td>
<td>13</td>
<td>0.762</td>
<td>$10^{-10}$</td>
<td>-0.7</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>0.517</td>
<td>$10^{-9}$</td>
<td>-0.4</td>
</tr>
<tr>
<td>Ticino sand</td>
<td>15</td>
<td>0.866</td>
<td>$10^{-14}$</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.622</td>
<td>$10^{-14}$</td>
<td></td>
</tr>
</tbody>
</table>

Note) *: determined in the formulation in terms of $e_v$ in %.
*: obtained from the simulation of the test results presented in Fig. 25 (after Kongkitkul et al., 2008).
$: compacted dry density (g/cm$^3$).

Fig. 13(b), the simulation is made of the $R$ and locally measured axial strain relation. Table 3 summarises the viscous property parameters used in the simulations presented in Figs. 8 through 13 together with those of other drained TC tests described in this paper. These parameters were determined so that the measured rate-dependency of stress-strain behaviour is best fitted by model simulation. Note that these simulations are not merely curve-fitting of respective test results. That is, once these parameters are determined based on results from tests performed along a certain loading history, the model is able to predict the rate-dependent stress-strain behaviour for any other arbitrary loading histories. Yet, it is convenient if approximated values of these parameters can be estimated only from particle characteristics without performing sophisticated shear tests as presented in this paper. This is indeed one of the objectives of the present study.

It may be seen from Figs. 8 through 13 that various trends of rate-dependent stress-strain behaviour, exhibit-
Fig. 16. Rate-sensitivity coefficients $\beta$ free from the effects of pore water plotted against: a) $D_{50}$, b) $U_c$ and c) FC

Fig. 17. Effects of specimen density and wet conditions (saturated or air-dried) on the $\beta$ value when the viscosity type is $P&N$

ANALYSIS OF VISCOSOUS PROPERTY

In this section, the correlations between the viscous property parameters: i.e., the rate-sensitivity coefficient ($\beta$, Eq. (1)); the decay parameter ($r_1$, Eq. (9b)); and the viscosity type parameter (Eq. (3)) at $e_{ir}=15\%$ ($\theta_{90}$), and the particle characteristics: i.e., $D_{50}$, $U_c$, fines content (FC) and the degree of particle angularity ($A^*$, Eq. (4)) and the correlations among these viscous property parameters are analysed.

Rate-sensitivity Coefficient $\beta$

The $\beta$ values listed in Table A1 are plotted against $D_{50}$ (Fig. 16(a)), $U_c$ (Fig. 16(b)) and FC (Fig. 16(c)). These data are all free from the effects of pore water and its change during shear. With poorly graded GMs consisting of relatively round and stiff particles that exhibit $P&N$ already in the pre-peak regime, only the data of air-dried specimens are plotted in Fig. 16, while they are compared with those of saturated specimens in Fig. 17. The $\beta$ value of saturated specimens of the same GM type is only marginally smaller than that of air-dried ones. This result
also indicates that the saturated specimens were essentially under fully drained conditions because of the following. Most of these $\beta$ values were measured when the specimens exhibited dilatant behaviour. Therefore, if under partially drained conditions, at higher strain rates, the saturated specimens are stronger and therefore exhibit a larger increase in the strength upon an increase in the strain rate than it is under fully drained conditions. Consequently, the $\beta$ values when saturated should have become larger than those when air-dried. With some other GM types, the $\beta$ values of saturated specimens have also been confirmed to be essentially the same as those of air-dried specimens (plotted in Fig. 16). For example, with poorly graded GMs consisting of relatively angular and stiff particles that exhibit TESRA in the pre-peak regime (i.e., Toyoura and Hostun sands), the $\beta$ values of saturated and air-dried specimens are essentially the same (Nawir et al., 2003).

The following trends may be seen from Fig. 16. Firstly, with both angular and round GMs, the effects of density on the $\beta$ value are generally insignificant. This trend is consistent with that of Toyoura sand (Nawir et al., 2003).

Secondly, the scatter among all the $\beta$ values is not small in Fig. 16(a-1). In Fig. 16(a-2), all the data points of the unbound GMs consisting of relatively angular and stiff (i.e., less crushable) particles that exhibit TESRA viscosity in the pre-peak regime are located in a zone between a pair of horizontal dotted lines. When limited to these data, in this very wide range of $D_{50}$ of about four orders of magnitude, the effects of $D_{50}$ on the $\beta$ value are not significant. This result indicates that the effect of particle size is basically insignificant.

Thirdly, in Fig. 16(a-2), in comparison with the data plotted in the zone between a pair of horizontal dotted lines, the $\beta$ values of GMs consisting of relatively round and stiff particles that exhibit P&N viscosity already in the pre-peak regime (i.e., Corundum A, Hime gravel, glass beads and Albany, Monterey, Ticino, SLB and Ottawa sands) are generally smaller. On the other hand, the $\beta$ values of GMs consisting of relatively crushable particles (i.e., crushed concrete, coral sand B and Inagi, Tanno, Narita and Omigawa sands) that exhibit a weaker trend of TESRA viscosity in the pre-peak regime with slower decay of $\Delta\sigma^c$ (i.e., larger $r_1$ values; Table 3 and as shown below) are generally larger. In this respect, Fig. 18 shows the relationships between the $\beta$ value and the degree of particle angularity, $A^*$, with an index of $U_c$ among a number of GMs. The data points located above a horizontal dotted line are those for GMs having either $U_c>4$ or relatively crushable particles or both. With the other data, the $\beta$ values are smaller and the variation is relatively small. Yet, when $A^*<about 500$, the $\beta$ value tends to decrease with a decrease in $A^*$ and the values when $A^*=0$ (i.e., glass beads and corundum A) are smallest. A noticeable scatter in the $\beta$ value for the same $A^*$ among these data is due largely to a variation in $U_c$, as discussed below. In summary, the effects of $A^*$ on the $\beta$ value are noticeable, while the effects of $U_c$ and particle crushability are much larger.

Fourthly, the major reason for a noticeable variation in the $\beta$ values of GMs consisting of relatively angular and stiff particles (plotted between two horizontal broken lines in Fig. 16(a-2)) is the effect of $U_c$. In fact, the $\beta$ values of these GMs tend to decrease with a decrease in $U_c$ when $U_c<about 3$, as indicated by a pair of broken curves in Fig. 16(b). The data points located above this zone are of GMs consisting of relatively crushable particles, while those located in the lower part in this zone and below are of GMs consisting of relatively round and stiff particles. In comparison, in Fig. 16(c), the scatter of the $\beta$ values when FC is equal and close to 0 is fairly large with both angular and round GMs. Furthermore, the physical meanings of $D_{50}$ and FC are somehow overlapping. For these reasons, the analysis based on FC is not made in the following.

In summary, the $\beta$ value is basically independent of $D_{50}$, while it tends to decrease with a decrease in $A^*$, $U_c$ and particle crushability. The effects of $U_c$ and particle crushability are much larger than those of $A^*$.

**Decay Parameter $r_1$**

In the simulations performed in this study, the value of this parameter (Eq. 9(b)) was assumed to be a constant with a given GM type irrespective of viscosity type, as stated earlier, and evaluated by simulation of the respective measured relationships between $R$ and $\varepsilon_0$ (in %). These $r_1$ values are listed in Table 3 and plotted in separate two different logarithmic scales against $D_{50}$ and $U_c$ in Figs. 19(a) and (b).

In Fig. 19(a), the $r_1$ value tends to decrease with an increase in $D_{50}$ with Silica sands, Nos. 3, 4, 5, 6 and 8. In Fig. 19(b-2), the data points in a zone surrounded by a broken curve are those of GMs consisting of relatively angular and stiff particles. In this zone, the $r_1$ value tends to decrease with a decrease in $U_c$. Moreover, the data points plotted in the upper part of Fig. 19(b-2) without names are those of GMs consisting of relatively crushable particles. A weak trend that the $r_1$ value increases with an increase in particle crushability may be seen. These effects of $D_{50}$ and $U_c$, as well as particle crushability, are general-
Fig. 19. Decay parameters $r_1$ (in terms of $e$, in %) plotted against: a) $D_{50}$ and b) $U_c$

Fig. 20. Relationships between $r_1$ and $A^*$

Fig. 21. Relationships between $\theta_{end}$ and $A^*$ of granular materials

Viscous properties of granular materials

ly much smaller than those of particle shape. That is, the data points in the lower part of Fig. 19(b-2) without names are those of GMs consisting of relatively round and stiff particles. The effects of particle shape on the $r_1$ value are obvious as a whole in the data plotted in Fig. 19(b-2). This trend can be seen more obviously from Fig. 20. That is, for the data with $U_c < 4.0$ (plotted below a horizontal broken line), the $r_1$ value is rather independent of $A^*$ when $A^* >$ about 750, but the $r_1$ value obviously decreases with a decrease in $A^*$ when $A^* <$ about 500. Furthermore, overall effects of $U_c$ are noticeable in Fig. 20. These trends are consistent with the following observations described earlier. Firstly, the $r_1$ value reflects the viscosity type in that $r_1$ is equal to 1.0 with Isotach type, while it decreases as the viscosity type changes from Isotach toward P&N. Secondly, more coherent geomaterials (e.g., sedimentary soft rock and cement-mixed soil as well as highly plastic clay) exhibit the Isotach type viscosity (with $r_1 = 1.0$). With unbound GMs, the viscosity type changes from Isotach toward P&N as the microstructure becomes less stable associated with; a) a decrease in the coordination number (i.e., the average number of contact points between particles per particle) due to a decrease in $U_c$ and a dry density (Duttine et al., 2008b); and b) a decrease in the inter-particle interlocking due to a decrease in $A^*$ (i.e., with a decrease in the particle angularity).

Viscosity Type Parameter $\theta_{end}$

The viscosity type parameter when $e_v = 15\%$ ($\theta_{end}$) was selected as the index representing the viscosity types (Fig. 2). Figure 21 shows the relationship between $\theta_{end}$ and $A^*$. 
with an index of $U_c$ for GMs with $U_c < 30.4$. As no GMs that exhibit Combined type at $e_v = 15\%$ were found in the present study, no data point with $u_{end} > 0$ is plotted in Fig. 21. The $\theta_{end}$ value is nearly constant, equal to 0.0 (i.e., TESRA viscosity) when $A^* > 750$. When $A^* < 750$, $\theta_{end}$ decreases with a decrease in $A^*$ at a high rate from 0.0 toward negative values, indicating that the trend of P&N viscosity type becomes stronger. The $\theta_{end}$ value becomes nearly $-1.0$ when $A^*$ reaches 0 (i.e., corundum A). When $A^*$ is around 250, the $\theta_{end}$ value tends to increase with an increase in $U_c$, although this trend is less obvious than the effect of $A^*$ (i.e., the effect of particle shape). No specific effects of $D_{50}$ and particle crushability on the $\theta_{end}$ value were found.

**Correlation among $r_1$, $\beta$ and $\theta_{end}$**

Figure 22 shows relationship between the values of $r_1$ and $\beta$ obtained by simulations assuming that these values are constant in respective TC tests. The $r_1$ value tends to decrease with a decrease in $\beta$, associated with changes in the viscosity type from Isotach toward P&N. A relatively large scatter seen in the data is due to that the whole data consist of the following three distinct groups showing different trends (Fig. 22(b)): 1) GMs consisting of relatively angular and stiff particles; 2) GMs consisting of relatively angular and crushable particles; and 3) GMs consisting of relatively round and stiff particles. The first two groups exhibit separate well-defined correlations, while the $\beta$ value for the same $r_1$ value is larger with more crushable GMs. No independent effects of $U_c$ can be seen on these correlations. With the third group (i.e., GMs consisting of relatively round and stiff particles), the $r_1$ values are very small with a large variation at a nearly constant $\beta$.

Figure 23 shows the relationships between $r_1$ and $\theta_{end}$ with an index of $A^*$. When $\theta_{end} = 0.0$ (i.e., TESRA viscosity), the $r_1$ value tends to decrease with a decrease in $D_{50}$ and $U_c$ (Figs. 19(a) and (b)). When $\theta_{end} < 0.0$ (i.e., P&N viscosity), the $r_1$ value obviously decreases with a decrease in $\theta_{end}$ associated with a decrease in $A^*$ (Fig. 21). Yet, for the same $\theta_{end}$ value, the $r_1$ value tends to decrease with a decrease in the $A^*$ value, showing that the $r_1$ and $\theta_{end}$ relation is not highly correlated.

Finally, Fig. 24 shows the $\beta$ and $\theta_{end}$ relation. This data set consists of the following two groups having totally different trends (as cited earlier). When $A^* > 750$, the viscosity type is TESRA (with $\theta_{end} = 0$) irrespective of $A^*$ value (Fig. 21), while the $\beta$ value significantly decreases...
with a decrease in $U_c$ and particle crushability (Fig. 16(b)). On the other hand, when $A^* < 750$, the viscosity type is P&N and this trend becomes stronger (i.e., the $\theta_{\text{null}}$ value becomes smaller) with a decrease in $A^*$ (Fig. 21), while the $\beta$ value, which is generally smaller than those for TESRA type, is not strongly affected by $A^*$ (Fig. 18).

In summary, the viscous properties of GMs are affected by, at least, particle shape (i.e., $A^*$), grading (i.e., $U_c$) and particle crushability, while the effects of particle size (i.e., $D_{50}$) are basically insignificant. Generally, as the particle becomes less angular, more uniform and stiffer (i.e., less crushable), the values of $\beta$, $r_1$ and $\theta$ generally decrease. More specifically, the $\beta$ value is affected by $U_c$ and particle crushability more significantly than $A^*$, while the values of $r_1$ and $\theta$ are affected by $A^*$ more significantly than $U_c$ and particle crushability.
CREEP DEFORMATION AND RESIDUAL DEFORMATION BY CYCLIC LOADING

Creep Deformation

Figure 25(a) shows results from drained TC tests ($\sigma' = 400$ kPa) at $\varepsilon_v = 0.0625\%$/min (constant) with SL for ten hours at several shear stress levels performed on similarly dense air-dried specimens of three relatively round GMs (corundum A, Albany sand and Hime gravel) and three relatively angular GMs (Toyoura sand, Silica No. 4 sand and coral sand A), all poorly graded consisting of stiff particles. Figure 25(b) compares the creep axial strains by SL for ten hours and the shear stress level (i.e., the ratio of $R = \sigma_v / \sigma'_h$ to its maximum value, $R_{_{_{\text{max}}}}$) at which the respective SL tests were performed. Despite that a variation in the relative density may affect this plot, it can be noted that, with similarly dense specimens of similarly poorly graded GMs consisting of stiff particles, the creep strain by sustained loading (SL) at the same ratio of the principal stress ratio to the peak stress ratio becomes smaller as the particles becomes less angular.

Figure 26 presents a data set from drained direct shear (DS) tests at a constant $\sigma'_h$ ($100$ kPa) with SL for two hours at different shear stress levels during otherwise ML at a constant shear displacement rate ($0.055$ mm/min). This data shows a similar trend as seen in Fig. 25. The test materials are two relatively angular GMs that exhibit the TESRA viscosity in the pre-peak regime and four relatively round GMs that exhibit the P&N viscosity in the pre-peak regime, all poorly graded consisting of stiff particles. The dilatancy rate is positive at most of the SL stages (Fig. 26(b)). Figure 26(c) compares the creep deformation by SL for two hours and the shear stress level (i.e., the ratio of $R_{_{_{\text{DS}}} = \tau / \sigma$ to its maximum value, $R_{_{_{\text{DS,peak}}}}$). Also in these DS tests, the creep deformation at the same shear stress level of the relatively round GMs is noticeably smaller than the relatively angular ones. The range of $D_{50}$ of the tested materials is small, $0.29$–$0.68$ mm, and therefore its effects on this trend must be insignificant.

Figure 27(a) shows the relationships between the creep axial strain by SL for 10 hours at a shear stress level equal to 0.85 obtained from Fig. 25(b) and the two viscosity type parameters obtained from the plots in Fig. 15: i.e., the $\theta$ value at $\varepsilon_v = 15\%$ ($\theta_{_{\text{end}}}$) and the $\theta$ value at the start of the respective SL tests, both obtained by the simulation of the overall $R$-$\varepsilon_v$ relations from the drained TC tests. Figure 27(b) shows similar relationships between the creep shear displacement by SL for 2 hours at a shear stress level equal to 0.85 obtained from Fig. 26(c) and the two viscosity type parameters: i.e., the value at the residual state ($\theta_{_{\text{end}}}$) and the $\theta$ value at the start of the respective SL tests, both obtained by the simulation of the overall $R_{_{\text{DS}}} - \varepsilon$ relations from the drained DS tests (Kongkitkul et al., 2008). It may be seen that the creep deformation generally decreases with a decrease in the viscosity type
parameters in both TC and DS tests. As the viscosity type parameter tends to decrease as the particle shape becomes less angular (Fig. 21), it implies that, at least with poorly graded GMs consisting of stiff particles, the creep deformation becomes smaller as the particles become less angular. This trend can be seen from Figs. 28(a) and (b). The straight lines in these figures were obtained by linear fitting. Furthermore, it may be seen from Figs. 27(a-2) and (b-2) that the creep deformation is accurately simulated by the proposed model.

Residual Deformation by Cyclic Loading

To examine whether the effects of particle shape on the creep strain by SL and the residual strain by cyclic loading (CL) are either similar or different, a series of stress-controlled drained TC tests were performed applying both SL and CL histories under otherwise the same conditions on air-dried dense specimens of four types of GMs consisting of stiff particles. The materials are all poorly graded GMs, relatively angular (Toyoura sand and model Chiba gravel H) or relatively round (corundum A and Hime gravel). Figure 29 shows the loading histories applied in a pair of drained TC tests, A and B, at $\sigma_0 = 40$ kPa on Toyoura sand and the test results. Creep shear strains by SL and residual shear strains by CL applied alternatively at different shear stress levels but at different sequences to a pair of very similarly dense specimens were compared. The sustained deviator stress ($q$) equal to 60, 90, 120 and 150 kPa (Figs. 29(a) and (b)). Each SL or CL history was applied for a period of 50,000 seconds, which was 500 cycles of CL at a frequency of 0.1 Hz.

Figure 30(a) compares the creep shear strain by SL and the residual shear strain by CL for a short duration (i.e., the first 100 seconds or the first one cycle) and for a long duration (i.e., the whole 50,000 seconds or the whole 500 cycles) from these two TC tests. The four data points of the respective relations are those obtained at the four deviator stress levels. The residual shear strain by the first cycle of CL is consistently smaller than the creep shear strain by SL for the first 100 seconds at any of these deviator stress levels. On the other hand, the residual shear strain by 500 cycles of CL is much larger than the creep shear strain by SL for the same duration (i.e., 50,000 seconds), in particular at a higher deviator stress levels. Figure 30(b) shows the data for a short duration (i.e., the first 100 seconds or the first one cycle) presented in Fig. 30(a) and similar data for the other three types of GMs having different particle shapes, while Fig. 30(c) shows the data for a long duration (i.e., the whole 50,000 seconds or the whole 500 cycle). With all these materials, the residual shear strain by the first one unload/reload cycle is much smaller than the creep shear strain by SL for 100 seconds (except for two data points at small shear stress level), but the ratio of the residual shear strain by CL to the creep shear strain for the same loading duration largely increases with an increase in the loading duration (i.e., with an increase in the number of loading cycles in the CL tests). This trend becomes stronger as the particle shape becomes less angular.

Kongkitkul et al. (2004) showed that the residual strain that takes place in polymer reinforcement during cyclic tensile loading is due totally to viscous properties. On the other hand, as shown above, the effects of particle shape on the creep strain by SL and the residual strain by CL are opposite. This fact suggests that the micro-structure that is rather stable under stationary boundary stresses becomes unstable by CL and this trend becomes stronger as the particles become less angular. It seems therefore that the residual strain that takes place during CL is not due totally to viscous properties, but it also includes a component by inviscid effects of CL, which should be a function of cyclic stress amplitude and loading cycles but not a function of loading frequency. It is known that the residual strain that takes place during a given CL history is not equal to a linear summation of two components by viscous properties and inviscid cyclic loading effects (Tatsuoka, 2007). The features of the developments of inelastic strain described in this paper are summarized in Table 4. The effect of void ratio indicated in this table is after Duttine et al. (2008b).

**CONCLUSIONS**

The following conclusions can be derived from the test results and analysis presented above:
1. The viscous properties of geomaterial can be characterized in terms of: a) the rate-sensitivity coefficient ($\beta$); b) the decay parameter ($r_1$), and c) the viscosity type parameter ($\theta$). These parameters are affected by particle characteristics, generally decreasing with a decrease in the uniformity coefficient ($U_c$), the particle crushability, and the degree of particle angularity ($A^*$). The effects of mean diameter ($D_{50}$) on these parameters are generally insignificant.

a) The $\beta$ value is insensitive to changes in dry density and wet conditions (i.e., air-dried or saturated, except for clays). The $\beta$ value is affected by $U_c$ and particle crushability more significantly than $A^*$ (i.e., the particle shape).

b) The $r_1$ values of poorly graded granular materials (GMs) consisting of relatively round and stiff particles, which exhibit the P&N viscosity in the prepeak regime, are significantly smaller than those of GMs consisting of relatively angular particles, which exhibit the TESRA viscosity. That is, the $r_1$ value is affected by $A^*$ more significantly than $U_c$ and particle crushability.

c) The $\theta$ value (i.e., the viscosity type) is affected by $A^*$ more significantly than $U_c$ and particle crushability. When $\theta<0$, the $r_1$ value tends to decrease with a decrease in the $\theta$ value.

2. With GMs consisting of stiff particles, as the particles become less angular:

a) the creep shear strain becomes smaller; and

b) the increase in the residual shear strain by CL with
Fig. 30. Comparison between residual shear strains by SL and CL from TC tests: a) air-dried dense Toyoura sand and b) and c) effects of the particle shape on the relative largeness between residual strains by SL and CL.

Table 4. General trends of inelastic properties of geomaterial (modified from Tatsuoka, 2007)

<table>
<thead>
<tr>
<th>Influencing factors</th>
<th>Viscosity type (θ)</th>
<th>Particle shape (in case of stiff particles)</th>
<th>Grading characteristics</th>
<th>Particle size (if saturated)</th>
<th>Particle crushability</th>
<th>Void ratio</th>
<th>Inter-particle bonding</th>
<th>Strain level</th>
<th>Summary Inter-particle contact point condition</th>
<th>Deformation by cyclic loading (inviscid)</th>
<th>Creep deformation</th>
</tr>
</thead>
</table>

an increase in the number of loading cycles becomes more significant than the increase in the creep shear strain for the same loading duration.

3. Residual strains by CL consist of two components due to viscous properties and inviscid CL effects. The inviscid CL effect becomes stronger with an increase in the number of loading cycle and the cyclic loading stress amplitude.

ACKNOWLEDGEMENTS

The corundum used in the present study was kindly provided by Prof. Gudehus, G., the University of Karlsruhe, Germany. The study was financially supported by the Grant-in-Aid for Scientific Research (B), KAKENHI No. 18360231, the Japanese Society for Promotion of Science. The help of the colleagues of the Geotechnical Laboratory of Tokyo University of Science in performing the experiment is deeply appreciated.

REFERENCES


**APPENDIX A1**

<table>
<thead>
<tr>
<th>Material</th>
<th>Grading properties</th>
<th>Wet condition</th>
<th>Density</th>
<th>Test method</th>
<th>Range of ε, kPa</th>
<th>β</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shinanogawa riverbed gravel</td>
<td>Dₐₙ = 11.25 mm,</td>
<td>Air-dried (compaction and TC test)</td>
<td>ρₖₑₚ = 2.06 g/cm³</td>
<td>Drained TC at εₛₑ = 40 kPa (large specimen)</td>
<td>0.00625–1.25%/min</td>
<td>0.0174</td>
<td>Present study</td>
</tr>
<tr>
<td></td>
<td>Dₐₙ = 1.537 mm,</td>
<td></td>
<td></td>
<td>Drained TC at εₛₑ = 400 kPa</td>
<td>0.00625–1.25%/min</td>
<td>0.0169</td>
<td>Present study</td>
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<tr>
<td></td>
<td>Dₐₙ = 0.321 mm,</td>
<td>Saturated</td>
<td>eₛₑ = 0.638</td>
<td></td>
<td>0.0142</td>
<td>0.0137</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dₐₙ = 0.180 mm,</td>
<td></td>
<td>eₛₑ = 0.686</td>
<td></td>
<td>0.0218</td>
<td>0.0194</td>
<td></td>
</tr>
<tr>
<td>Hime gravel (air-pluviated)</td>
<td>Dₐₙ = 0.321 mm,</td>
<td>Saturated</td>
<td>eₛₑ = 0.864</td>
<td>Drained TC at εₛₑ = 400 kPa</td>
<td>0.00625–1.25%/min</td>
<td>0.0211</td>
<td>Present study</td>
</tr>
<tr>
<td></td>
<td>Dₐₙ = 0.180 mm,</td>
<td></td>
<td>eₛₑ = 0.642</td>
<td></td>
<td>0.0168</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toyoura sand (air-pluviated)</td>
<td>Dₐₙ = 1.508 mm,</td>
<td>Saturated</td>
<td>eₛₑ = 0.824</td>
<td>Drained TC at εₛₑ = 400 kPa</td>
<td>0.00625–1.25%/min</td>
<td>0.0222</td>
<td>Present study</td>
</tr>
<tr>
<td></td>
<td>Dₐₙ = 0.508 mm,</td>
<td></td>
<td>eₛₑ = 0.731</td>
<td></td>
<td>0.0220</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silica No. 3 sand (air-pluviated)</td>
<td>Dₐₙ = 1.395 mm,</td>
<td>Saturated</td>
<td>eₛₑ = 0.862</td>
<td>Drained TC at εₛₑ = 400 kPa</td>
<td>0.00625–1.25%/min</td>
<td>0.0204</td>
<td>Present study</td>
</tr>
<tr>
<td></td>
<td>Dₐₙ = 1.644 mm,</td>
<td></td>
<td>eₛₑ = 0.691</td>
<td></td>
<td>0.0205</td>
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<td></td>
</tr>
<tr>
<td>Material</td>
<td>Grading properties</td>
<td>Wet condition</td>
<td>Density(^1)</td>
<td>Test method</td>
<td>Range of (\varepsilon), (\beta)</td>
<td>Ref.</td>
<td></td>
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<td>-------------------------------</td>
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</tr>
<tr>
<td>Silica sand No. 5 (air-pluviated)</td>
<td>(D_{10}=0.554\ mm, \ U_i=2.243, \ G_i=2.646, \ e_{max}=1.079, \ e_{min}=0.671)</td>
<td>Saturated</td>
<td>(e_i=0.857)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0239</td>
<td>Present study</td>
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<tr>
<td>Silica sand No. 6 sand (air-pluviated)</td>
<td>(D_{10}=0.290\ mm, \ U_i=2.427, \ G_i=2.642, \ e_{max}=1.174, \ e_{min}=0.671)</td>
<td>Saturated</td>
<td>(e_i=0.980)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0247</td>
<td>Present study</td>
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<tr>
<td>Silica sand No. 8 sand (air-pluviated)</td>
<td>(D_{10}=0.099\ mm, \ U_i=2.235, \ G_i=2.633, \ e_{max}=1.431, \ e_{min}=0.761)</td>
<td>Saturated</td>
<td>(e_i=1.173)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0308</td>
<td>Present study</td>
<td></td>
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<tr>
<td>Mixed silica sand (air-pluviated)</td>
<td>(D_{10}=0.811\ mm, \ U_i=13.084, \ G_i=2.639, \ e_{max}=0.899, \ e_{min}=0.473)</td>
<td>Saturated</td>
<td>(e_i=0.582)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0304</td>
<td>Present study</td>
<td></td>
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<tr>
<td>Coral sand A (air-pluviated)</td>
<td>(D_{10}=0.170\ mm, \ U_i=2.066, \ G_i=2.645, \ e_{max}=0.806, \ e_{min}=0.484)</td>
<td>Saturated</td>
<td>(e_i=0.735)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0199</td>
<td>Present study</td>
<td></td>
</tr>
<tr>
<td>Coral sand B (air-pluviated)</td>
<td>(D_{10}=0.372\ mm, \ U_i=2.153, \ G_i=2.785, \ e_{max}=1.052, \ e_{min}=0.708)</td>
<td>Saturated</td>
<td>(e_i=0.905)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0334</td>
<td>Present study</td>
<td></td>
</tr>
<tr>
<td>Tanno sand (air-pluviated)</td>
<td>(D_{10}=0.168\ mm, \ U_i=30.375, \ G_i=2.465, \ e_{max}=1.872, \ e_{min}=0.977)</td>
<td>Saturated</td>
<td>(e_i=1.122)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0454</td>
<td>Present study</td>
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<tr>
<td>Inagi sand (air-pluviated)</td>
<td>(D_{10}=0.180\ mm, \ U_i=20.600, \ G_i=2.836, \ e_{max}=1.348, \ e_{min}=0.784)</td>
<td>Saturated</td>
<td>(e_i=1.073)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0416</td>
<td>Present study</td>
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<tr>
<td>Inagi sand (moist compacted)</td>
<td>Moist((w=7–13.5%))</td>
<td>Saturated</td>
<td>(e_i=0.793)</td>
<td>Drained PSC at (\sigma_i=50\ kPa)</td>
<td>0.025–0.25%/min, 0.0436</td>
<td>Kawaharazono (2007); Hirakawa et al. (2008)</td>
<td></td>
</tr>
<tr>
<td>Narita sand (moist compacted)</td>
<td>(D_{10}=0.167\ mm, \ U_i=2.0,mm, \ G_i=16.6, \ P_C=19.1%)</td>
<td>Saturated</td>
<td>(e_i=0.848)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0206</td>
<td>Present study</td>
<td></td>
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<tr>
<td>Ishihama beach sand (air-pluviated)</td>
<td>(D_{10}=0.344\ mm, \ U_i=2.119, \ G_i=2.728, \ e_{max}=0.944, \ e_{min}=0.593)</td>
<td>Saturated</td>
<td>(e_i=0.874)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0206</td>
<td>Present study</td>
<td></td>
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<tr>
<td>Omigawa sand (air-pluviated)</td>
<td>(D_{10}=0.172\ mm, \ U_i=4.619, \ G_i=2.751, \ e_{max}=1.329, \ e_{min}=0.729)</td>
<td>Saturated</td>
<td>(e_i=0.723)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0181</td>
<td>Present study</td>
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<tr>
<td>Albany silica sand (air-pluviated)</td>
<td>(D_{10}=0.302\ mm, \ U_i=2.221, \ G_i=2.671, \ e_{max}=0.804, \ e_{min}=0.503)</td>
<td>Air-dried</td>
<td>(e_i=0.735)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0178</td>
<td>Present study</td>
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<tr>
<td>S.L.B. (Silver Leighton Buzzard) sand (air-pluviated)</td>
<td>(D_{10}=0.681\ mm, \ U_i=1.43, \ G_i=2.66, \ e_{max}=0.79, \ e_{min}=0.49)</td>
<td>Air-dried</td>
<td>(e_i=0.762)</td>
<td>Drained TC at (\sigma_i=50\ kPa)</td>
<td>0.00625–1.25%/min, 0.0193</td>
<td>Present study</td>
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<td>Ottawa sand (air-pluviated)</td>
<td>(D_{10}=0.174\ mm, \ U_i=1.76, \ G_i=2.665, \ e_{max}=0.864, \ e_{min}=0.515)</td>
<td>Air-dried</td>
<td>(e_i=0.755)</td>
<td>Drained TC at (\sigma_i=50\ kPa)</td>
<td>0.00625–1.25%/min, 0.0243</td>
<td>Present study</td>
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<td>Monterey sand (air-pluviated)</td>
<td>(D_{10}=0.484\ mm, \ U_i=1.4, \ G_i=2.0, \ e_{max}=0.86, \ e_{min}=0.55)</td>
<td>Air-dried</td>
<td>(e_i=0.765)</td>
<td>Drained TC at (\sigma_i=50\ kPa)</td>
<td>0.00625–1.25%/min, 0.0198</td>
<td>Present study</td>
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<td>Ticino sand (air-pluviated)</td>
<td>(D_{10}=0.527\ mm, \ U_i=1.52, \ G_i=2.68, \ e_{max}=0.96, \ e_{min}=0.59)</td>
<td>Air-dried</td>
<td>(e_i=0.866)</td>
<td>Drained TC at (\sigma_i=50\ kPa)</td>
<td>0.00625–1.25%/min, 0.0174</td>
<td>Present study</td>
<td></td>
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<td>Corundum A (air-pluviated)</td>
<td>(D_{10}=1.416\ mm, \ U_i=1.623, \ G_i=3.900, \ e_{max}=1.066, \ e_{min}=0.865)</td>
<td>Air-dried</td>
<td>(e_i=0.935)</td>
<td>Drained TC at (\sigma_i=400\ kPa)</td>
<td>0.00625–1.25%/min, 0.0150</td>
<td>Present study</td>
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<tr>
<td>Material</td>
<td>Grading properties</td>
<td>Wet condition</td>
<td>Density(1)</td>
<td>Test method</td>
<td>Range of (\varepsilon_r)</td>
<td>(\beta)</td>
<td>Ref.</td>
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<tr>
<td>Corundum B (air-dried compacted)</td>
<td>(D_{50}=0.00163 \text{ mm,} \quad U_d=28.153, \quad G_r=4.137, \quad e_{\text{min}}=3.166, \quad e_{\text{max}}=1.863)</td>
<td>Air-dried</td>
<td>(e_r=1.714)</td>
<td>Drained TC at (\sigma'_r=50 \text{ kPa})</td>
<td>0.00625–1.25%/min</td>
<td>0.0304</td>
<td>Present study</td>
</tr>
<tr>
<td>MACH (air-pluviated)</td>
<td>(D_{50}=1.089 \text{ mm,} \quad U_r=6.343, \quad G_r=2.846, \quad e_{\text{min}}=0.626, \quad e_{\text{max}}=0.410)</td>
<td>Air-dried</td>
<td>(e_r=0.479)</td>
<td>Drained TC at (\sigma'_r=400 \text{ kPa})</td>
<td>0.00625–1.25%/min</td>
<td>0.0200</td>
<td>Present study</td>
</tr>
<tr>
<td>Glass beads A (air pluviated)</td>
<td>(D_{50}=0.4 \text{ mm,} \quad U_r=1.205, \quad G_r=2.497, \quad e_{\text{min}}=0.726, \quad e_{\text{max}}=0.577)</td>
<td>Air-dried</td>
<td>(e_r=0.542)</td>
<td>Drained TC at (\sigma'_r=50 \text{ kPa})</td>
<td>0.00625–1.25%/min</td>
<td>0.0114</td>
<td>Present study</td>
</tr>
<tr>
<td>Glass beads B (air pluviated)</td>
<td>(D_{50}=0.2 \text{ mm,} \quad U_r=1.188, \quad G_r=2.497, \quad e_{\text{min}}=0.783, \quad e_{\text{max}}=0.593)</td>
<td>Air-dried</td>
<td>(e_r=0.582)</td>
<td>Drained TC at (\sigma'_r=50 \text{ kPa})</td>
<td>0.00625–1.25%/min</td>
<td>0.0146</td>
<td>Present study</td>
</tr>
<tr>
<td>Glass beads C (air pluviated)</td>
<td>(D_{50}=0.1 \text{ mm,} \quad U_r=1.093, \quad G_r=2.497, \quad e_{\text{min}}=0.787, \quad e_{\text{max}}=0.598)</td>
<td>Air-dried</td>
<td>(e_r=0.606)</td>
<td>Drained TC at (\sigma'_r=50 \text{ kPa})</td>
<td>0.00625–1.25%/min</td>
<td>0.0174</td>
<td>Present study</td>
</tr>
<tr>
<td>Jamuna river sand (air-pluviated)</td>
<td>(D_{50}=0.16, \quad U_r=2.1, \quad F_C=7%; \quad G_r=2.7, \quad e_{\text{min}}=1.173, \quad e_{\text{max}}=0.690)</td>
<td>Air-dried</td>
<td>(e_r=0.775)</td>
<td>Drained PSC at (\sigma'_r=490 \text{ kPa})</td>
<td>0.00125–0.125%/min</td>
<td>0.0273</td>
<td>Yasin et al. (2003)</td>
</tr>
<tr>
<td>Original Chiba gravel (moist compacted)</td>
<td>Crushed sandstone; (D_{50}=7.8 \text{ mm,} \quad D_{10}=9.6 \text{ mm,} \quad U_r=11.2, \quad G_r=2.71)</td>
<td>Moist ((w=5%; \quad S_c=77%))</td>
<td>Dense, (e_r=0.19)</td>
<td>Drained TC at (\sigma'_r=490 \text{ kPa}) (large specimens)</td>
<td>0.0008–0.06%/min</td>
<td>0.0335</td>
<td>Anh Dan et al. (2004)</td>
</tr>
<tr>
<td>Model Chiba gravel A (air-dried compacted)</td>
<td>Crushed sandstone; (D_{50}=0.8 \text{ mm,} \quad D_{10}=5.0 \text{ mm,} \quad U_r=2.1, \quad G_r=2.74, \quad e_{\text{min}}=0.727, \quad e_{\text{max}}=0.363)</td>
<td>Air-dried</td>
<td>(e_r=0.584) and 0.556 ((\rho_c=1.760) and 1.770 g/cm(^3))</td>
<td>Drained TC at (\sigma'_r=40 \text{ kPa})</td>
<td>0.008–0.00008%/min</td>
<td>0.0244</td>
<td>Hirakawa (2003); Hirakawa et al. (2003)</td>
</tr>
<tr>
<td>Crushed concrete aggregate</td>
<td>(D_{50}=5.5–6.5, \quad U_r=19, \quad F_C=1.2–2.1%; \quad G_r=2.65, \quad (\rho_c=1.78 \text{ g/cm}^3))</td>
<td>Moist ((w=16.69, \quad 17.34%; \quad w_{opt}=16.9%))</td>
<td>(\rho_c=1.72, \quad 1.75 \text{ g/cm}^3)</td>
<td>Drained TC at (\sigma'_r=20 \text{ kPa})</td>
<td>0.001–0.1%/min</td>
<td>0.0536</td>
<td>Agil et al. (2005); Tatsuoka et al. (2006a)</td>
</tr>
<tr>
<td>Fujinomori clay(^3)</td>
<td>(D_{50}=0.017 \text{ mm,} \quad U_r=10, \quad P_I=33, \quad \text{LL}=62%)</td>
<td>Air-dried</td>
<td>(e_r=1.093)</td>
<td>Drained TC at (\sigma'_r=77 \text{ kPa})</td>
<td>0.002–0.2%/min</td>
<td>0.0444</td>
<td>Li et al. (2004); Tatsuoka et al. (2006a)</td>
</tr>
<tr>
<td>Kaolin(^3)</td>
<td>(D_{50}=0.0013 \text{ mm,} \quad U_r=4.32, \quad P_I=41.6, \quad \text{LL}=79.6%)</td>
<td>Air-dried</td>
<td>(e_r=1.38)</td>
<td>Drained TC at (\sigma'_r=100 \text{ kPa})</td>
<td>0.000046–0.0012%/min</td>
<td>0.029</td>
<td>Deng and Tatsuoka (2007); Tatsuoka et al. (2006a)</td>
</tr>
</tbody>
</table>

1) \(e_r\)=consolidated void ratio.
2) \(w_{as}\) and \(S_{as}\); measured after each test.
3) \(\beta\) value at oven-dried state was inferred based on the respective \(\beta\)-S relations.