MEASUREMENTS OF UNSATURATED HYDRAULIC CONDUCTIVITY FUNCTIONS OF TWO FINE-GRAINED MATERIALS

SAID TAIBI(i), KATIA VANESSA BICALHO(ii), CHAHIRA SAYAD-GAIDI(iii) and JEAN-MARIE FLEUREAU(iv)

ABSTRACT

Water unsaturated hydraulic conductivity ($k_w$) functions of two remolded fine-grained materials were measured over a wide range of degrees of saturation ($S$) with two methods. The instantaneous profile method (IPM) was used for $S_i > 50\%$. An original vapor equilibration method (also known as the vapor equilibrium technique, VET) was used for $S_i < 50\%$. Both materials compacted at the standard Proctor optimum water content and maximum density, have saturated hydraulic conductivities ($k_{sw}<10^{-7}$ m/s). The VET couples the total soil suction ($s$) control from desiccators with saturated salt solutions with water mass measurements from a digital laboratory balance. The $k_w$ measurements of the two techniques are consistent and complementary. The effect of hysteresis on the $k_w$ functions at higher $s$ values was also investigated. The experimental results suggest that the hysteretic effect on the $k_w$-$S$ and $k_w$-$s$ relationships cannot be neglected, and that the measured $k_w$ are significantly dependent on the initial $S_i$. The VET tests on the specimens that were initially dried give the lowest values of $k_w$ and the tests on the specimens that were initially saturated give the highest values of $k_w$. The relative hydraulic conductivity values are very small ($k_w < 2 \times 10^{-7}$) in this saturation range ($S_i < 50\%$) for the tested materials.

Key words: drying and wetting paths, instantaneous profile method, laboratory tests, unsaturated hydraulic conductivity, vapor equilibrium technique (IGC: D4/E7)

INTRODUCTION

Reliable measurements and predictions of $k_w$ functions for low permeability soils are essential for describing unsaturated flow in natural or compacted soils that are often used to construct liners and covers for waste containment. Although several laboratory experimental techniques are now available to determine the water and air hydraulic conductivity functions of unsaturated rigid and swelling soils, there are still very few experimental data in literature regarding the unsaturated hydraulic properties of low permeability soils ($k_s \leq 10^{-9}$ m/s) over a large $S_i$ range. In addition, a characteristic of $k_w$ that is significant when modeling the hydraulic behavior of earth barriers is the hysteresis of $k_w$ functions (Topp and Miller, 1966; Corey, 1957; Hillel, 1982; Fredlund and Xing, 1994; Meerdink et al., 1996; Fleureau and Taibi, 1995): depending on the hydraulic path (wetting or drying) followed, different functions can be obtained. As a result, the objectives of this paper are: (i) to show measurements of the $k_w$ functions in a large saturation range for low permeability materials by using two laboratory testing techniques; (ii) to evaluate whether hysteresis exists in the $k_w$-$S_i$ and $k_w$-$s$ relationships at high soil suction values, for the tested materials.

The objectives are attained by performing several laboratory tests and analyzing the experimental results. Two testing techniques are used for determining the $k_w$ functions over a wide $S_i$ range of a sandy-clay: the instantaneous profile method (IPM) for $S_i > 50\%$ and an original vapor equilibrium technique (VET) for $S_i < 50\%$. The VET couples the total soil suction control from desiccators with saturated salt solutions with water mass measurements from a digital laboratory balance (drying and wetting paths). The VET was also used to verify the effect of hysteresis on measured $k_w$ functions of two fine-grained materials (i.e., a sandy-clay mixture and a mud-cement mixture) under high total soil suctions.

The soil water retention curves, SWRC, (i.e., the relationship between the soil suction and its water content) of the tested materials were also determined by using these techniques. In the VET, total suctions are controlled by the salts whereas the IPM uses tensiometers that measure directly matric suctions. In most soils, osmotic suction generally represents only a small fraction of the total suction for water content changes normally encountered in most engineering problems involving unsaturated soils (Aitchison, 1961; Fredlund and Rahardjo,

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1993) and the matric and total suction can be considered as comparable for the tested sand-clay mixture. In the case of mud-cement samples, osmotic suction can be negligible compared to matric suction only at the very strong suctions used in the VET, but would certainly not be it in the field of weak suctions of the IPM, taking into account the pollutants likely to be in material. For simplicity reasons, in this paper, the word “suction” is used synonymously with soil matric suction. This paper describes the two laboratory testing techniques and analyzes the experimental results of the two tested materials.

**MATERIALS AND METHODS**

**Materials**

Two fine-grained remolded materials with different hydrodynamic properties were tested: a sandy-clay and a ‘mud-cement’ mixture. The sandy-clay consists of a mixture of 90% of kaolin, 5% of sand and 5% of silica based on dry weights. The liquid limit of the sand-clay is 38%, the plastic limit is 19%, clay fraction is 53%, Casagrande classification is CL, specific gravity is 2.63, Proctor optimum water content is about 18% and the corresponding dry density, \( \gamma_d \), normalized with the unit weight of the water at 4°C, \( \gamma_w \), (i.e., \( \gamma_d/\gamma_w \)) is 1.67. The mud-cement mixtures consist of mixtures of an organic mud and a CPA cement. The mud is classified as organic high plasticity silt based on the USCS. X-ray diffraction showed that the mud is predominantly made of quartz (30–40%), calcite (20–30%), feldspar (3–7%), dolomite (2–3%), and the clay fraction (about 10%) is mixed layers of smectite (20%), kaolinite (10%) and illite (7%). The organic matter content varies from 5 to 10%. The liquid limit is 118%, the plastic limit is 58%, and specific gravity of the solids is 2.5. Further details regarding the properties and mineralogical compositions of the ‘mud-cement’ mixture are provided by Boutouil (1998).

The saturated water hydraulic conductivity, \( k_{sat} \), of the materials, measured under steady state and unsteady state conditions by means of constant pressure head and pulse tests (Brace et al., 1968; Hsieh et al., 1981; Taibi et al., 2003) are about \( 1.5 \times 10^{-10} \text{ m/s} \) for the sandy-clay compacted at the standard Proctor optimum water content and \( 10^{-4} \text{ m/s} \) for the ‘mud-cement’ mixture, respectively.

**Water Unsaturated Hydraulic Conductivity \( (k_u) \) Functions**

Two laboratory experimental techniques were used for determining the \( k_u \) functions of fine-grained materials: IPM (infiltration in a vertical column of soil) and an original VET that couples suction control from desiccators with saturated salt solutions with water mass measurements from a digital laboratory balance. The IPM was used to determine \( k_u \) functions for the compacted sandy-clay in the \( S_r \) range of 50% to approximately 90% with corresponding total (or matric) soil suction range of 6 MPa to 30 kPa whereas the VET was used to determine \( k_u \) functions for low degrees of saturation \( (S_r < 50\%) \) with relatively high total soil suctions \( (4 \text{ MPa} < s < 345 \text{ MPa}) \).

**IPM (Infiltration in a Vertical Column of Soil)**

The IPM applied to different surface water flow conditions (evaporation and infiltration) has been used by many researchers (Wind, 1968; Vachaud et al., 1974; Hamilton et al., 1981; Daniel, 1983; Ed Diny, 1993; Tamari et al., 1993; Chiu and Shackelford, 1998; Gaidi, 2002; Arya 2002) to estimate the retention and conductivity curves for different soils in a large water content range. The IPM is an unsteady-state method. It uses a cylindrical specimen of soil that is subjected to a continuous water flow from one end of the specimen. The hydraulic head gradient and flow rate at various points along the specimen can be obtained using one of several procedures (Klute, 1972; Fredlund and Rahardjo, 1993). Using the first procedure, the water content and pore-water pressure head distributions can be measured independently. The water content distribution can be used to compute the flow rates. The pore-water pressure head gradient can be calculated from the measured pore-water pressure head distribution. The gravitational head gradient is obtained from the elevation difference. Using the second procedure, the water content distribution is measured while the pore-water pressure head is inferred from the soil-water characteristic curve.

Unsaturated hydraulic conductivity is derived from Darcy’s law using the pressure head gradient and mean flow between two measurement points. Morath et al. (1997) observed that the calculated hydraulic conductivity using the IPM is very sensitive to errors of calibration and position of the tensiometers in the column of soil, and to sample non-homogeneity. All the variations of the IPM are based on the same theoretical principles, and primarily differ in the method used to remove or add water and in the technique of suction or moisture measurements (Bicalho et al., 2007).

The IPM used is an instrumented vertical column of soil with HMS 9000 probes and tensiometers measuring, respectively, the resulting volumetric water content and pore water pressure in the soil column at several depths and time intervals under infiltration conditions imposed at the bottom of the soil column \( (\zeta = 0) \) and zero-flow condition imposed at the top of the soil column \( (\zeta = l) \). A schematic of the experimental equipment used is illustrated in Fig. 1. The equipment consists of an acrylic cylinder \( (220 \text{ mm in length and } 50 \text{ mm in inner diameter}) \) containing the compacted soil specimen, four sets of soil moisture sensors (HMS 9000 probes) placed along the soil specimen at different depths \( (15 \text{ mm from the bottom face (probe a), } 95 \text{ mm (probe b), } 155 \text{ mm (probe c), and } 195 \text{ mm (probe d)}) \) for the measurement of the volumetric water content at the given position and time, a set of tensiometers inserted in the soil specimen, connected to a pressure transducer and recorded on a personal com-
computer-based data acquisition system, and a water source for wetting the soil column with a digital water pressure-volume controller connected to one end of the specimen column (at \( z = 0 \)). The vertical column has small holes located at four depths in which the probe and tensiometers are inserted. Each set of tensiometers and probe is placed at the same elevation along the column so that the pore water pressure and volumetric water content recorded data can be directly correlated. The HMS 9000 probes are based on the principle of electrical capacity measurements and were calibrated for the tested sandy-clay soil. The tests were conducted on specimens of the sandy-clay compacted directly in the test column at the standard Proctor optimum water content and corresponding dry unit weight using a static compaction press.

**VET (Desiccators with Saturated Salt Solutions and a Digital Balance)**

For higher suctions and continuous air interface phase the transfer of water occurs in the vapor phase, and desiccators with saturated salt solutions are routinely used to determine the SWRC data for total suction values ranging from 3 to 400 MPa (Fleureau et al., 1993; Delage et al., 1998; Tang and Cui, 2005; Taibi et al., 2005). This relatively low cost laboratory technique consists in placing the soil specimens in a totally closed desiccator that contains a given saturated salt solution at the bottom. The saturated salt solutions are used to control the relative humidity of the atmosphere in the desiccators containing the soil specimens. The imposed total soil suction is related to the relative humidity \( H \) of the vapor space where the specimen is equilibrated by the following equation, which is derived from Kelvin’s law (Fredlund and Rahardjo, 1993):

\[
s = \frac{-RT}{M_w (1/\rho_w)} \ln (H)
\]

where \( R \) = universal gas constant [8.31432 J/mol K]; \( T \) = absolute temperature [K]; \( M_w \) = molecular weight of water [18.016 g/mol]; \( \rho_w \) = density of water [Mg/m³] as a function of \( T \); and \( H \) = measured relative humidity defined as \( u_r / u_{v0} \), where \( u_r \) = the partial pressure of pore water vapor in the soil specimen and \( u_{v0} \) = the saturation pressure of water vapor above a flat surface of water at temperature \( T \). The value of \( H \) depends on the temperature, type and concentration of the used salt solution (Tang and Cui, 2005). Therefore, the accuracy of this technique depends on the calibration used to determine the \( H \) generated by the used salt solution at the measured temperature. All the measurements were performed at constant temperature (about 20°C ± 2°C).

The laboratory experimental technique used for determining the \( k_w \) functions of low permeability soils couples the total soil suction control from desiccators with saturated salt solutions with water mass measurements from a digital balance (precision 10⁻³ g). The basic experimental setup consists of several desiccators with different saturated salt solutions at the bottom and a digital laboratory balance (Fig. 2). The suction difference (\( \Delta s \)) between the soil specimen (s) and that imposed by the relative humidity generated by the saturated salt solution in the desiccator (s₀) causes the water to flow in or out of the soil specimen until the potential equilibrium is reached. During the wetting or drying paths, the balance monitors the pore water gain (wetting path) or loss (drying path). The soil specimens are laterally sealed with paraffin for creating one-dimensional flow condition. During the unsteady...
state flow in the sample, one measures the variation of the average water content with time by weighing the soil specimen until the specimen’s final suction reaches equilibrium. At this time, the flow stops (hydraulic flow rate (v) is equal to zero).

The governing equation for one-dimensional flow, neglecting gravity, is:

\[ \frac{\partial \psi}{\partial t} = D \frac{\partial^2 \psi}{\partial z^2} \]  

(2)

where \(\psi\) = suction head (length); \(t\) = time, \(z\) = distance and \(D\) = diffusivity. In general, Eq. (2) is non-linear and difficult to solve analytically. However, if the increments of \(s\) are sufficiently small, it is possible to assume that \(D\) is constant and that the \(s-\theta\) relationship is linear between two steps, which makes it possible to linearize Eq. (2) and obtain its analytical solution. Considering the initial and boundary conditions as:

\[ \psi(z, 0) = \psi_0 \]  

(3)

\[ \psi(0, t) = \psi(t, l) = \psi_0 \]  

(4)

Using the method suggested by Gardner (1956), the solution of Eq. (2) is:

\[ \psi(z, t) = \psi_0 + \frac{4 \Delta \psi}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \left( \frac{n\pi}{l} z \right) \exp \left[ -\left( \frac{n^2\pi^2}{l^2} \right) Dt \right] \]  

(5)

\[ V_w(t) = V_{wt} \left[ 1 - \frac{8}{\pi^2} \exp \left( -\frac{\pi^2}{l^2} Dt \right) \right] \]  

(6)

\[ \ln \left[ \frac{V_{wt} - V_w(t)}{V_{wt}} \right] = \ln \left( \frac{8 V_{wt}}{\pi^2} \right) - \frac{n^2\pi^2}{l^2} Dt \]  

(7)

\[ D = \frac{\pi^2 \mu}{l^2} \]  

(8)

where \(V_w(t)\) = change in water volume at time \(t\), \(V_{wt}\) = total change in water volume, \(l\) = length of the soil specimen, and \(\mu\) = slope of the straight line in \([t, \ln(V_{wt} - V_w(t))]\) space.

The coefficient of diffusivity \(D\) is related to the hydraulic conductivity \(k_w\) by (Child and Collins, 1950; Klute, 1952): \(D = k_w (\Delta \psi/\partial \theta)\), with \(\psi\): the capillary potential and \(\theta\): volumetric water content, defined as the ratio of pore water volume \(V_w\) to the total volume of sample \(V_t\), \(\theta = (V_w/V_t)\). When the sample is placed in the desiccator, its capillary potential varies from \(\psi_1\) (initial capillary potential) to \(\psi_2\) (imposed capillary potential in the desiccator), then \(\Delta \psi = \psi_2 - \psi_1\), and its volumetric water content varies from \(\theta_1\) to \(\theta_2\), then \(\Delta \theta = \theta_2 - \theta_1\).

The chosen \(\Delta \psi\) must be small enough to meet the considered assumption \((\partial \psi/\partial \theta) = (\Delta \psi/\Delta \theta)\), but large enough to provide a measurable volume of flow. The mean diffusivity is then written as: \(D = k_w (\Delta \psi/\Delta \theta)\), or \(k_w = D (\Delta \psi/\Delta \theta)\).

The volume of pore water \(V_w\) is related to its mass \(W_w\) by \(V_w = (W_w/\rho_w)\), where \(\rho_w\) is the water density. The variation of the volumetric water content \(\Delta \theta\) can be related to the variation of the change in water mass of the sample \(W_{wt}\) by: \(\Delta \theta = (W_{wt}/\rho_w V_t)\) and the \(k_w-D\) relationship is defined by:

\[ k_w = \frac{D}{\Delta \psi} \frac{W_{wt}}{\rho_w V_t} \]  

(9)

This method assumes that \(k_w\) is constant during the flow process. The mass measurements throughout the test and the dry mass and total volume of the soil specimen measured at the end of the test are used to back-calculate the mean specimen volumetric water content at each step. The external total volume of the specimens (between 1 cm and 3 cm) was determined by immersing the specimens for 2 hours in a non-wetting oil (commercial Ker-dane, BP) to fill the largest pores without swelling and weighing them before and after imbition (Fleureau et al., 1993).

The \(k_w\) is determined for each increment of suction and the experiment is repeated for different magnitudes of suction or water content to yield a water hydraulic conductivity-suction \((k_w-s)\) relationship. On wetting path, the \(k_w-s\) relationship is obtained by proceeding through a series of steady states with smaller and smaller suctions. The method is also used for obtaining the drying \(k_w-s\) relationship (i.e., series of steady states with increasing suction). The same soil specimen was continuously moistened/dried so that there is no hysteresis associated with alternately wetting and drying a soil.

The time required to reach equilibrium for the soil specimen is obtained by taking the mass measurements throughout the test. The frequency of weighing varies from 1 per day at the beginning of the test, to 3 per day at the end. Equilibrium is reached when the relative variation of the weight of the sample \(\Delta m/m_0\) versus time (Figs. 6(a) and 7(a)) does not exceed 0.01% per day, \(m_0\) is the initial weight of the sample. Taibi et al. (2005) mention that the equilibrium time depends on the soil properties, the soil specimen size and the suction difference. In order to decrease the equilibrium time, the tests are carried out on very small soil specimens (10 mm in height and 35 mm in diameter).

The samples used in the VET for the measurement of \(k_w\) functions were first statically compacted in a mold at the standard Proctor optimum water content (or saturation \(S_{op}\)) and corresponding density. To evaluate whether hysteresis exists in the \(k_w-S_s\) and \(k_w-S_r\) relationships for the sandy-clay, three procedures were used to prepare the soil specimens: (i) for the drying path starting from the initial degree of saturation \(S_{op} = S_{op(1)}\), some of the specimens were used ‘as compacted’, (ii) for the wetting path starting from \(S_{op(1)} = 0\%), some of the compacted specimens were allowed to dry in the oven at 105°C ± 5°C for 24 hours, and (iii) for the drying path starting from \(S_{op(1)} = 100\%), some of the specimens were saturated by applying vacuum and flushing de-aired water into the specimen for a period of 120 minutes. VET tests were also conducted using specimens of ‘mud-cement’ mixtures, either saturated (for the drying path starting from \(S_{op(1)} = 100\%)\) or dried in the oven at 105°C ± 5°C for 24 hours (for the wetting path starting from \(S_{op(1)} = 0\%).

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RESULTS AND DISCUSSION

Measured Unsaturated Hydraulic Conductivity

IPM (Infiltration in a Vertical Column of Soil) Results

Examples of the resulting volumetric water content and pore water pressure profiles at various time intervals for the sandy-clay due to a pressure of 20 kPa applied at the bottom of the soil column are presented in Figs. 3(a) and (b), respectively.

Plotting the measured infiltration velocity against time shows that the steady state ($v=3 \times 10^{-4}$ cm/s) was reached after about 6 days for the tested clay. The soil specimen's height was measured during the tests by using a dial gauge and a negligible change in the height of the sample was observed (i.e., $e = \Delta h/h < 1\%$).

Figures 4(a) and (b) present the measured $k_w$-$S_r$ and $k_w$-s relationships, respectively, for the tested sandy-clay (wetting path, $S_o=S_p$). The measured $k_w$ values range from $6 \times 10^{-11}$ m/s to $10^{-14}$ m/s for the tested sandy-clay. The corresponding s values change from 3 kPa to 16 MPa. The tensiometers used in the IPM were designed to measure matric suctions ranging from 3 kPa to approximately 90 kPa. However, problems were met with these devices and eventually suctions were derived from the SWRC, using the correspondence between water contents and suctions. The data of the SWRC of the sandy-clay were determined independently using different methods (i.e., tensiometric plates (1–20 kPa), osmotic technique (0.05–1.5 MPa) and vapor equilibrium technique with saturated salt solutions (4–300 MPa)) to control the soil suction (Fig. 5). Details of these methods are provided by Fleureau et al. (1993). The SWRC data for the sandy-clay obtained from the VET tests discussed in this paper are also presented in Fig. 5. The experimental results derived from these different techniques are consistent and complementary.
The IPM technique cannot adequately capture conditions close to or at saturation, therefore, the saturated hydraulic conductivity \( k_{\text{sat}} = 1.5 \times 10^{-10} \text{ m/s} \) was independently measured under steady state and unsteady state conditions by means of constant pressure head and pulse tests.

VET (Desiccators with Saturated Salt Solutions and a Digital Balance) Results

Some examples of the variation of the water mass, water volume and the resulting \( \ln (V_{\text{wT}} - V_{\text{w}}(t)) \) against time for the tested ‘mud-cement’ (drying path) and sandy-clay (wetting path) are presented in Figs. 6 and 7, respectively. The initial \( (S_{r0}, s_0) \) and final \( (S_{rf}, s_f) \) conditions for each stage of imposed suction are also presented in the figures. The values of suction are determined by using the VET and the corresponding saturation values by using a non-wetting oil (commercial Kerdane, BP) to determine the external total volume of the specimens (Fleureau et al., 1993). Equilibrium time was reached after about 12 days for the small tested ‘mud-cement’ specimens (10 mm in height and 35 mm in diameter) and the used salt solutions (Fig. 6). In the case of the sandy-clay (wetting tests), the
For each soil specimen, the hydraulic conductivity is analytically determined from the graph of $\ln (V_{wT} - V_w(t))$ versus time. Examples of the resulting plot for the drying path of the saturated 'mud-cement' mixture and wetting path are shown in Fig. 8.
path of the sandy-clay are presented in Figs. 6(c) and 7(c), respectively. The experimental results presented in Figs. 6(c), and 7(c) confirm the assumed linearity of the ln \((V_w - V_d)\)-time relationship (coefficient of determination \(R^2\) > 0.87) for the tested materials.

The measured \(k_w\) values (drying and wetting paths starting from different initial degrees of saturation, \(S_{r0}\)) for the mud-cement mixture and the compacted sandy-clay are plotted as a function of saturation and suction in Figs. 8(a), 9(a), and 8(b), 9(b), respectively. Taking into account the experimental conditions, hydraulic conductivity was given with an uncertainty of about 10\%. The experimental results suggest that the hysteretic effect on the \(k_w-S_r\) and \(k_w-s\) relationships cannot be neglected in this saturation range (\(S_r < 50\%\)), and that the \(k_w-S_r\) relationship appears to exhibit less hysteresis than the \(k_w-S_r\) relationship at relatively high suctions (4 MPa \(\leq s \leq 330\) MPa).

Figure 10 presents a dimensionless plot of the logarithm of the relative hydraulic conductivity, \(k_{rw} = k_w/k_{sat}\) versus \(S_r\) for the IPM and VET tests conducted on the sandy-clay mixture (C) soil and the VET tests conducted on the "mud-cement" mixture (M). Figure 10 shows that the values of \(k_w\) measured by these tests are significantly dependent on the initial degree of saturation of the specimens. The VET tests (wetting path) on the specimens which were initially dried (\(S_{r0} = 0\%\)) give the lowest values of \(k_w\) and the VET tests (drying path) on the specimens which were initially saturated (\(S_{r0} = 100\%\)) give the highest values of \(k_w\). The trend observed in the measured \(k_w\) values is:

\(k_w\) [for tests on initially dry samples] < \(k_w\) [for tests on samples at the initial optimum water content] < \(k_w\) [for tests on initially saturated samples].

The IPM test results on the sandy-clay (C), following the wetting path starting from the initial degrees of saturation corresponding to the standard Proctor optimum water content (\(S_{o0} = S_{op}\)), indicate that \(k_{rw}\) has a small value (\(k_{rw} \approx 0.05\)) while the saturation is relatively high (\(S_r > 0.8\)). These results are consistent with the measured \(k_w\) values presented in previous studies (Taibi, 1994; Bicalho, 1999) for fine-grained soils. The VET test results of the sandy-clay mixture (C) and the mud-cement mixture (M) indicate that \(k_{rw} < 2 \times 10^{-3}\) m/s for \(S_r < 50\%\).

The predicted results for the tested clay obtained by an empirical expression proposed originally by Bicalho et al. (2000) for defining \(k_{rw-S}\) relationship at higher \(S_r\) on the wetting cycle of a compacted silt is also presented in Fig. 10. The expression was determined by curve fitting to the experimental results and is given by:
Table 1. Summary of the specimen preparation for the hydraulic conductivity tests (initial values)

<table>
<thead>
<tr>
<th>Test methods</th>
<th>Material</th>
<th>Initial state</th>
<th>Number of specimens</th>
<th>Diameter/Height (mm)</th>
<th>Mean dry density</th>
<th>Mean initial degree of saturation $S_r$ (%)</th>
<th>Mean initial water content $w_0$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.P.M</td>
<td>Sandy-clay</td>
<td>CSPOW (*)</td>
<td>01 (a-1)</td>
<td>50/220</td>
<td>1.67</td>
<td>72</td>
<td>16</td>
</tr>
<tr>
<td>V.E.T</td>
<td>Sandy-clay</td>
<td>CSPOW (*)</td>
<td>01 (b-1)</td>
<td>35/10</td>
<td>1.67</td>
<td>81</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>CSPOW (*) and dried in the oven at 105/110°C</td>
<td>02 (c-1, c-2)</td>
<td>35/10</td>
<td>1.6</td>
<td>100</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mud-cement</td>
<td>compacted then dried in the oven at 105/110°C</td>
<td>01 (d-1)</td>
<td>35/10</td>
<td>1.1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>compacted and saturated</td>
<td>01 (f-1)</td>
<td>35/10</td>
<td>1.1</td>
<td>100</td>
<td>58</td>
</tr>
</tbody>
</table>

(*)CSPOW = Compacted at the standard Proctor optimum water content

Table 2. The used saturated salt solutions: (1) $K_2SO_4$; (2) $CaSO_4$; (3) $ZnSO_4 \cdot 7H_2O$; (4) $KCl$; (5) $NH_4Cl$; (6) $NaCl$; (7) $NaNO_2$; (8) $Ca(NO_3)_2 \cdot 4H_2O$; (9) $KCN$; (10) $CaCl_2 \cdot 6H_2O$; (11) $H_2SO_4$

$$k_{rw}(S_r) = a + (1-a) \left[ \frac{(S_r - b)}{(1-b)} \right]^n$$  \hspace{1cm} (10)

where $a$, $b$ and $n$ are empirical parameters. The obtained empirical parameters for the $k_{rw}$-S function for the sandy-clay are: $a = 0.00000006$, $b = 0.001$ and $n = 11$. The results of the experimental investigation indicate that the expression proposed by Bicalho et al. (2000) is in reasonable agreement with the values of measured $k_{uw}$ derived from both the IPM and VET tests (drying and wetting paths starting from the same initial degree of saturation, $S_{r0}$) for the tested sandy-clay.

CONCLUSIONS

Very small $k_w$ values of two fine-grained materials (i.e., a sandy-clay mixture, and a ‘mud-cement’ mixture) were measured using two laboratory testing techniques: (i) VET (vapor equilibrium technique) that couples the total soil suction control from desiccators with saturated salt solutions with water mass measurements from a digital laboratory balance (on drying and wetting paths); the VET was used for measuring $k_w$ values under very high suctions $\leq 4$ MPA. (ii) IPM (infiltration in a vertical column of soil) used for measuring $k_w$ values at suction $S_r > 50$% for the sandy-clay mixture. The $k_w$ results of the two techniques are consistent and complementary in terms of suction ranges. The measured $k_w$ values range from $10^{-9}$ to $10^{-17}$ m/s. The VET is able to measure very small $k_w$ values ($10^{-15} < k_w < 10^{-17}$ m/s). The IPM test results on the sandy-clay (C) following the wetting path starting from $S_{r0} = S_{r0}$ indicate that $k_w$ has a small value ($k_w = 0.05$) while the $S_r$ is relatively high ($S_r = 0.8$), and the VET test results on the sandy-clay (C) and the mud-cement mixture show that $k_w$ values are very small ($k_w < 2 \times 10^{-3}$) at $S_r < 50\%$. 

$$
\text{(a) Mud-cement, (b) Sandy clay}
$$
The VET on drying and wetting paths is used to evaluate the effect of hysteresis on the \( k_w \) functions under high suction values for the tested materials. This study shows that the measured \( k_w \) (VET tests) of the two materials is significantly dependent on the initial \( S \). The VET tests (wetting path) on initially dried specimens give the lowest values of \( k_w \) and the tests (drying path) on the initially saturated specimens give the highest values of \( k_w \). The experimental results suggest that the \( k_w-S \) relationship exhibits less hysteresis than the \( k_w-S \) relationship at relatively high suctions (4 MPa \(< s < 330 \) MPa) where the corresponding degrees of saturation are \( S_s < 50\% \) for the tested material. However, the effect of hysteresis on the \( k_w-S \) and \( k_w-s \) relationships in a large \( S \) range for low permeability soils is still a matter of debate.

NOTATION

IPM = instantaneous profile method;  
VET = vapor equilibrium technique;  
SWRC = soil water retention curves;  
\( C \) = sandy-clay mixture;  
\( M \) = “mud-cement” mixture;  
\( k_w \) = water unsaturated hydraulic conductivity;  
\( k_s \) = relative water hydraulic conductivity;  
\( k_w \) = saturated water hydraulic conductivity;  
\( a \), \( b \), and \( n \) = Bicalho et al. (2000) empirical parameters for \( k_r (S) \) function;  
\( S_r \) = degree of saturation;  
\( S_s \) = initial \( S \);  
\( S_o \) = \( S \) at optimum water content;  
\( \varepsilon \) = \( \Delta l/l \), axial strain;  
\( \Delta l \) = soil specimen height change;  
\( \ell_0 \) = initial soil specimen height;  
\( s \) = soil suction;  
\( s_o \) = soil suction at optimum water content;  
\( H \) = relative humidity;  
\( R \) = the universal gas constant (i.e., 8.31432 J/mol K);  
\( T \) = the absolute measured temperature in K;  
\( M_w \) = the molecular weight of water (i.e., 18.016 g/mol);  
\( \rho_w \) = density of water in kg/m\(^3\) as a function of \( T \);  
\( \psi \) = volumetric head [length];  
\( \theta \) = volumetric water content, it represents the fraction of the total volume of soil that is occupied by the water contained in the soil;  
\( \theta = (V_w/V) \)

REFERENCES


