GEOTECHNICAL PARAMETERS FROM V_P AND V_S MEASUREMENTS IN UNSATURATED SOILS

ENRICO CONTEⁱ⁾, RENATO MARIA COSENTINIⁱⁱ⁾ and ANTONELLO TRONCONEⁱⁱⁱ⁾

ABSTRACT

This paper presents a simple procedure for evaluating the coefficients of volume change of unsaturated soils from measurements of the shear and dilatational wave velocities. The procedure is based on some analytical expressions that have been derived in the present study using the theory of linear elasticity for low-frequency excitations. The soil parameters required as input are the porosity and degree of saturation of the soil, Poisson's ratio of the soil skeleton, and three physical parameters that generally assume constant values. The results of some applications of the proposed method to a set of experimental data published in the literature are also shown.

Key words: dynamic, geophysical explorations, partially saturated soils, pore pressure, wave propagation (IGC: C8/D7/E8)

INTRODUCTION

In the last few decades, geotechnical and earthquake engineering applications have stimulated the development of more and more reliable geophysical tests for the mechanical characterization of the soils in-situ and/or in the laboratory (Stokoe and Santamarina, 2000; Valle-Molina, 2006). Their use is for measuring shear and dilatational wave velocities, V_S and V_P , with the objective of relating them to some physical or constitutive parameters of the soil taking advantage of appropriate theoretical formulations or empirical relationships. Owing to the fact that in these tests the soil is subjected to small perturbations, the material parameters obtained in this way are commonly referred to the undisturbed state of the soil. In addition, linear elasticity is the most frequently used constitutive assumption to model the mechanical behaviour of the soil. In this context, for a mono-phase medium, V_S and V_P are related to the small-strain shear and constrained moduli of the medium, respectively (Richart et al., 1970).

For two-phase media, which is the case of saturated soils, the theory generally used to analyse wave propagation in geomaterials subjected to small amplitude excitations, is that originally developed by Biot (1956). Based on this theory, Foti et al. (2002) proposed a procedure for estimating soil porosity from measured shear and dilatational wave velocities. This procedure is very useful from an engineering viewpoint, especially in the case of coarse-grained soils in which in-situ undisturbed sampling can be only achieved using sophisticated techniques (Hofman et

al., 2000).

Wave propagation in unsaturated soils is obviously much more difficult to deal with due to the complex interactions between the solid skeleton and the fluids (air and water) filling the pores. A simplification is usually made when the soil is very close to saturation. In such circumstances, in fact, air contained in the pores is present in the form of occluded bubbles that can be considered as one with pore water. In this connection, an accepted assumption is to retain the formal structure of Biot's theory, while introducing an appropriate bulk modulus for the water-air mixture (Ishihara, 1970; Kokusho, 2000; Tamura et al., 2002). Brutsaert (1964) was perhaps the first to develop a model for describing wave propagation through porous media saturated by two fluids. Further contributions were later provided by Gray (1983), Garg and Nayfeh (1986), Berryman et al. (1988), Tuncay and Corapcioglu (1997), Wei and Muraleetharan (2002), and Lo et al. (2005). While all these studies are highly valuable from a theoretical point of view, they have not led to simple-to-use expressions for the wave velocities that are of practical interest. In addition, some material parameters involved in these expressions are difficult to determine experimentally. This limits the use of these models for engineering purposes.

Recently, Conte et al. (2009) presented a simplified formulation that is based on the theory of linear poroelasticity in conjunction with the constitutive relationships originally proposed by Fredlund and Morgenstern (1976) to describe the volume changes undergone by unsaturated soils owing to changes in total stress and suction. The ex-

ⁱ⁾ Associate Professor, Dipartimento di Difesa del Suolo, Università della Calabria, Italy (conte@dds.unical.it).

ⁱⁱ⁾ Research Assistant, ditto.

ⁱⁱⁱ⁾ Assistant Professor, ditto.

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pressions of V_s and V_p obtained applying this formulation to the case of low-frequency excitations, which is the most frequently encountered condition in the geophysical tests (Santamarina et al., 2001), are used in the present study to develop a procedure for evaluating the volumetric deformation coefficients of unsaturated soils at their undisturbed or natural state, once the measured values of P-wave and S-wave velocities, porosity and degree of saturation of the soil are available. The results of some applications of the method to a set of experimental data published in the literature are also shown.

P-WAVE AND S-WAVE VELOCITIES FOR LOW-FREQUENCY EXCITATIONS

The stress-strain equations adopted in the present study to determine the wave velocities for unsaturated soils subjected to small amplitude excitations are:

$$\begin{aligned} \sigma = & G(\text{grad } \mathbf{u}^s + \text{grad}^T \mathbf{u}^s) + H\mathbf{I} \text{div } \mathbf{u}^s \\ & + n^w[\chi L + (1 - \chi)C]\mathbf{I} \text{div } \mathbf{u}^w \\ & + n^a[\chi C + (1 - \chi)N]\mathbf{I} \text{div } \mathbf{u}^a \end{aligned} \quad (1)$$

$$p_w = W \text{div } \mathbf{u}^s - n^w L \text{div } \mathbf{u}^w - n^a C \text{div } \mathbf{u}^a \quad (2)$$

$$p_a = M \text{div } \mathbf{u}^s - n^w C \text{div } \mathbf{u}^w - n^a N \text{div } \mathbf{u}^a \quad (3)$$

where σ is the total stress tensor, p_w and p_a are the pore-water and pore-air pressures caused by the excitation, $\mathbf{u}^s$, $\mathbf{u}^w$ and $\mathbf{u}^a$ are the displacement fields for the solid, water and air phases, respectively, G is the shear modulus of the soil skeleton, χ is a material parameter the expression of which is provided by Eq. (6), n^w is the volume fraction of water and n^a is that of air. These latter are related to the porosity, n , and the degree of saturation of the soil, S_r , by the equations:

$$n^w = nS_r \quad (4a)$$

$$n^a = n(1 - S_r) \quad (4b)$$

In Eqs. (1) to (3), the sign convention of continuum mechanics is used according to which a dilatant volumetric strain is positive and stresses are positive in tension. Moreover, $\mathbf{I}$ is the identity tensor, the symbol grad stands for the spatial gradient, div indicates the divergence operator and the superscript T denotes the transpose. The other parameters appearing in Eqs. (1)–(3) take the following form:

$$H = 2a_s G - \chi W - (1 - \chi)M \quad (4c)$$

$$W = -\frac{K^w A}{D} + n^w L + n^a C \quad (4d)$$

$$M = -\frac{K^a B}{D} + n^w C + n^a N \quad (4e)$$

$$L = \frac{K^w[n^a m_1^s + K^a(m_1^s m_2^w - m_2^s m_1^w)]}{D} \quad (4f)$$

$$N = \frac{K^a[n^w m_1^s + K^w(m_1^s m_2^w - m_2^s m_1^w)]}{D} \quad (4g)$$

$$C = \frac{K^w K^a (m_1^s m_2^w - m_2^s m_1^w)}{D} \quad (4h)$$

$$A = n^a m_1^w + K^a (m_1^s m_2^w - m_2^s m_1^w) \quad (4i)$$

$$B = n^w m_1^a + K^w (m_1^s m_2^w - m_2^s m_1^w) \quad (4j)$$

$$D = (m_1^s m_2^w - m_2^s m_1^w)(n^a K^w + n^w K^a) + n^a n^w m_1^s \quad (4k)$$

$$a_s = \frac{\nu}{1 - 2\nu} \quad (4l)$$

in which K^w and K^a are the bulk modulus of water and air, respectively, and ν is Poisson's ratio of the soil skeleton. Following Fredlund and Morgenstern (1976), m_1^s and m_2^s are the coefficients of volume change of the soil structure with respect to a change in the mean net total stress, $(\sigma_m + p_a)$, and in matric suction, $(p_a - p_w)$ respectively, which are the stress state variables usually considered to describe the behaviour of unsaturated soils, and σ_m is the mean total stress (i.e., $\sigma_m = (\sigma_x + \sigma_y + \sigma_z)/3$, where σ_x , σ_y and σ_z are the total normal stresses in x , y and z directions). Similarly, m_1^w and m_2^w are the coefficients of volume change for the water phase, and m_1^a and m_2^a are those for the air phase. Continuity requirement leads to the following equations relating these coefficients:

$$m_1^s = m_1^w + m_1^a \quad (5a)$$

$$m_2^s = m_2^w + m_2^a \quad (5b)$$

As pointed out by Fredlund and Rahardjo (1993), the volumetric deformation coefficients reduce to that of the soil structure, m_v , when the pores are completely filled with water (saturated soil). On the other hand, when the soil approaches a dry condition, the coefficients of water volume change go to zero. Moreover, the following identities hold (Conte et al., 2009):

$$\chi = m_2^s / m_1^s \quad (6)$$

$$m_1^w = m_2^s \quad (7)$$

Equations (1) to (3) were derived under the assumptions of isotropic linearly elastic behaviour of the soil skeleton, infinitesimal strains, isothermal conditions, solid particle incompressibility and validity of the constitutive relationships first proposed by Fredlund and Morgenstern (1976). The basic equations that were considered to derive these equations are presented in APPENDIX I.

P-wave and S-wave velocities can be obtained from the differential equations governing wave propagation in three-phase porous media. These equations are the motion equation for the soil and those for the fluid phases. They can be obtained as an extension of those presented by Coussy (2004) for a porous medium saturated by one fluid. By neglecting the body forces and making use of Darcy's law for describing the flow of air and water through the soil, the governing equations are:

$$\text{div } \sigma - \rho_s(1 - n)\ddot{\mathbf{u}}^s - \rho_w n^w \ddot{\mathbf{u}}^w - \rho_a n^a \ddot{\mathbf{u}}^a = 0 \quad (8)$$

$$n^w(\ddot{\mathbf{u}}^w - \dot{\mathbf{u}}^s) = -\frac{k_w}{\rho_w g}(\text{grad } p_w + \rho_w \ddot{\mathbf{u}}^w) \quad (9)$$

$$n^a(\ddot{\mathbf{u}}^a - \ddot{\mathbf{u}}^s) = -\frac{k_a}{\rho_a g}(\text{grad } p_a + \rho_a \ddot{\mathbf{u}}^a) \quad (10)$$

where ρ_s , ρ_w and ρ_a stand for the mass densities of the soil particles, water and air, respectively, g is the gravity acceleration, and k_w and k_a are the coefficient of permeability for the water and air phases, respectively. The dot notation is used in these equations to denote the time derivatives.

Considering the permeability characteristics of the soils and the frequency content involved in most geophysical tests (Santamarina et al., 2001), it is usually assumed that any excitation falls in the low frequency range, where the effects of the relative motion between the solid and the fluid phases may be disregarded (Biot, 1956). In this connection, Miura et al. (2001) conducted an interesting study on the frequency-dependent characteristics of the waves in various saturated soils from clay to soft rocks. The study highlighted that the coefficient of permeability is the main soil parameter controlling the wideness of the low frequency range. In particular, the smaller the coefficient of permeability, the wider is this range. As a consequence, the above assumption should be even more accepted for unsaturated soils than for saturated soils owing to the fact that the former soils are generally characterized by much smaller values of the coefficient of permeability than the latter ones. From a mathematical viewpoint, this is equivalent to imposing $\mathbf{u}^a = \mathbf{u}^w = \mathbf{u}^s = \mathbf{u}$, where $\mathbf{u}$ denotes the displacement field of the medium. Thus, the governing Eqs. (8) to (10) reduce to:

$$\text{div } \boldsymbol{\sigma} - \rho \ddot{\mathbf{u}} = 0 \quad (11)$$

with $\rho = (1-n)\rho_s + S_r n \rho_w + (1-S_r)n\rho_a$. Moreover, owing to the constitutive Eqs. (1) to (3), Eq. (11) takes the form:

$$\left[2G(1+a_s) + \chi \frac{K^w A}{D} + (1-\chi) \frac{K^a B}{D} \right] \text{grad div } \mathbf{u} - G(\text{grad div } \mathbf{u} - \nabla^2 \mathbf{u}) - \rho \ddot{\mathbf{u}} = 0 \quad (12)$$

where ∇^2 is the Laplacian operator. Lastly, by applying Helmholtz's decomposition to Eq. (12) leads to two uncoupled equations that govern the propagation of S-wave and P-wave in three-phase media under undrained conditions (Cosentini, 2006). The propagation velocities of these waves are denoted by V_s and V_p , respectively, and are provided by the following equations:

$$V_s^2 = \frac{G}{\rho} \quad (13)$$

$$V_p^2 = \frac{2G(1-\nu)/(1-2\nu) + C_1/C_2}{\rho} \quad (14)$$

where

$$C_1 = K^w K^a (-\chi^2 + m_2^w/m_1^s) + n[\chi^2 K^w (1-S_r) + (1-\chi)^2 K^a S_r]/m_1^s \quad (15a)$$

$$C_2 = n[S_r K^a + (1-S_r)K^w](-\chi^2 + m_2^w/m_1^s) + n^2 S_r (1-S_r)/m_1^s \quad (15b)$$

Equation (14) can be cast in a more convenient form from an engineering viewpoint by introducing some additional simplifications. For this purpose, the parameter χ is replaced by S_r as an approximation that was already considered in several studies (Bishop, 1959; Bishop and Blight, 1963; Jommi, 2000; Wei and Muraleetharan, 2002; Gallipoli et al., 2003). Moreover, under the reasonable assumptions that the air mass density may be ignored ($\rho_a \approx 0$) and the bulk modulus of air is negligible with respect to that of water ($K^a/K^w \approx 0$), and taking advantage of the following identity (Fredlund and Rahardjo, 1993):

$$m_1^s = \frac{3(1-2\nu)}{2(1+\nu)G} \quad (16)$$

the expression for V_p^2 simplifies as follows:

$$V_p^2 = \frac{\frac{2(1-\nu)}{1-2\nu} G + \frac{S_r^2}{m_2^w - \frac{3(1-2\nu)S_r^2}{2(1+\nu)G} + \frac{nS_r}{K^w}}}{(1-n)\rho_s + S_r n \rho_w} \quad (17)$$

where G is provided by Eq. (13) as a function of V_s , i.e.:

$$G = [(1-n)\rho_s + S_r n \rho_w] V_s^2 \quad (18)$$

In these latter equations, ρ_w , ρ_s and K^w may be considered as physical constants that take standard values, and V_p and V_s can be measured using routine geophysical tests that are performed in-situ or in the laboratory. As a result, Eq. (17) represents a relationship in which the unknown variables are the porosity n , Poisson's ratio ν , degree of saturation S_r , and the coefficient of volume change m_2^w . This latter is defined as the slope of the soil-water characteristic curve at a given suction, i.e., $m_2^w = \partial n^w / \partial (p_a - p_w)$ (Fredlund and Rahardjo, 1993).

It is worth noting that for dry soils ($S_r = 0$) or completely saturated soils ($S_r = 1$), Eqs. (13) and (17) yield the well-known expressions for V_s and V_p derived from the linear poroelasticity theory. They are:

$$V_s^2 = \frac{G}{(1-n)\rho_s} \quad (19a)$$

$$V_p^2 = \frac{2G(1-\nu)/(1-2\nu)}{(1-n)\rho_s} \quad (19b)$$

for dry soils (Richart et al., 1970), and

$$V_s^2 = \frac{G}{(1-n)\rho_s + n\rho_w} \quad (20a)$$

$$V_p^2 = \frac{2G(1-\nu)/(1-2\nu) + K^w/n}{(1-n)\rho_s + n\rho_w} \quad (20b)$$

for saturated soils. In particular, these latter equations coincide with those proposed by Foti et al. (2002) for evaluating the porosity of saturated soils from V_s and V_p measurements, when the soil particles are assumed to be incompressible (Eqs. (6a) and (17) in their paper).

Table 1. Index properties of the soils tested by Inci et al. (2003)

Soil Property	Soil 1	Soil 2	Soil 3
Particle size (% by weight)			
Sand	56	3	2
Silt	27	38	29
Clay	17	59	69
Atterberg limits			
LL (%)	16	40	81
IP (%)	7	17	55
Specific gravity			
	2.68	2.68	2.69

COMPARISON WITH EXPERIMENTAL RESULTS

The results from an experimental study conducted by Inci et al. (2003) are here considered to validate the expressions for V_P and V_S derived in the previous section. The authors are not aware of other existing studies containing all the data necessary for allowing this validation to be performed. Inci et al. (2003) employed an ultrasonic pulse transmission apparatus to measure V_P and V_S of unsaturated soil samples prepared in the laboratory. The soil samples were compacted at wet of optimum, optimum and dry of optimum water content according to the standard or modified Proctor procedure, and subsequently were allowed to dry in order to vary the degree of saturation. The tests were carried out on cylindrical and cubic samples of three different soils. Soils 1 and 2 were natural soils, whereas Soil 3 was a mixture of Soil 2 with 25% bentonite by weight. The main index properties of these soils are summarized in Table 1. As can be noted, Soil 1 is a clayey silty sand with low plasticity. Soil 2 and Soil 3 can be defined as a silty clay of medium and high plasticity, respectively. Wave velocity measurements were performed by transducers for P and S waves, the maximum frequency of which was 50 and 100 kHz, respectively. The lateral dimension of the samples was always sufficiently higher than the wavelength, so that the effects of the sample boundary on the wave propagation velocities can be neglected (Santamarina et al., 2001). No total confining stress was applied to the samples during the geophysical tests. More details of the soil properties and the performed tests can be found in the paper by Inci et al. (2003). By summarizing, for each soil sample, the authors provided the measured P-wave and S-wave velocities and the values of porosity and degree of saturation.

Unfortunately, no soil-water characteristic curve was experimentally obtained by Inci et al. (2003) for the soils considered. Therefore, the real values of m_2^w are not available. Nevertheless, in a related study, Inci et al. (2003) measured the total suction, s , in samples of the same soils as those described above, and related s to the gravimetric water content, w , of each sample. The authors also presented some analytical curves to fit the experimental data (Fig. 3 of their paper). Generally, these curves are defined for a narrow range of suction and are character-

ized by low values of the correlation coefficient. Their analytical form is:

$$s = -\alpha \ln(w) + \beta \quad (21)$$

where α and β are fitting parameters depending on the soil type and the condition under which the samples were prepared. In the present study, these fitting curves were used for evaluating approximately m_2^w . Specifically, Eq. (21) was first applied for obtaining an average value of the total suction for each sample as a function of the gravimetric water content. This latter parameter was determined using the well-known phase relationships that are based on the porosity, degree of saturation and specific gravity. After inverting Eq. (21) and performing its derivative, the coefficient of compressibility, b_m , was evaluated at the above-mentioned total suction under the assumption that the osmotic suction is constant, that is $b_m = \partial w / \partial s = \partial w / \partial (p_a - p_w)$. The values of b_m obtained in this way were then turned into those of the corresponding deformation coefficient in terms of volumetric water content, m_2^w , using the relationship suggested by Fredlund and Rahardjo (1993), which involves the specific gravity and porosity of the soil.

As it will be explained in the next section, the value of Poisson's ratio considered in the calculations was assumed as $\nu = 0.5\nu_u$, where ν_u is provided by the following equation (Richart et al., 1970):

$$\nu_u = \frac{0.5V_p^2 - V_s^2}{V_p^2 - V_s^2} \quad (22)$$

as a function of the measured P-wave and S-wave velocities. Lastly, the water properties ρ_w and K^w were assumed to be equal to 1 t/m³ and 2.25 GPa respectively, and ρ_s was determined using the specific gravity values reported in Table 1.

A comparison between the values of V_P as measured by Inci et al. (2003) with those predicted using Eq. (17) is shown in Fig. 1. It is relevant to observe that no comparison between predicted and measured V_S is presented. This is because the values of G are not available for the tested soils (usually, G is just obtained from V_S measurements). However, the measured values of V_S were used in conjunction with those of n , S_r and the above-specified soil parameters m_2^w and ν , to calculate V_P by Eqs. (17) and (18). As can be seen, the proposed equation generally underestimates the experimental values of V_P . Nevertheless, the agreement between calculated and measured results can be considered fairly good for most of them, especially if one takes into account the fact that the values of m_2^w employed in the calculations were roughly evaluated.

EVALUATING m_2^w FROM V_P AND V_S MEASUREMENTS

The derived expressions of V_S and V_P (Eqs. (13) and (17)) are employed in this section for evaluating the coefficient of volume change m_2^w of unsaturated soils. To

Fig. 1. Measured P-wave velocity against predicted P-wave velocity using Eq. (17) for the soils tested by Inci et al. (2003)

this purpose, after substituting Eq. (18) into Eq. (17), this latter is inverted to obtain m_2^w as a function of n , S_r , v , V_s and V_p . The resulting expression is:

$$m_2^w = \frac{(1-2v)S_r^2}{[(1-n)\rho_s + S_r n \rho_w]} \times \left[\frac{3}{2(1+v)V_s^2} + \frac{1}{(1-2v)V_p^2 - 2(1-v)V_s^2} \right] - \frac{nS_r}{K^w} \quad (23)$$

Fig. 2. Influence of Poisson's ratio on the coefficient of volume change m_2^w

Results from this equation are plotted in Fig. 2 versus Poisson's ratio of the soil skeleton for three different soil samples, whose values of V_p and V_s as well as those of the porosity and degree of saturation were measured by Inci et al. (2003). The parameters ρ_s , ρ_w and K^w were assumed to be equal to 2.68 t/m^3 , 1 t/m^3 and 2.25 GPa , respectively. For each soil sample, the value of v was considered to vary from zero to the value provided by Eq. (22) which corresponds to Poisson's ratio of a mono-phase medium, without making a distinction among the constituent phases. For v that is greater than v_u , Eq. (23) leads to inconsistent values of m_2^w . Figure 2 shows that m_2^w changes slightly with v when this latter parameter is sufficiently smaller than v_u . On the other hand, m_2^w increases rapidly as v approaches v_u , and is infinitely high at $v = v_u$. Therefore, v_u as calculated by Eq. (22) can be considered as an upper limit for the values of Poisson's ratio of the soil skeleton. Similar considerations can also be made for the saturated soils for which the value of v_u is close to 0.5 (Foti et al., 2001). In conclusion, the results of Fig. 2 indicate that v may be selected in a narrow range from zero to a sufficiently lower value than v_u , without yielding substantial variations in the calculated values of m_2^w . Generally, the smaller the degree of saturation of the soil, the narrower is this range. As a consequence, the assumption of $v = 0.5v_u$ (that is the mean value of v in the range from 0 to v_u) may be considered proper for the soils. This assumption was made in all the analyses performed in this study. In light of these considerations, it turns out that the value of v may be estimated from V_p and V_s measurements (using Eq. (22)), and therefore the unknown parameters in Eq. (23) reduce in practice to n , S_r , V_p and V_s .

The complete set of experimental data published by Inci et al. (2003) was considered to calculate the values of m_2^w by Eq. (23) in which the measured values of n , S_r , V_s and V_p were used. The values of m_2^w that were obtained in

this way for different values of the degree of saturation, are compared in Fig. 3 to those derived from Eq. (21) using the procedure described in the previous section. These latter values of m_2^w are distinguished in Fig. 3 by the abbreviation FC (i.e., from the fitting curves provided by Inci et al., 2003). As previously explained, Poisson's ratio of the soil skeleton was assumed equal to $0.5\nu_u$, for each soil sample. Figure 3 shows that the values of the volumetric coefficient m_2^w as calculated using Eq. (23) are always smaller than those deduced from the fitting curves provided by Inci et al. (2003). This aspect is worth analysing by further experimental studies in which the measured values of V_s and V_p are related to the soil state in terms of wetting, drying or scanning soil-water characteristic curve, from which m_2^w is evaluated as the slope at a given suction. As it is known, these curves present a different slope under the same suction. In particular, the scanning curves, on which the state of an undisturbed soil should lie (Fredlund, 2000), have in general much gentler slopes than the other curves, and hence are characterized by lower values of m_2^w . In addition, it is relevant to observe that the soil-water characteristic curves as measured in the laboratory are substantially flat at very low values of

S_r . This implies that the real values of m_2^w go to zero as S_r approaches zero (Fredlund and Rahardjo, 1993). As shown in Fig. 3, the values of m_2^w as calculated by Eq. (23) account for this requirement.

EMPIRICAL RELATIONSHIPS FOR A PRELIMINARY EVALUATION OF n AND S_r

The degree of saturation and porosity of fine-grained soils such as silts and clays, can be readily obtained in the laboratory on undisturbed specimens. The determination of these parameters in the case of coarse-grained soils is obviously much more difficult because undisturbed sampling of such materials can be only achieved using sophisticated techniques such as freezing. In addition, n and S_r cannot be evaluated from the results of routine in-situ tests, with the exception of the saturated soils for which $S_r = 1$ and n can be related to V_p and V_s (Foti et al., 2002). On the other hand, some empirical formulas relating these soil parameters to measured V_p and V_s are available, which could be useful from a practical viewpoint. For example, Watkins et al. (1972) presented the following relationship between porosity and compressional wave velocity (in m/s):

$$n = -0.175 \ln(V_p) + 1.56 \quad (24)$$

It was obtained as a least squares fit to the experimental data from different unsaturated materials. The predictive capability of Eq. (24) is assessed here by comparing the calculated porosity with the values of n obtained experimentally by Inci et al. (2003) for the soils described in the previous section. The values of V_p used in Eq. (24) are those measured by the same authors. The results are presented in Fig. 4. The agreement between predicted and measured porosity may be considered reasonable with the exception of the majority of the samples of Soil 1, for which Eq. (24) significantly overestimates the values of n . This could be due to the high percentage of sand in this soil (as documented in Table 1), which could have affected the values of n obtained in the laboratory.

Based on the experimental data provided by Inci et al.

Fig. 3. Comparison of m_2^w as obtained from the fitting curves presented by Inci et al. (2003), to m_2^w as calculated by Eq. (23)

Fig. 4. Measured porosity versus predicted porosity by Eq. (24)

Fig. 5. Empirical relationship of the degree of saturation of the soil, S_r , to the ratio V_p/V_s

Fig. 6. Empirical relationship of S_r to the parameter v_u expressed by Eq. (22)

(2003), the degree of saturation is graphically related to the ratio V_p/V_s in Fig. 5. For completeness, some data published by Leong et al. (2004) are also reported in this figure. The results show that V_p/V_s is practically insensitive to S_r in the range from 0 to 60%, where this ratio takes a fairly constant value that is slightly less than 2. On the contrary, for higher values of S_r than 60%, V_p/V_s clearly increases with increasing S_r . Although these data do not allow a suitable relationship between S_r and V_p/V_s to be defined, they may provide some indications concerning S_r on the basis of only V_p and V_s measurements. For example, if the observed trend is confirmed by further experimental data, it could be concluded that when V_p/V_s is sufficiently greater than 2, the corresponding value of S_r is likely greater than 60%.

Ishihara et al. (1998) proposed the following equation relating V_p/V_s to the B coefficient first defined by Skempton (1954):

$$\left(\frac{V_p}{V_s}\right)^2 = \frac{4K^w}{3nGB} \quad (25)$$

This equation can be used in conjunction with a relationship between B and S_r to evaluate the degree of saturation from V_p and V_s measurements. A relationship of this kind was presented by Head (1986) in a graphical form. Unfortunately, owing to the fact that the modulus G appearing in Eq. (25) is a function of S_r (Eq. (18)), an iterative procedure is required to obtain S_r from these equations. The authors are not aware of other existing empirical formulas that relate S_r to V_s and/or V_p . An attempt in this respect was performed in present study on the basis of the available data. In this connection, Fig. 6 shows the experimental values of S_r provided by Inci et al. (2003) versus the parameter v_u that is a function of V_p and V_s according to Eq. (22). Although the data are highly dispersed especially at low saturation levels, they show a general tendency of the soil specimens with higher v_u to present a higher value of S_r . To express analytically this trend, the simple equation is considered:

$$S_r = 2v_u \quad (26)$$

which is also plotted in Fig. 6. It is characterized by a

Fig. 7. Comparison of m_2^w as calculated using Eq. (23) with the porosity and degree of saturation provided by Inci et al. (2003), to m_2^w as calculated using Eq. (23) with the porosity and degree of saturation estimated by Eqs. (24) and (26), respectively

linear correlation coefficient of 0.76.

Lastly, Fig. 7 compares the coefficient m_2^w as calculated by Eq. (23) in which the experimental values of n and S_r provided by Inci et al. (2003) were used, to the same volumetric coefficient as derived from Eq. (23) using, however, the values of n and S_r predicted by Eqs. (24) and (26), respectively. For the sake of evidence, this latter volumetric coefficient is indicated as $\bar{m}_2^w$ in Fig. 7. To present the results in a proper scale, the highest values of m_2^w and $\bar{m}_2^w$ were not included in this figure. Their presence in the graph has, in any case, confirmed the conclusions reported below. In the calculations, the measured values of V_s and V_p were used, and v was assumed equal to $0.5v_u$. As can be seen, there is a reasonable agreement between the results, in spite of the significant variations in the data shown in Figs. 4 and 6. The range of the data is also indicated in Fig. 7, from which the confidence limits can be evaluated. In conclusion, a high level of accuracy in n and S_r does not seem to be required when m_2^w is evaluated by Eq. (23).

A PROCEDURE FOR ESTIMATING THE VOLUMETRIC DEFORMATION COEFFICIENTS OF UNSATURATED SOIL

A simple-to-use procedure is suggested for an estimation of the coefficients of volume change which define the constitutive relationships proposed by Fredlund and Morgenstern (1976) for unsaturated soils (Eqs. (A-1) to (A-3), in APPENDIX I). The procedure requires that the measured values of V_S and V_P are available together with those of n and S_r . In the absence of these latter data, as it occurs in the case of coarse-grained soils or when in-situ geophysical tests are only performed, some empirical formulas relating n and S_r to V_S and V_P could be used for a preliminary estimation of these parameters. Obviously, more reliable formulas than those considered in the previous section (if available) should be used to this purpose. As already highlighted, ρ_w , ρ_s and K^w are physical constants for which fairly standard numerical values can be assumed. In addition, Poisson's ratio of the soil skeleton may be selected in the range from zero to a sufficiently lower value than ν_u , with this latter being a function of only V_S and V_P (Eq. (22)).

Once these data are known, the coefficients of volume change can be evaluated using the equations presented in the previous sections. In particular, m_1^S is obtained by the following expression:

$$m_1^S = \frac{3(1-2\nu)}{2(1+\nu)[(1-n)\rho_s + S_r n \rho_w] V_S^2} \quad (27)$$

which derives from Eq. (16) in which G is provided by Eq. (18). Moreover, m_1^S is related to m_2^S by Eq. (6) from which it results:

$$m_2^S = \chi m_1^S \quad (28)$$

where χ may be replaced by S_r , as an approximation. Equation (28) also provides an estimation for m_1^w , owing to Eq. (7). Lastly, m_2^w can be evaluated using Eq. (23).

As an example, Fig. 8 presents the coefficients of volume change for the soils considered in the previous sections, as calculated using the described procedure in conjunction with the experimental data provided by Inci et al. (2003). For each soil sample, it was assumed that Poisson's ratio is $0.5\nu_u$. Figure 8 shows that the differences among the calculated volumetric coefficients are large at low values of S_r , and decrease with increasing S_r . Moreover, the results shown in Fig. 8 meet the requirement that all volume change coefficients approach the same value as the degree of saturation approaches 100% (Fredlund and Rahardjo, 1993).

GENERAL COMMENTS

The primary objective of this work is for presenting a procedure for an evaluation of the coefficients of volume change of unsaturated soils from V_P and V_S measurements. Since the techniques used to measure V_P and V_S are based on the application of small perturbations to the soil, the material parameters obtained in this way are commonly referred to the initial or undisturbed state of the soil. Some theoretical expressions relating the volumetric deformation coefficients to V_P and V_S have been derived in the previous sections, using a simplified formulation that is based on the theory of linear elasticity and other approximations. Owing to these assumptions, the derived expressions cannot in principle account for important factors such as stress state, stress history and stress path. On the other hand, the effects of these factors are implicitly included in the measured values of V_P and V_S . Thus, it is commonly accepted that there is a relation between the above-mentioned factors and the soil parameters that are evaluated through a process of inversion of V_P and V_S measurements on the basis of some theoretical expressions derived from the elasticity theory (Ishihara, 1970; Hardin and Drnevich, 1972; Iwasaki et al., 1978; Knox et al., 1982; Santamarina et al., 2001; Mancuso et al., 2002; Ng and Yung, 2008, and others). To define such a relation for unsaturated soils, a systematic experimental study should be conducted in which V_P and V_S measurements are carried out on soil samples with known porosity, degree of saturation, stress state, stress history and stress path. The authors expect these studies

Fig. 8. The coefficients of volume change for the soils tested by Inci et al. (2003), as evaluated using the proposed procedure

to be performed in the future.

CONCLUDING REMARKS

A simple-to-use procedure has been presented for an evaluation of the coefficients of volume change of unsaturated soils from V_p and V_s measurements by geophysical tests, such as cross-hole, down-hole or laboratory tests. The other data required as input are the porosity and degree of saturation of the soil, three physical parameters that generally assume standard numerical values, and Poisson's ratio of the soil skeleton. This latter may however be selected in a specified range.

The proposed procedure is of interest from an engineering viewpoint. Nevertheless, owing to the fact that the experimental data used in support of the developed theoretical formulation are incomplete, further experimental verifications are necessary.

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REFERENCES

- Berryman, J. G., Thigpen, L. and Chin, R. C. Y. (1988): Bulk elastic wave propagation in partially saturated porous solids, *Journal of Acoustic Society of America*, **84**(1), 360–373.
- Biot, M. A. (1956): Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Lower frequency range, *Journal of Acoustic Society of America*, **28**(2), 168–178.
- Bishop, A. W. (1959): The principle of effective stress, *Teknisk Ukeblad*, **106**(39), 859–863.
- Bishop, A. W. and Blight, G. E. (1963): Some aspects of effective stress in saturated and partly saturated soils, *Géotechnique*, **13**(3), 177–197.
- Brutsaert, W. (1964): The propagation of elastic waves in unconsolidated unsaturated granular mediums, *Journal of Geophysical Research*, **69**(2), 243–257.
- Conte, E., Cosentini, R. M. and Troncone, A. (2009): Shear and dilatational wave velocities for unsaturated soils, *Soil Dynamics and Earthquake Engineering*, **29**(6), 946–952.
- Cosentini, R. M. (2006): Elastic wave propagation in unsaturated porous media, *Ph. D. Thesis*, University of Reggio Calabria, Italy (in Italian).
- Coussy, O. (2004): *Poromechanics*, John Wiley & Sons, Chichester, UK, 152–156, 220.
- Foti, S., Lai, G. and Lancellotta, R. (2002): Porosity of fluid-saturated porous media from measured seismic wave velocities, *Géotechnique*, **52**(5), 359–373.
- Fredlund, D. G. (2000): The 1999 R. M. Hardy Lecture: the implementation of unsaturated soil mechanics into geotechnical engineering, *Canadian Geotechnical Journal*, **37**, 963–986.
- Fredlund, D. G. and Morgenstern, N. R. (1976): Constitutive relations for volume change in unsaturated soils, *Canadian Geotechnical Journal*, **13**, 261–276.
- Fredlund, D. G. and Rahardjo, H. (1993): *Soil Mechanics for Unsaturated Soils*, John Wiley & Sons, New York, 349–369.
- Gallipoli, D., Gens, A., Sharma, R. and Vaunat J. (2003): An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour, *Géotechnique*, **53**(1), 123–135.
- Garg, S. K. and Nayfeh, A. H. (1986): Compressional wave propagation in liquid and/or gas saturated elastic porous media, *Journal of Applied Physics*, **60**(9), 3045–3055.
- Gray, W. G. (1983): General conservation equations for multiphase systems: 4. Constitutive theory including phase change, *Advances in Water Resource*, **6**, 130–140.
- Hardin, B. O. and Drnevich, V. P. (1972): Shear modulus and damping in soils: measurements and parameter effects, *Journal of Soil Mechanics and Foundation Engineering*, ASCE, **98**(6), 603–624.
- Head, K. H. (1986): *Manual of Soil Laboratory Testing*, Pentech Press, London, **3**, 754–755.
- Hofman, B. A., Sego, D. C. and Robertson, P. K. (2000): In situ ground freezing to obtain undisturbed samples of loose sand, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, **126**(11), 979–989.
- Inci, G., Yesiller, N. and Kagawa, T. (2003): Experimental investigation of dynamic response of compacted clayey soils, *Geotechnical Testing Journal*, **26**(2), 1–17.
- Ishihara, K. (1970): Approximate forms of wave equations for water-saturated porous materials and related dynamic modulus, *Soils and Foundations*, **10**(4), 10–38.
- Ishihara, K., Huang, Y. and Tsuchiya, H. (1998): Liquefaction resistance of nearly saturated sand as correlated with longitudinal velocity, *Poromechanics—A tribute to Maurice A. Biot* (eds. by J. F. Thimus, Y. Abousleiman, A. H. D. Cheng, O. Coussy and E. Detournay), Balkema, Rotterdam, 583–586.
- Iwasaki, T., Tatsuoaka, F. and Takagi, Y. (1978): Shear moduli of sands under cyclic torsional shear loading, *Soils and Foundations*, **18**(1), 39–56.
- Jommi, C. (2000): Remarks on the constitutive modelling of unsaturated soils, *International Workshop on Unsaturated Soils*, Trento, Italy, 139–153.
- Knox, D. P., Stokoe, K. H. II and Kopperman, S. E. (1982): Effect of state of stress in velocity of low amplitude shear waves propagating along principal stress directions in dry sand, *Report GR 82-23*, University of Texas, Austin, USA.
- Kokusho, T. (2000): Correlation of pore-pressure B -value with P -wave velocity and Poisson's ratio for imperfectly saturated sand or gravel, *Soils and Foundations*, **40**(4), 95–102.
- Leong, E.-C., Yeo, S.-H. and Rahardjo, H. (2004): Measurement of wave velocities and attenuation using an ultrasonic test system, *Canadian Geotechnical Journal*, **41**, 844–860.
- Lo, W.-C., Majer, E. and Sposito, G. (2005): Wave propagation through elastic porous media containing two immiscible fluids, *Water Resources Research*, **41**, 1–20.
- Mancuso, C., Vassallo, R. and d'Onofrio, A. (2002): Small strain behaviour of a silty sand in controlled-suction resonant column—torsional shear tests, *Canadian Geotechnical Journal*, **39**(1), 22–31.
- Miura, K., Yoshida, N. and Kim, Y.-S. (2001): Frequency dependent property of waves in saturated soil, *Soils and Foundations*, **41**(2), 1–19.
- Ng, C. W. W. and Yung, S. Y. (2008): Determination of the anisotropic shear stiffness of an unsaturated decomposed soil, *Géotechnique*, **58**(1), 23–35.
- Richart, F. E., Hall, J. R. and Woods, R. D. (1970): *Vibrations of Soils and Foundations*, Prentice Hall, Englewood Cliffs, 74–86.
- Santamarina, J. C., Klein, K. A. and Fam, M. A. (2001): *Soils and Waves*, John Wiley & Sons, New York, 185–195, 232–242, 275–277.
- Skempton, A. W. (1954): The pore-pressure coefficients A and B , *Géotechnique*, **4**(4), 143–147.
- Stokoe, K. H. and Santamarina, J. C. (2000): Seismic-wave-based testing in geotechnical engineering, *Proc. International Conference of Geotechnical and Geological Engineering*, Melbourne, Australia.
- Tamura, S., Tokimatsu, K., Abe, A. and Sato, M. (2002): Effects of air bubbles on B -value and P -wave velocity of a partly saturated sand, *Soils and Foundations*, **42**(1), 121–129.
- Tuncay, K. and Corapcioglu, M. Y. (1996): Body waves in poro-

- lastic media saturated by two immiscible fluids, *Journal of Geophysical Research*, **101**(B11), 25,149–25,159.
- 37) Valle-Molina, C. (2006): Measurements of V_p and V_s in dry, unsaturated and saturated sand specimens with piezoelectric transducers, *Ph.D. Dissertation*, University of Texas, Austin, USA.
- 38) Watkins, J. S., Walters, L. A. and Godson, R. H. (1972): Dependence of in-situ compressional-wave velocity on porosity in unsaturated rocks, *Geophysics*, **37**(1), 29–35.
- 39) Wei, C. and Muraleetharan, K. K. (2002): A continuum theory of porous media saturated by multiple immiscible fluids: I. Linear poroelasticity, *International Journal of Engineering Science*, **40**, 1807–1833.

APPENDIX I: Derivation of the stress-strain equations for unsaturated soils subjected to small excitations

The formulation presented in this study is based on the constitutive relationships introduced by Fredlund and Morgenstern (1976), which can be expressed in the following compressibility form in accordance with the sign convention of continuum mechanics (i.e., a dilatant volumetric strain is positive and stresses are positive in tension):

$$d\varepsilon_v = m_1^s d(\sigma_m + p_a) - m_2^s d(p_a - p_w) \quad (\text{A-1})$$

$$dn^w = m_1^w d(\sigma_m + p_a) - m_2^w d(p_a - p_w) \quad (\text{A-2})$$

$$dn^a = m_1^a d(\sigma_m + p_a) - m_2^a d(p_a - p_w) \quad (\text{A-3})$$

where $d\varepsilon_v$ is the volume change of the soil structure with respect to the initial volume of the soil V , m_1^s and m_2^s denote the associated coefficients of volume change with respect to a change in the mean net total stress, $(\sigma_m + p_a)$, and in matric suction, $(p_a - p_w)$, respectively, σ_m is the mean total stress, and p_w and p_a are the pore-water and pore-air pressures, respectively. Similarly, dn^w is the volume change of water with respect to V , m_1^w and m_2^w are the coefficients of water volume change, dn^a is the volume change of the air phase, and m_1^a and m_2^a are the relating volumetric coefficients. Following Fredlund and Rahardjo (1993), Eq. (A-1) can also be formulated in a linear elasticity form in which the net total stress and suction are considered as stress state variables. This leads to

the following equation (Cosentini, 2006):

$$d\sigma = E d\varepsilon - \chi dp_w \mathbf{I} - (1 - \chi) dp_a \mathbf{I} \quad (\text{A-4})$$

where σ is the total stress tensor, ε is the strain tensor, E is the tensor of the elastic moduli, $\mathbf{I}$ stands for the identity tensor and χ is a parameter that is related to the volume change coefficients of the solid phase by the equation:

$$\chi = m_2^s / m_1^s \quad (\text{A-5})$$

Moreover, the change in the volume fraction of water can also be expressed as:

$$dn^w = \frac{dm^w}{\rho_w} - n^w \frac{dp_w}{K^w} \quad (\text{A-6a})$$

where m^w is the water mass content per unit volume of the soil, ρ_w is the mass density of water, and K^w is the bulk modulus of water. This equation derives from the relationship $dM_w = d(\rho_w V_w)$, in which M_w and V_w are the mass content and the volume of water, respectively. Likewise, for the air phase we have:

$$dn^a = \frac{dm^a}{\rho_a} - n^a \frac{dp_a}{K^a} \quad (\text{A-6b})$$

where m^a , ρ_a , and K^a stand for the above-defined parameters referred to air.

Lastly, the mass balance equations are invoked. Under the hypothesis of small perturbations, these equations are (Coussy, 2004):

$$\frac{dm^w}{\rho_w} = -n^w \text{div}(\mathbf{u}^w - \mathbf{u}^s) \quad (\text{A-7a})$$

$$\frac{dm^a}{\rho_a} = -n^a \text{div}(\mathbf{u}^a - \mathbf{u}^s) \quad (\text{A-7b})$$

in which the symbol div is the divergence operator, and $\mathbf{u}^s$, $\mathbf{u}^w$ and $\mathbf{u}^a$ are the displacement fields for the solid, water and air phases, respectively.

Substituting Eqs. (A-6a) to (A-7b) into (A-2), (A-3) and (A-4), and performing a series of algebraic operations yield the stress-strain equations presented in main text (i.e., Eqs. (1), (2) and (3)) (Conte et al., 2009).