FREQUENCY RESPONSE CHARACTERISTICS OF BENDER ELEMENT TEST SYSTEM IDENTIFIED BY FREQUENCY-SWEPT SIGNAL INPUT—INFLUENCE ON ACCURACY OF RECEIVED WAVEFORM RECONSTRUCTION—

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ABSTRACT

On the basis of the method for received waveform reconstruction that the authors had previously proposed on bender element test, this paper discusses the required characteristics of frequency response of the test system and frequency-swept signal, which are essential for the method, from the view point of reconstruction accuracy. In order to argue influence on characteristics of frequency response, frequency responses of several bender element test systems are experimentally identified by using eleven kinds of frequency-swept signals. Test results show that the frequency response of the system is substantially determined by the kinds of sample and identification of frequency response is affected by characteristics of input frequency-swept signal. Then reconstructed received waveform is calculated and compared to observed waveform. A normalized cross-correlation function is proposed to estimate quantitatively the degree of similarity between two waveforms and applied to the reconstructed and observed waveforms. This comparative analysis reveals that accurate identification of frequency response in a given frequency range leads to accurate waveform reconstruction. Also, test results show an additional advantage of this reconstruction technique in a noisy environment.

Key words: bender element, laboratory test, secondary wave velocity, test procedure (IGC: D7)

INTRODUCTION

Bender elements, introduced by Shirley and Hampton (1978), were embedded in laboratory test apparatus by Dyvik and Madshus (1985) to estimate the shear wave velocity propagating in soil sample. The shear wave velocity $V_s$ on the bender element test is given by the following equation:

$$ V_s = \frac{L}{\Delta t} \quad (1) $$

where $L$ is the travel distance and $\Delta t$ is travel time of shear wave. While many previous researches (e.g., Brignoli et al., 1996; Leong et al., 2005) indicate that the tip-to-tip distance of bender elements is reasonable for the travel distance $L$, it still remains unclear how the travel time $\Delta t$ can reasonably be determined because distortion of received waveform called near-field effect often complicates the measuring of arrival time (Sanchez-Salinero et al., 1985). Even though some approaches to detect reasonable value have been proposed so far (e.g., Viggiani and Atkinson, 1995; Greening and Nash, 2004), one-period sine wave of which frequency is determined by trial-and-error to cancel near-field effect is still applied to input waveform to estimate reasonable arrival time.

Meanwhile, the authors have proposed so far an alternative method which reconstructs received waveform for any input wave by operating convolution of frequency response of bender element test system and the input waveform and demonstrated the waveform reconstruction on several apparatuses of which frequency responses were identified by frequency-swept signal (Ogino et al., 2008). Frequency response of a system being independent of the input waveform, this method allows experimenters to restore the received wave and estimate the travel time for any input wave after experiments were implemented. The authors have also proposed two types of frequency-swept signal of which frequency characteristics can be changed to identify the frequency response of the test system. However, the authors still have not mentioned as to how the identified frequency response and, eventually, the reconstructed waveform are affected by frequency-
where, \( Z \) is the frequency response of bender element test system, \( X(f) \) and \( Y(f) \) are the Fourier transform of arbitrary input wave and corresponding received wave, respectively. Equation (2) represents the relationship between input and its response on a linear system, which is equivalent to convolution integral in the time domain. Then frequency response \( Z(f) \) can be written as follows:

\[
Z(f) = X^{-1}(f) \cdot Y(f) \tag{3}
\]

If the input wave \( X(f) \) is frequency-swept signal, \( X(f)^{-1} \) is given by following equation:

\[
X_{FSS}(f) = X_{FSS}(f) \cdot Y(f) \tag{4}
\]

where, \( X_{FSS} \) is the Fourier transform of frequency-swept signal and asterisk represents complex conjugate. Then \( Z(f) \) can be identified by a bender element test for frequency-swept input signal and described as Eq. (5).

\[
Z(f) = X_{FSS}(f) \cdot Y(f) = X_{FSS}(f) \cdot Y(f) \tag{5}
\]

where, \( Y(f) \) is the corresponding received signal. Finally, Eq. (5) represents a cross-correlation between frequency-swept signal and its received signal. Two types of frequency-swept signals, called LSSP and TSP in the paper, are proposed and discretely given by the following equations. LSSP is given by Eq. (6).

\[
x_{LSSP}(t) = \begin{cases} 
A \sin \left( 2\pi \left( f_0 + \frac{t}{t_i} \right) t \right) & 0 \leq t \leq t_i \\
0 & t_i < t \leq N \Delta t
\end{cases} \tag{6}
\]

where, \( x_{LSSP}(t) \) is the time history of LSSP, \( A \) is amplitude, \( f_0 \) and \( \Delta f \) are the origin and width of the frequency sweep, respectively, \( t_i \) is duration time of frequency sweep, \( N \) is number of data, \( k \) is integer number from 0 to \( N \), \( \Delta f \) is discrete time, and \( f_i \) is the sampling frequency. Also, TSP is given by Eq. (7) in frequency domain (Aoshima, 1981).

\[
X_{TSP}(k) = \begin{cases} 
A \exp \left( \frac{m \pi k^2}{N^2} \right) & 0 \leq k \leq \frac{N}{2} \\
0 & \frac{N}{2} + 1 < k \leq N
\end{cases} \tag{7}
\]

where, \( X_{TSP}(k) \) is the Fourier transform of TSP, \( m \) is a parameter relating the degree of sweeping, and \( f(k) \) is the discrete frequency. For LSSP, the frequency linearly increases with time for \( t_i \) in Eq. (6), while the phase shifts in proportion to square of frequency for TSP in Eq. (7).

Being equivalent to a received wave for an ideal impulse input, frequency response fundamentally involves travel time in itself. Some techniques directly estimating travel time from phase shift of frequency response called frequency domain technique have been shown by Greening and Nash (2004) and Ogino et al. (2008). In practical situation, meanwhile, determination of travel time still often depends on trial-and-error transmitting of sine pulse with various frequencies, which is called time domain technique. The reconstruction technique proposed in this paper allows us to choose a practical way in time domain as well as the theoretical determination in frequency domain. Hence, this technique is especially effective in the situation where received waveform for particular wave input is needed.

**CHARACTERISTICS OF FREQUENCY-SWEPT SIGNAL**

Characteristics of frequency-swept signal are determined by parameters in the above equations. LSSP has six parameters of \( A, f_0, \Delta f, t_i, N \), and \( f_i \) in Eq. (6) and TSP has four parameters of \( A, m, N \), and \( f_i \) in Eq. (7). Figure 1 shows six typical waveform of frequency-swept signals obtained from Eq. (6) or Eq. (7) with varying values of parameters as shown in the figure. Names of the signals in the figure such as LSSP1, and TSP1, are corresponding to Table 1 shown below. As entire length of frequency-swept signal including the trailing zeros is defined to be the product of number of data \( N \) and interval of sampling \( f_i^{-1} \), the lengths of waves except for TSP5 amount approximately to 40 ms, while that of TSP5 amounts to 100 ms in Fig. 1. Meanwhile, sweeping part of the wave is elongated as \( t_i \) increases for LSSP and as \( m \) increases for TSP. In particular, the parameter \( t_i \) directly represents the length of sweeping part for LSSP. Figure 2 shows Fourier spectra of the frequency-swept signals shown in Fig. 1. For LSSP1 and LSSP3, spectra have large amplitude in the range from 0.5 to 20 kHz, while from 5 to 15 kHz for LSP7. The upper and lower limits of the range coincide with \( f_0 \) and \( f_0 + 2\Delta f \), respectively. This coincidence indicates that LSSP involves frequency component in the range from \( f_0 \) to \( 2\Delta f \), not from \( f_0 \) to \( \Delta f \). It also seems that frequency characteristics of LSSP are related only to \( f_0 \) and \( \Delta f \), not affected by \( t_i \). Meanwhile in TSP, the spectrum is flat through the given fre-
frequency because absolute value of $X_{TSP}(k)$ in Eq. (7) are defined to be constant. Equation (7) also defines the upper and lower limits of frequency as $f(1) = f_s/N$, and $f(N/2) = f_s/2$, respectively. Hence, frequency characteristics of TSP, as well as the entire length of wave, are determined by $f_s$ and $N$.

Although the frequency characteristics of frequency-swept signal are improved as the values of $N$ and $f_s$ increase because the base and Nyquist frequency of waves are given by $f_s/N$, and $f_s/2$, respectively, the upper or lower limits of these parameters practically depend on performance of hardware that generates the waveforms. In the case of generating waveforms from a DA converter, for example, sampling frequency and buffer of DA converter confines the upper limit of $N$ and $f_s$. Since discrete data involve frequency component from the base to Nyquist frequency, frequency range in which spectrum of LSSP has large amplitude in Fig. 2 should be obtained within this range. Namely, $f_0$ should be more than base frequency and $f_0 + 2\Delta f$ should be less than Nyquist frequency. The upper and lower limits of frequency range for TSP is consistently equivalent to the base and Nyquist frequency shown above. The upper limit of $A$ is also confined by maximum output voltage of DA converter. Wave energy of frequency-swept signal increases as the value of $t_t$ or $m$ as well as $A$ increases.

**LABORATORY EQUIPMENTS AND EXPERIMENTAL PROCEDURES**

Laboratory bender element tests were performed on several samples. Three kinds of samples: Toyoura sand, Kasaoka clay and, Akita peat were subjected to the tests. Toyoura sand and Kasaoka clay are commercially available sample whose particle densities are 2.64 t/m$^3$ and 2.77 t/m$^3$, respectively. Liquid limit, plasticity index, and clay content of Kasaoka clay are 62%, 26%, and 45%. Akita peat is taken from a peat layer of 2 m beneath the ground surface in Akita city. Particle density, ignition loss, and degree of decomposition of Akita peat are 1.64 t/m$^3$, 27%, and 45%, respectively.
Table 1. Parameters of frequency-swept signals used in this paper

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>f_s(kHz)</th>
<th>f_0(kHz)</th>
<th>Δf(kHz)</th>
<th>A</th>
<th>t_0(ms)</th>
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</thead>
<tbody>
<tr>
<td>LSSP1</td>
<td>4096</td>
<td>100</td>
<td>0.5</td>
<td>10.0</td>
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<td>2</td>
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<tr>
<td>LSSP2</td>
<td>4096</td>
<td>100</td>
<td>0.5</td>
<td>10.0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>LSSP3</td>
<td>4096</td>
<td>100</td>
<td>0.5</td>
<td>10.0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>LSSP4</td>
<td>4096</td>
<td>100</td>
<td>0.5</td>
<td>10.0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>LSSP5</td>
<td>4096</td>
<td>100</td>
<td>0.5</td>
<td>2.0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>LSSP6</td>
<td>4096</td>
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<td>0.5</td>
<td>5.0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>LSSP7</td>
<td>4096</td>
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<td>5.0</td>
<td>5.0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>TSP1</td>
<td>4096</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td>50</td>
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<td>2048</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>150</td>
</tr>
</tbody>
</table>

Fig. 3. Schematic diagram of waveform reconstruction analysis

The degree of similarity between reconstructed and observed waveforms can be evaluated by following normalized cross-correlation function, which is a conventional approach to template matching:

\[
R(k) = \frac{CC_{yz}(k)}{\sqrt{|CC_{yy}(k)|\cdot|CC_{zz}(k)|}}
\]

where, \(R(k)\) is the normalized cross-correlation function, \(k\) is the time lag of two waveforms, \(CC_{yz}(k)\) is the cross-correlation function between the observed and the reconstructed wave, \(CC_{yy}(k)\) and \(CC_{zz}(k)\) are the autocorrelation functions of observed and reconstructed waves, respectively. For discrete data, these functions can be represented by following equations:

\[
CC_{yz}(k) = \frac{1}{N}\sum_{m} y(m)z(m-k)
\]

\[
CC_{yy}(k) = \frac{1}{N}\sum_{m} y(m)y(m-k)
\]

\[
CC_{zz}(k) = \frac{1}{N}\sum_{m} z(m)z(m-k)
\]

where, \(y(m)\) and \(z(m)\) \((m=0, 1, 2, ..., N)\) are discrete data of observed and reconstructed wave, respectively. Circular shift is applied to parameter \(m\) when the value of \(m-k\) is negative. For example, \(y(m-k)=y(N)\) for \(m-k=-1\) and \(y(m-k)=y(N-1)\) for \(m-k=-2\), and so on. The value of Eq. (8), which inevitably takes in the range from \(-1\) to \(1\), means that reconstructed waveform is in complete agreement with observed one for \(R(k)=1\), while completely inverted for \(R(k)=-1\) at the lag of two waveforms being \(k\). Since there must be no time lag between the observed and reconstructed waveforms, degree of similarity is described by \(R(0)\).
RESULTS AND DISCUSSIONS

Test System and Frequency Response

Frequency response of bender element test system can be identified by frequency-swept input signals. Typical test results on Toyoura sand consolidated in oedometer are shown in Figs. 4 and 5. Figure 4 shows the time history of transmitted (top) and received (bottom) waveforms. Fourier spectrum of frequency response given by Eq. (5) is shown in Fig. 5. A sharp peak at 30 kHz and frequency range from 5 to 50 kHz in which amplitudes exist can be seen in the spectrum.

As entire system of bender element test is composed of test apparatus, bender elements, and specimen, frequency response of the system is considered to be affected by those elements (Lee and Santamarina, 2005; Blewett et al., 2000). Figure 5 summarizes the peak frequencies and frequency ranges on various test systems composed of different samples or apparatus, different dimensions of bender elements or specimens and under different consolidation stress. Note that $h$, $h_{TBE}$ and $h_{RBE}$ are height of specimen, transmitter and receiver element, respectively and, frequency range is defined as the range of normalized amplitudes $A/A_{\text{max}} \geq 0.05$. Frequency response of the system composed of Toyoura sand and oedometer shows remarkably wide range of 5 to 50 kHz and high resonant frequency at 25 to 30 kHz as compared to other systems. Meanwhile, it appears that resonant frequency and upper limit of the range significantly decrease in the case of the system composed of Akita peat, especially and triaxial apparatus. This implies that the characteristics of the system response move to high-frequency side as stiffness of the lateral boundary condition of specimen and in particular, stiffness of specimen itself increases. On the other hand, it seems that consolidation stress and height of specimen or bender elements affect these characteristics relatively less. Even though resonant frequency goes high slightly as consolidation stress increases on peat sample, the values of resonant frequencies are at most 5 kHz on clay or peat samples through any height of specimen or bender elements.

Identified Frequency Response and Characteristics of Frequency-swept Signal

Although frequency response of test system is determined by the elements composing the test system as shown above, identification accuracy of frequency response is strongly affected by the characteristics of frequency-swept signal. Figure 6 shows Fourier spectra of six frequency sweeps for input signal and identified frequency responses of a system. Although the system is quite the same through six figures, there is seemingly little similarity among these frequency responses. The ranges in which amplitudes of frequency response exist are different among the figures, corresponding to the range in which amplitudes of input signals exist. There are no amplitudes in the frequency responses over 20 kHz for LSSP1 and LSSP3 input or 5 kHz for LSSP5 input, while in the range less than 5 kHz and over 15 kHz for LSSP7 input. Further, it seems that the shape of frequency response is less affected by degree of sweeping because there is no significant difference between Fig. 7(c) and (d) or Fig. 7(e) and (f), in which only the degree of sweeping is different.

However, these frequency responses are partially in good agreement with each other. Figure 7 again shows the frequency responses with each scaling of Y axes in Figs. 7(a), (b) and (e) being adjusted so that each peak of...
amplitude agrees with that of Fig. 7(e). In the frequency ranges in which amplitude exists, good fits are obtained for these four spectra. This means that frequency responses shown in Fig. 7 are part of the whole system’s response just in frequency range in which those amplitudes exist. Moreover, the frequency range is strongly associated with characteristics of frequency-swept signal input to the system. It follows that frequency-swept signal should have a sufficiently wide frequency range for accurate identification of frequency response of the system.

The practical frequency range required for input signal can be obtained from characteristics of the system response. Normalized Fourier spectra of TSP5, which involves amplitudes less than 10 kHz, and identified frequency response of another system are shown in Fig. 8. Due to small stiffness of peat sample, there is no amplitude in the frequency response over 4 kHz while amplitudes exist to 10 kHz in TSP5. This means that the system has no response over 4 kHz and the frequency range of TSP5 is sufficient to cover that of system response in this case, even though the upper limit of frequency range on TSP5 is lower as compared to that of TSP1 or TSP3 in Fig. 7. Since the frequency range of system response varies depending on kinds of sample, apparatus, etc. as seen in Fig. 6 as well as Figs. 7 and 8, it follows that characteristics of frequency-swept signal should be controlled to cover the range, that is to say, parameters $f_0$ and $\Delta f$ or $N$ and $f_s$ should be determined to cover the range.

Reconstruction Accuracy of Received Waveform and Characteristics of Frequency Response

Inaccurate identification of frequency response leads to serious error in reconstruction of received waveform. Four reconstructed received waveforms for one-period sine wave input calculated from frequency responses shown in Fig. 9 are compared with observed wave in Fig. 10. Although frequency response from TSP1 input, which has the widest frequency range (see Fig. 9), obtains an accurate result, there are significant differences in accuracy among these calculations. It appears that accuracy of reconstruction decreases as the frequency range of frequency response used for waveform calculation narrows. In fact, the waveform reconstructed from frequency response identified by LSSP5 input, whose high frequency range is cut off, is far from the observed waveform. In particular, high frequency component represented as many spike-like voltage changes in the ob-
served waveform disappeared in the reconstructed waveforms as the frequency range moves to low frequency side, namely, TSP1, LSSP3, LSSP7, and LSSP5 in that order.

Degree of similarity between observed and reconstructed waveform can also quantitatively be represented as value of $R(0)$ in Eq. (8). Figure 11 summarizes values of $R(0)$ for one-period sine inputs of 10 kHz as well as 3, 5, 15 and 20 kHz. Spectra of frequency responses used for calculations are also shown in the figures. Compared to $R(0)$ among in the case of TSP1, LSSP3, and LSSP5 input at 10 kHz, the values of $R(0)$ increase in the order of apparent similarity to observed waveform in Fig. 10. Furthermore, decrease of $R(0)$ and amplitude of frequency response is in agreement over the range of 5 kHz in Fig. 11(b) or over 10 kHz in Fig. 11(c), indicating that accuracy of waveform reconstruction definitely decreases out of the range in which amplitude of frequency response exists. Considering waveform reconstruction is calculated by product of frequency response and Fourier transform of input wave (see Eq. (2)), this is because components of reconstructed wave in the range that amplitude of frequency response does not exist are calculated to be zero regardless of the amplitude of input wave.

This indicates that accuracy of waveform reconstruction should increase as overlapped area of given input wave and frequency response in the spectrum increases. Fourier spectra for one-period sine waves of 3, 5, 10, 15 and 20 kHz, which is generally used for bender element test, are shown in Fig. 12. As well as at the frequencies of sine waves, amplitude of spectrum exists around it. The ranges reach over twice the frequency. It follows that frequency response should be identified in at least over twice wide frequency range even for reconstruction for specific one-period sine wave input. Meanwhile, values of $R(0)$ for TSP2, TSP3, and TSP4 shown in Fig. 11(a) and LSSP1, LSSP2, and LSSP4 shown in Fig. 11(d), which are different only in the length of sweeping, indicate that long sweep does not necessarily result in accurate reconstruction. Thus, for reconstruction accuracy, frequency response of the system should be identified to cover the frequency range in which amplitude of given input wave exists. In other words, spectrum of frequency-swept signal should be controlled to be flat and to cover it. In that sense, it is concluded that TSP is more appropriate for identification of frequency response than LSSP.

An Additional Advantage of Waveform Reconstruction

Under noisy environment, there is an additional advantage to this reconstruction technique. Figure 13 shows observed and reconstructed received waveforms for one-period sine input of 5 kHz in a triaxial apparatus. In this situation there is much noise from a motor controlling axial displacement, thus high frequency noise causes disturbance of observed waveform and low frequency noise lifts the base line. However, these noises decrease to negligible level in the reconstructed waveform. Improvement of signal-noise ratio is more significant compared with the observed waveform which is averaged by 4-cycle
Repeated measurements in Fig. 13. In addition, significant cross-talk at the beginning of received waveform is also reduced in the reconstructed waveform. This high signal-noise ratio is due to large energy of frequency-swept signal. As frequency-swept signal is transmitted over a long time with increasing/decreasing frequency, amplitude of the received wave is larger than that for one-period sine wave input. This large amplitude results in less noise in identified frequency response and reconstructed waveform.

Improving signal-noise ratio of received waveform contributes to accurate detection of shear wave arrival. Figure 13 also shows a reference travel time which is calculated from shear modulus evaluated by cyclic loading at axial strain amplitude of 0.005% on the same specimen. The arrival time from cyclic loading test corresponds to the rising point of received waveform when the start time is obtained by the beginning of transmitted waveform. The rising point can clearly be detected in the reconstructed waveform as a crossing point of the baseline, while it is difficult in the observed waveforms if the reference arrival time is absent. Hence, this reconstruction technique is expected to be an effective method for accurate and easy detection of the arrival time under noisy environment as well as the situation in which level
of received wave amplitude is very low.

CONCLUSIONS

Bender element tests for identifying frequency response are implemented on several test systems to study relationship among frequency characteristics of frequency-swept signal and test system, and accuracy of waveform reconstruction. The main conclusions from this study are as follows:

1. Frequency response of test system is affected by test conditions and elements composing the system. In particular, the system response moves to high-frequency side as stiffness of lateral boundary condition of the specimen or stiffness of specimen increases.

2. For accurate identification of frequency response, characteristics of frequency-swept signal should be determined to cover the frequency range of the response. Spectrum of identified frequency response has amplitudes only in the frequency range in which amplitude of the frequency-swept signal exists even if test system has responses out of the range.

3. In terms of frequency characteristics, TSP is more appropriate for identifying frequency response than LSSP because TSP has flat spectrum over the entire frequency range.

4. This reconstruction method provides more clear received waveform than the observed one in noisy environment. Noises from outside or inside of the system are reduced to negligible level due to large energy of frequency-swept signal.

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