A NEW MODEL FOR UNSATURATED SOIL USING SKELETON STRESS AND DEGREE OF SATURATION AS STATE VARIABLES

FENG ZHANG and TOMOYUKI IKARIYA

ABSTRACT

In this paper, based on experimental results a new constitutive model, using skeleton stress and degree of saturation as independent state variables, is proposed for unsaturated soil, in which the influence of the degree of saturation can be properly described. In the model, a very simple moisture characteristics curve considering moisture hysteresis of unsaturated soil is also proposed. The moisture characteristics curve can not only be applied to secondary drying process but also primary drying process originated from slurry soil. The constitutive model is able to describe not only the behavior of unsaturated soil but also saturated soil because the skeleton stress can smoothly shift to effective stress if saturation changes from unsaturated condition to saturated condition. Meanwhile the overconsolidation, one of the main features of soils that are discussed in the models for saturated soils, is also considered together with the degree of saturation. Other mechanical features such as structure of soil and stress-induced anisotropy can be easily incorporated into the proposed model within the framework of the present research. It is known from the simulation that the main features of unsaturated soil in isotropic consolidation test and triaxial compression test under drained and exhausted condition with different confining stress and suction can be qualitatively described.

Key words: constitutive model, degree of saturation, overconsolidation, skeleton stress, unsaturated soil (IGC: D6)

INTRODUCTION

Much research has been done on the mechanical behavior of unsaturated soils experimentally, empirically, and theoretically. Some constitutive models for unsaturated soils have been developed since the work by Alonso et al. (1990), in which Barcelona Basic Model (BBM) was proposed using LC conception and normally regarded as one of the basic models for unsaturated soils. The constitutive models based on the framework of BBM, can be found in the works such as Kohgo et al. (1993a, b), Cui and Delage (1996) and Sun et al. (2000). In these models, stress-suction-strain relations of unsaturated soil were considered explicitly while the degree of saturation was not considered directly. On the other hand, constitutive models considering the influence of the degree of saturation were also proposed in the same period, which can be found in the literature such as Kato et al. (1996), Karube et al. (1997), Gallipoli et al. (2003), Sheng et al. (2008), Muraleetharan and Liu (2005), Sun et al. (2007a) and Sheng et al. (2008).

Most of the models in literature usually use stress and suction as independent state variables. In the work by Ohno et al. (2007), however, a very interesting assumption based on various experimental results was proposed that void ratio-logarithmic effective stress relation $e$-ln $p$ is dependent on the degree of saturation and the dependency is modeled with a simple relation. In the model proposed by Ohno et al. (2007), therefore, only the effective stress that was defined by a combination of net stress, suction and ‘effective degree of saturation’, was used as state variable, based on which a Cam-clay type model for unsaturated soil was proposed. The advantage of using effective stress or skeleton stress instead of using net stress and suction as independent state variables in modeling unsaturated soils is that it is much easier and smoother to describe the behavior of soil from unsaturated state to saturated state and vice verse if the moisture characteristics of the soil are properly described and incorporated into the constitutive model. Based on this type of model, it is possible to conduct a numerical analysis related to boundary value problems of saturated and unsaturated grounds in a unique way. A pioneer research on this field can be found in the work by Kanazawa et al. (2008).

In this paper, based on the experimental results conducted by the same authors and other results available in literature, a constitutive model using the skeleton stress and degree of saturation as independent state variables is proposed for unsaturated soil, in which the influence of the degree of saturation can be properly described. Meanwhile, other mechanical features such as overconsolidation, soil structure formed in sedimentary process of the soil, and stress-induced anisotropy that are often discussed for saturated soil and can be suitably described...
with unique framework (Zhang et al., 2007), can also be easily considered in the same way for saturated and unsaturated states. In the proposed model, a very simple moisture characteristics curve considering moisture hysteresis of unsaturated soil is also proposed from the hint of the work by Muraleetharan and Liu (2005). The moisture characteristics curve can not only be applied to the secondary drying process but also the primary drying process originated from slurry soil.

It is true that the research on unsaturated soil considering overconsolidation effect which is one of the main features of soils discussed in the models for saturated soils, can be found in literature, e.g., the work by Kohgo et al. (1993b) and Kohgo et al. (2007a) in which the effective stress is defined by net stress and suction but not related directly to the degree of saturation and the subloading concept is used; or the work by Russell and Khalili (2006) with the concept of bounding surface. For simplicity, in this paper only the overconsolidation and the degree of saturation are considered as state variables. Other mechanical features such as the structure of soil (Asaoka et al., 1998) and the stress-induced anisotropy (Sekiguchi, 1977; Hashiguchi and Chen, 1998; Zhang et al., 2007), however, can be easily incorporated into the proposed model within the framework of the present research.

**DISCUSSION OF VOID RATIO-SKELETON STRESS RELATION BASED ON TEST RESULTS**

In discussing the mechanical behavior of an unsaturated soil, stress-strain relation is a key factor. The stress, here, might be total stress, net stress or effective stress defined by Bishop, according to preference of researchers who deal with constitutive model of unsaturated soils. Through out this paper, however, effective stress is defined by the concept of skeleton stress as shown in following relations:

\[
\sigma_{ij}^s = \sigma_{ij}^t - U \delta_{ij} \tag{1}
\]

\[
U = S_r u_e + (1 - S_r) u_a \tag{2}
\]

where \(U\) is mean pore pressure, \(\sigma_{ij}^s\) is skeleton stress tensor, \(S_r\) is degree of saturation, \(\sigma_{ij}^t\) is total stress tensor and \(u_e\) is air pressure. Equation (1) means that the skeleton stress tensor is the difference of total stress tensor with mean pore pressure. Equation (1) can also be rewritten as

\[
\sigma_{ij}^s = \sigma_{ij}^t - u_e \delta_{ij} + S_r (u_e - u_a) \delta_{ij} = \sigma_{ij}^t + S_r s \delta_{ij} \tag{3}
\]

where \(\sigma_{ij}^n\) is net stress tensor and \(s\) is suction. Equation (3) is just the definition of the effective stress defined by Bishop if taking the value \(\chi\) in Bishop’s definition as \(S_r\). The physical explanation for Eqs. (1) and (3), however, is different.

In many researches, the effective stress has been defined by the degree of saturation. Many of these relationships, however, were developed based on shear strength data under drying process (Alonso et al., 2010; Lu et al., 2010). In the work by Khalili and Zargarbashi (2010), a multi-stage drying and wetting shear tests were conducted for several unsaturated remolded soils along or around critical state to check the variation of effective stress along not only the drying path but also the wetting path. It is found from their research that \(\chi-s\) relation and \(S_r-s\) relation are similar in a sense but quite different in the stage transited from drying to wetting. Therefore, it is admitted that there still exists some doubt about the possibility that the degree of saturation may not be suitable to be used directly as the parameter for representing the effective stress. In spite of the fact, as a simplification, Eq. (1) is used to define the effective stress in this paper. Also for simplicity, throughout the context, the skeleton stress tensor \(\sigma_{ij}^s\) will be abbreviated as \(\sigma_{ij}\) without specification in the following context.
Based on test results available in the literature and the results obtained by the authors, it is more natural and reasonable to use the skeleton stress as an independent state variable to describe the mechanical behavior of unsaturated soil. Figure 1 shows the test results of an oedometer test for unsaturated soils by Honda (2000), in which the influence of degree of saturation on e-ln \( p \) relation is carefully investigated. The results are re-plotted in void ratio-mean skeleton stress space. The oedometer test was conducted under constant suction but different initial degree of saturation. The soil sample is a clay with a plasticity index of 29.6 and liquid limit of 43.0. In preparing the sample, compaction was first conducted under undrained condition and a prescribed degree of saturation was set for each specimen. It is known from the figure that in both cases of constant suctions, the lower the initial degree of saturation is, the higher the void ratio will be under the same mean skeleton stress. Moreover, normally consolidated lines (N.C.L.) for different initial degree of saturation are parallel to each other in normally consolidated region.

Figure 2 shows the test results of void ratios at critical state in triaxial compression test on unsaturated silt under drained and exhausted condition in different degree of saturation, re-plotted from the work by Cui and Delage (1996). The samples were statically compacted silt and the testing device used in compression test utilized Osmotic technique for suction control using polyethylene glycol solution (PEG). In Fig. 2, the numbers represent the degrees of saturation of the samples at critical state of the triaxial compression tests plotted in e-ln \( p \) space. Due to the difficulty to monitor the degree of saturation during loading, only those values at critical state were evaluated based on the values of shear stress ratio, mean skeleton stress, void ratio and water content at critical state. The e-ln \( p \) lines can then be plotted through two points where the void ratios are the same but the mean skeleton stresses are different. Without losing general tendency, it is possible to plot other lines passing through the marked points with the same gradient, which means that the e-ln \( p \) lines align parallel to each other for different degree of saturations. Though the compression tests were conducted under different suctions and confining stresses, a tendency can be identified clearly, that is, the lower the degree of saturation is, the higher the void ratio will be under the same mean skeleton stress at critical state. This phenomenon can also be observed in the work by Kodaka et al. (2006) as shown in Fig. 3 in which the tested results of void ratios at critical state of triaxial compression tests on unsaturated DL clay, a manmade clay with 90% of silt and very low value of plasticity index, under drained and exhausted condition with different saturation and confining stress, are re-plotted. The samples used in the tests were also statically compacted to prescribed initial degree of saturation of 70%. The phenomenon shown in Figs. 2 and 3 were also observed in the triaxial compression tests on unsaturated Kaolin samples which were prepared from slurry with primary drying process in suction load-
DERIVATION OF CONSTITUTIVE MODEL

Based on the discussion in previous section, it is natural to establish a quantitative relation for void ratio-logarithmic mean skeleton stress e-ln p relation, using the degree of saturation as a state variable. Here, it is assumed that normally consolidated line in unsaturated state (N.C.L.S.) is parallel to the normally consolidated line in saturated state (N.C.L.) but in a higher position than N.C.L., as shown in Fig. 6, which means that under the same mean skeleton stress, unsaturated soil can keep higher void ratio than that of saturated soil. The N.C.L.S. and C.S.L.S. are given in the following relations as:

\[ N.C.L.S.: \ e = N(S) - \lambda \ln \frac{p}{p_t} \] (5)

\[ C.S.L.S.: \ e = \Gamma(S) - \lambda \ln \frac{p}{p_t} \] (6)

where \(N(S)\) and \(\Gamma(S)\) are the void ratios at N.C.L.S. and C.S.L.S. under a reference mean skeleton stress \(p_t\). Usually \(p_t = 98\) kPa and certain degree of saturation. \(p = \sigma_0 / 3\) and \(q = \frac{1}{2}(\sigma_0 - p\delta_v)(\sigma_0 - p\delta_u)/2\) are the mean skeleton stress and the second invariant of deviatoric skeleton stress tensor. \(M\) is the stress ratio at critical state and has the same value for saturated and unsaturated states. Therefore, similar to the derivation of Cam-clay model for saturated soils, the void ratio \(e\) subjected to shearing is assumed to be,

\[ e = \chi(\eta, S) - \lambda \ln \frac{p}{p_t} \] (7)

where \(\chi(\eta, S)\) is a function of shear stress ratio \(\eta\) and the degree of saturation \(S\) and can be expressed with simple functions as:

(i) For Cam-clay type (Roscoe et al., 1963):

\[ e = N(S) - \frac{N(S) - \Gamma(S)}{M} \eta - \lambda \ln \frac{p}{p_t} \] (8)

(ii) For Modified Cam-clay type (Schofield and Wroth, 1968):

\[ e = N(S) - \frac{N(S) - \Gamma(S)}{\ln 2} \ln \frac{M^2 + \eta^2}{M^2} - \lambda \ln \frac{p}{p_t} \] (9)

Under an saturated isotropic normally consolidated state, that is, \(s = 0, p = p_0, \eta = 0, S = 1, N = N(S) = 1\), \(e\) takes a value of \(e_0\) and can be expressed as,

\[ e_0 = N - \lambda \ln \frac{p_0}{p_t} \] (10)

From Eqs. (9) and (10)

\[ -\Delta e = e_0 - e = N - N(S) \]

\[ + \frac{N(S) - \Gamma(S)}{\ln 2} \ln \frac{M^2 + \eta^2}{M^2} + \lambda \ln \frac{p}{p_0} \] (11)
where \( \lambda \) is compression index. Similar to the original Cam-Clay model, elastic change of void ratio of unsaturated soil can be calculated with swelling index \( \kappa \) as:

\[
-\Delta e^e = \kappa \ln \frac{p}{p_0}
\]  

(12)

From Eqs. (11) and (12), it is clear that unlike most constitutive models for unsaturated soils, both the compression index and swelling index are independent from suction or degree of saturation. Elastic volumetric strain can then be calculated as:

\[
e^e = -\frac{\Delta e^e}{1 + e_0} = -\kappa \ln \frac{p}{p_0}
\]  

(13)

By differentiating Eq. (13), the following relation can be derived,

\[
d e^e = \frac{\kappa}{1 + e_0} \frac{dp}{p}
\]  

(14)

Plastic part of the change of void ratio can be given as

\[
-\Delta e^p = N - N(S_0) + \frac{N(S_0) - \Gamma(S_0)}{2} \ln \frac{M^2 + \eta^2}{M^2} + (\lambda - \kappa) \ln \frac{p}{p_0}
\]  

(15)

Therefore volumetric strain can also be divided into elastic and plastic parts and the plastic part can be expressed as,

\[
e^p = -\frac{\Delta e^p}{1 + e_0} = \frac{N - N(S_0)}{1 + e_0} + \frac{N(S_0) - \Gamma(S_0)}{1 + e_0} \ln \frac{M^2 + \eta^2}{M^2} + (\lambda - \kappa) \ln \frac{p}{p_0}
\]  

(16)

Equation (16) is only suitable for normally consolidated soil. For overconsolidated soil, concept of subloading surface proposed by Hashiguchi and Ueno (1977) can easily be applied to unsaturated soil as:

\[
e^p = N - N(S_0) + \frac{N(S_0) - \Gamma(S_0)}{2} \ln \frac{M^2 + \eta^2}{M^2} + (\lambda - \kappa) \ln \frac{p}{p_0}
\]  

(17)

where \((p^*, q^*)\) represents a normally consolidated stress state through which a normal yielding surface passes, as shown in Fig. 7. According to the similarity between the normal yielding surface and subloading yielding surface, the following relations can easily be obtained.

\[
\eta = q^* = \frac{q}{p^*} = \frac{p^*}{p} = \frac{p_{Nc}}{p_0}
\]  

\[p^* = \frac{p}{p_0} - \frac{p_{Nc}}{p_{Nc}} = \frac{p_{Nc}}{p_0} \]  

(18)

(19)

Substituting Eqs. (18) and (19) into Eq. (17), the following relation is obtained,

\[
e^p = N - N(S_0) + \frac{N(S_0) - \Gamma(S_0)}{2} \ln \frac{M^2 + \eta^2}{M^2} + (\lambda - \kappa) \ln \frac{p}{p_0}
\]  

(20)

where,

\[
C_v = \frac{\lambda - \kappa}{1 + e_0}, \quad \rho_e = (\lambda - \kappa) \ln \frac{p_{Nc}}{p_{Nc}}
\]  

(21)

\(\rho_e\) represents a void ratio difference between normally consolidated state and overconsolidated state under the same mean skeleton stress. As shown in Fig. 6, it is known that N.C.L.S. moves upward in parallel from N.C.L. when the degree of saturation decreases. Therefore, a new state variable \(\rho\), which represents a void ratio difference between N.C.L. and N.C.L.S. under the same mean skeleton stress, is expressed as,

\[
\rho = N(S_0) - N
\]  

(22)

Equation (20) then can be rewritten as,

\[
e^p = C_v \ln \frac{p}{p_0} + \frac{N(S_0) - \Gamma(S_0)}{2} \ln \frac{M^2 + \eta^2}{M^2} - \frac{\rho_e(\lambda - \kappa)}{1 + e_0}
\]  

(23)

Therefore, yielding function can be written as,

\[
f = \ln \frac{p}{p_0} + \frac{N(S_0) - \Gamma(S_0)}{2} \ln \frac{M^2 + \eta^2}{M^2} - \frac{\rho_e(\lambda - \kappa)}{1 + e_0}
\]  

(24)

By the definition of the critical state and some algebraic calculations, it is easy to obtain a useful relation as,

\[
d e^p = \lambda \frac{\partial f}{\partial p} |_{\eta = M} = 0 \Rightarrow N(S_0) - \Gamma(S_0) = (\lambda - \kappa) \ln 2
\]  

(25)
Substituting Eqs. (21) and (25) into Eq. (24), the following relation is obtained,

\[ f = \ln \frac{p}{p_0} + \ln \frac{M^2 + \eta^2}{M^2} - \frac{\rho_s}{1 + \epsilon_0} \frac{1}{C_p} + \frac{\rho_s}{1 + \epsilon_0} \frac{1}{C_p} \epsilon_s^p \frac{1}{C_p} = 0 \]

\[ f_s = \ln \frac{p}{p_0} + \ln \frac{M^2 + \eta^2}{M^2} \]  \hspace{1cm} (26)

where

\[ f_s = \ln \frac{p}{p_0} + \ln \frac{M^2 + \eta^2}{M^2} \]  \hspace{1cm} (27)

From consistency equation \( df = 0 \), it is known that

\[ df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} - d\left( \frac{\rho_s}{1 + \epsilon_0} \frac{1}{C_p} + d\left( \frac{\rho_s}{1 + \epsilon_0} \frac{1}{C_p} \right) \right) \]

\[ \epsilon_s^p = 0 \]  \hspace{1cm} (28)

In Eq. (28), it is necessary to give evolution equations for the development of the state variables \( \rho_s \) of overconsolidation and \( \rho_s \) of saturation, and the flow rule for plastic strain tensor in the following ways:

(i) Associate flow rule: \( d\epsilon_{ij}^p = A \frac{\partial f}{\partial \sigma_{ij}} \) \hspace{1cm} (29)

(ii) \( d\left( \frac{\rho_s}{1 + \epsilon_0} \right) \) = \(- A \frac{\rho}{p} \rho = \alpha \rho_s + \beta \rho_s \)

\[ N(S) = \frac{N_i - N}{S_i - S_i}; \quad N_i = N(S) \]  \hspace{1cm} (30)

(iii) \( \rho_s = N(S) - N = QS_i - S_i \); \[ Q = \frac{N_i - N}{S_i - S_i} \]

\[ d\rho_s = - Q dS \]

Where, \( S_i \) and \( S_i \) are the degrees of saturation under residual and saturated conditions. Equation (31) means that \( N(S_i) \) changes linearly with the degree of saturation. Parameters \( \alpha \) and \( \beta \) control the development of the state variables \( \rho_s \). \( N_i \) is the void ratios at \( N.C.L.S. \) under the reference mean skeleton stress \( p \), when the degree of saturation is in residual state, that is, \( N_i = N(S_i) \).

Volumetric strain increment can be divided into elastic and plastic parts as,

\[ d\epsilon_{ij}^v = d\epsilon_{ij}^p + d\epsilon_{ij}^v \] \hspace{1cm} (32)

Using Hooke’s theory with stiffness tensor \( E_{ijkl} \), incremental stress tensor can be expressed as,

\[ d\sigma_{ij} = E_{ijkl} (d\epsilon_{kl}) = E_{ijkl} d\epsilon_{kl} = E_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \] \hspace{1cm} (33)

Substituting this equation into Eq. (28), the following relation can be obtained,

\[ \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\epsilon_{kl} - \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\epsilon_{kl} = \frac{\partial f}{\partial \sigma_{ij}} \frac{1}{p} \]

\[ + \frac{Q}{1 + \epsilon_0} dS_i - A \frac{1}{C_p} \frac{\partial f}{\partial \sigma_{mn}} \] \hspace{1cm} (34)

which results in,

\[ A = \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\epsilon_{kl} + \frac{1}{C_p} \frac{Q}{1 + \epsilon_0} dS_i \]

\[ h_p + \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \] \hspace{1cm} (35)

where

\[ h_p = \frac{\partial f}{\partial \sigma_{mn}} \frac{\rho}{p} \] \hspace{1cm} (36)

Therefore it is easy to define the loading criteria as:

\[ A > 0 \] \hspace{1cm} loading

\[ A = 0 \] \hspace{1cm} neutral

\[ A < 0 \] \hspace{1cm} unloading

Substituting Eq. (35) into Eq. (29),

\[ d\epsilon_{ij}^p = \frac{h_p}{C_p} + \frac{\partial f}{\partial \sigma_{mn}} E_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{ij} \] \hspace{1cm} (38)

Meanwhile,

\[ d\sigma_{ij} = E_{ijkl} (d\epsilon_{kl}) \] \hspace{1cm} (39)

\[ = \frac{1}{C_p} \frac{Q}{1 + \epsilon_0} dS_i \] \hspace{1cm} (40)

\[ D = \frac{h_p}{C_p} + \frac{\partial f}{\partial \sigma_{mn}} E_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \]

Where

\[ A = \frac{1}{C_p} \frac{Q}{1 + \epsilon_0} dS_i \] \hspace{1cm} (41)

MOISTURE CHARACTERISTICS CURVE

In order to properly consider the influence of the degree of saturation on stress-strain-dilatancy relationships of unsaturated soils, it is necessary to give a precise description of the moisture characteristics, taking into consideration the moisture hysteresis. Therefore, a suitable moisture characteristics curve for suction-saturation relation should include skeleton curves and scanning curves so that at any moisture state \( (S_i, s) \), it is possible to obtain an incremental relation between suction and the degree of saturation as,

\[ dS_i = k_s dS \] \hspace{1cm} (43)

Where \( k_s \) is the tangential stiffness of suction-saturation relation.

In this paper, skeleton curves for the moisture characteristics with tangential and arc-tangential functions are
given in three different ways according to the state of the moisture as,

(i) Primary drying curve from slurry:

\[ S_0 = S_0^0 - \frac{2}{\pi} \left( S_0^0 - S_1^0 \right) \tan^{-1} \left( (e^{c_3} - 1)/e^{c_3} \right) \]  \hspace{1cm} (43)

or

\[ s = \frac{1}{c_1} \ln \left[ 1 + e^{c_3} \tan \left( \frac{\pi}{2} \frac{S_0^0 - S_1^0}{S_0^0 - S_1^0} \right) \right] \]  \hspace{1cm} (44)

(ii) Secondary drying curve experienced drying-wetting process:

\[ S_0 = S_0^1 - \frac{2}{\pi} \left( S_0^1 - S_1^0 \right) \tan^{-1} \left( (e^{c_3} - 1)/e^{c_3} \right) \]  \hspace{1cm} (45)

or

\[ s = \frac{1}{c_1} \ln \left[ 1 + e^{c_3} \tan \left( \frac{\pi}{2} \frac{S_0^1 - S_1^0}{S_0^1 - S_1^0} \right) \right] \]  \hspace{1cm} (46)

(iii) Wetting curve:

\[ S_0 = S_0^1 - \frac{2}{\pi} \left( S_0^1 - S_1^0 \right) \tan^{-1} \left( (e^{c_3} - 1)/e^{c_3} \right) \]  \hspace{1cm} (47)

or

\[ s = \frac{1}{c_2} \ln \left[ 1 + e^{c_3} \tan \left( \frac{\pi}{2} \frac{S_0^1 - S_1^0}{S_0^1 - S_1^0} \right) \right] \]  \hspace{1cm} (48)

where \( S_0 \) is a parameter corresponding to drying AEV and \( S_0^0 \) is a parameter corresponding to WEV, as shown in Fig. 8. \( c_1 \) and \( c_2 \) are scaling factors that control the shape of the curves. \( S_0^0 \) is the degree of saturation of a slurry under fully saturated condition and is equal to 1.0.

As to the scanning curve in the process of drying-wetting process between the skeleton curves, the incremental relation between suction and saturation is expressed as,

\[ k_s = k_{s0}^{-1} + k_{s1}^{-1} \]  \hspace{1cm} (49)

\( k_{s0} \) is the gradient of suction-saturation relation under the condition that inner variable \( r \) equals to 0. \( k_{s1} \) is expressed as:

\[ k_{s1} = k_{s0}^{k_s} \left( 1 + c_1 \frac{1-r}{r} \right) \]  \hspace{1cm} (50)

where \( c_1 \) is a scaling factor controlling the curvature of the scanning curve. \( k_{s1} \) is the gradient of the corresponding skeleton curve on which the moisture state \( (S_0, s) \) is locating under the condition that \( r \) equals to 1, as shown in Fig. 8. According to the illustration in Fig. 8, the inner variable \( r \) is defined as,

\[ r = \begin{cases} \frac{\delta_s}{\delta_r} & ds > 0 \\ \frac{\delta_s}{\delta_r} & ds \leq 0 \end{cases} \]  \hspace{1cm} (51)

Eq. (49) means that the stiffness of \( k_s \) consists of two parts, \( k_{s0} \) and \( k_{s1} \) in a way that its value looks like the value of a spring consisted from two series springs. It is easily understood from Eqs. (49) and (50) that if \( r = 0 \), \( k_{s1} \) will be infinite and \( k_s = k_{s0} \). While if \( r = 1 \) and \( k_{s0} \gg k_{s1} \), \( k_s \) will be equal to \( k_{s1} \), which coincides with the gradient of the skeleton curve. This explanation can also be easily understood by means of the illustration shown in Fig. 8.

Eight parameters are involved in the proposed moisture characteristics curve, among which three parameters \( c_1, c_2 \) and \( c_3 \) are determined with curve fitting method while other five parameters, \( k_{s0}, S_0^0, S_0^1, S_1^0 \) and \( S_0 \) have definite physical meaning and can be determined by the test of moisture characteristics easily.

Figure 9 shows a theoretical prediction of moisture characteristics curve of a fictional unsaturated silt. It is very clear that all main features of the moisture characteristics can be properly described. The values of the parameters are listed in Table 1.

In the area between the primary drying curve and the secondary drying curve, the upper bound for the scanning curve is changed from the secondary drying curve to the primary drying curve; and the lower bound for the scanning curve is changed from the wetting curve to a combined curve made from the wetting curve in the region \( S_0 \leq S_1^0 \) and the line \( s = 0 \) in the region \( S_0 > S_1^0 \). The scanning rule, however, is totally the same as those for the secondary drying-wetting process discussed in Eqs. (49) to (51). So many hysteresis curves have been

![Fig. 8. Image of moisture characteristic curve of unsaturated soil](image1)

![Fig. 9. Simulated moisture characteristics curve of unsaturated fictional silt](image2)
Table 1. Parameters of moisture characteristics curve of unsaturated fictional silt

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated degrees of saturation $S_s^r$</td>
<td>0.82</td>
</tr>
<tr>
<td>Residual degrees of saturation $S_r$</td>
<td>0.64</td>
</tr>
<tr>
<td>Parameter corresponding to drying AEV (kPa) $S_d$</td>
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</tr>
<tr>
<td>Parameter corresponding to wetting WEV (kPa) $S_w$</td>
<td>320</td>
</tr>
<tr>
<td>Initial stiffness of scanning curve (kPa) $k_{sc}$</td>
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<td>Parameter of shape function $c_1$</td>
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<tr>
<td>Parameter of shape function $c_2$</td>
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</tr>
<tr>
<td>Parameter of shape function $c_3$</td>
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</tbody>
</table>

proposed in previous researches that it is almost impossible to list up all of them. What should be emphasized in this research which is different from previous models is, that it is possible to describe the hysteresis relation for both saturated and unsaturated states smoothly with relatively small number of parameters.

PERFORMANCE OF THE PROPOSED MODEL

Nine parameters are involved in the proposed model, among which five parameters, $M$, $N$, $\lambda$, $\kappa$, and $\nu$ are the same as the ones in Cam-clay model. The other three parameters $a$, $b$ and $\beta$, the parameters that control the losing rate of overconsolidation contributed both from the state parameters $\rho_e$ and $\rho_r$ when the soil subjected to shearing or compression, have clear physical meanings and can be easily determined based on conventional triaxial compression tests under drained and exhausted conditions. The ninth parameter, $N$, the void ratio at N.C.L.S. under $p = p_r$ and residual unsaturated state $N_r = N(S^r)$, is just a physical state like $N$.

In order to check the performance of the proposed model, a fictional silt with a moisture characteristics curve shown in Fig. 9, whose parameters are listed in Table 1, is simulated under different loading conditions in isotropic consolidation tests and triaxial compression tests.

Simulation of Isotropic Consolidation Tests

Figure 10 shows the simulated volumetric change of the normally consolidated silt subjected to different suction in isotropic consolidation test. The loading path includes three steps as listed below:

Step I: Applying suction to the silt initially under saturated condition

Step II: Keeping the suction in a constant value and applying net mean stress from 0 to 10,000 kPa

Step III: Reducing the suction to zero (submergence)

The reason for using this loading path is to ensure that the influence of suction on volumetric change in isotropic consolidation can be independently investigated because all the loading tests are started from the samples under the same initial condition. The material parameters used in the model are listed in Table 2. The correctness of the simulation is evident because so many test results have been reported in the literature. The volumetric contraction due to submergence in the Step III, however, is not completely equal to the difference between $\varepsilon_r - \ln p_{net}$ curves with suction and without suction. In the figure, mean net stress is expressed as $p_{net} = \sigma_{ii}^n/3$.

Figure 11 shows more detailed performance of volumetric contraction due to submergence at different values of net stress in the isotropic consolidation test with a constant suction of 1500 kPa. It can be seen that a residual difference between $\varepsilon_r - \ln p_{net}$ curves with suction and

Table 2. Material parameters of fictional silt in oedometer tests

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression index $\lambda$</td>
<td>0.050</td>
</tr>
<tr>
<td>Swelling index $\kappa$</td>
<td>0.010</td>
</tr>
<tr>
<td>Critical state parameter $M$</td>
<td>1.0</td>
</tr>
<tr>
<td>Void ratio $N$ ($p' = 98$ kPa on N.C.L.)</td>
<td>1.14</td>
</tr>
<tr>
<td>Poisson's ratio $\nu$</td>
<td>0.30</td>
</tr>
<tr>
<td>Parameter of overconsolidation $a$</td>
<td>5.00</td>
</tr>
<tr>
<td>Parameter of suction $b$</td>
<td>5.50</td>
</tr>
<tr>
<td>Parameter of overconsolidation $\beta$</td>
<td>1.0</td>
</tr>
<tr>
<td>Void ratio $N_r$ ($p' = 98$ kPa on N.C.L.S.)</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Fig. 10. Simulated relations between void ratio and net stress in isotropic compression under different constant suctions

Fig. 11. Simulated processes of volumetric contraction due to submergence of unsaturated soil
without suction after the suction is completely released, is negligible at relatively low net stress but will increase to some extent. The reason why this phenomenon occurs is that the development of state values $\rho_s$ of overconsolidation and $\rho_e$ of saturation are independent. Therefore, $\rho_e$ will be zero at the end of submergence ($s = 0$) but $\rho_s$ is not necessary to be zero. After the submergence has been completed ($s = 0$), if a loading such as shearing continues, then according to the definition of $\rho_s$, the soil will reach C.S.L., where $\rho_s$ will definitely become zero. The correctness of this independence is still need to be verified by test results. At present stage, it just predicts the possibility of the existence of the residual difference.

![Graphs of skeleton stress, strain, and dilatancy relations at different confining stresses and suctions](image)

(a) $\sigma_3 = 50$ kPa  
(b) $\sigma_3 = 100$ kPa  
(c) $\sigma_3 = 400$ kPa

Fig. 12. Skeleton stress, strain and dilatancy relations at different confining stresses and suctions (Cui and Delage, 1996)
Simulation of Triaxial Compression Tests under Different Suction and Confining Stress

Figure 12 shows the skeleton stress, strain and dilatancy relations of triaxial compression test on unsaturated silt under drained and exhausted conditions under different confining stresses and suctions (Cui and Delage, 1996). Because the silt samples were statically compacted under certain vertical stresses, it would be in slightly overconsolidated state if the confining stress is relatively low, e.g., \( \sigma_3 = 50 \) kPa. The results are simulated with the proposed model. Because there is no detailed data related to moisture characteristics curve, the moisture characteristics curve of the fictional unsaturated silt shown in Fig. 9 is assumed for the silt. Therefore, the following predictions by the model do not aim to describe precisely the test results but an overall behavior of the unsaturated silt qualitatively.

The material parameters of the fictional silt in triaxial tests are listed in Table 3. In simulating the triaxial compression tests, suction loading before shearing is calculated at first so that all the samples simulated are started from the same initial conditions except the state variable of overconsolidation which is dependent on vertical stress once applied in the static compaction of the samples. The initial values of state variables are listed in Table 4.

Figures 13 to 15 show the simulated skeleton stress, strain and dilatancy relations and skeleton stress paths in triaxial compression tests on unsaturated silts under drained and exhausted condition with different confining stresses. During shearing, the confining stress and the suction are kept constant. The volumetric strains shown in the figures are countered from the point on which the shearing started.

In the case of \( \sigma_3 = 50 \) kPa, due to the slightly overconsolidated condition, a typical strain hardening and strain softening accompanied by positive dilatancy can be seen in the simulated results, especially in the case of high suction. Meanwhile, it is known from the figures that the higher the suction is, the higher the stress difference will be in all cases. The simulated results agree qualitatively well with the test results shown in Fig. 12, though the fictional silt is not the same material as the real silt. Due to the definitions of skeleton stress and moisture characteristics curve, if suction is kept constant during shear, then the degree of saturation will also be constant and consequently the skeleton stress path will be the same as total stress path if the air pressure is kept constant. In reality, however, during shearing with constant suction, drainage may occur and therefore degree of saturation may change slightly. As the result, the skeleton stress path may be a little different from the total stress path. Fortunately, the difference is usually small enough to be negligible (see the results in the work by Kodaka et al., 2006).

---

Table 3. Material parameters of fictional silt in triaxial tests

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression index ( k )</td>
<td>0.050</td>
</tr>
<tr>
<td>Swelling index ( \kappa )</td>
<td>0.010</td>
</tr>
<tr>
<td>Critical state parameter ( M )</td>
<td>1.0</td>
</tr>
<tr>
<td>Void ratio ( N (p' = 98 \text{ kPa on N.C.L}) )</td>
<td>1.14</td>
</tr>
<tr>
<td>Poisson’s ratio ( \nu )</td>
<td>0.30</td>
</tr>
</tbody>
</table>

| Parameter of overconsolidation \( a \) | 5.0   |
| Parameter of suction \( b \)          | 0.50  |
| Parameter of overconsolidation \( b \) | 1.0   |
| Void ratio \( N_s (p' = 98 \text{ kPa on N.C.L.S}) \) | 1.28 |

Table 4. Initial value of state variables in triaxial tests

<table>
<thead>
<tr>
<th>State variable of saturation ( \rho_s )</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>State variable of overconsolidation ( \rho_c )</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\( \sigma_1 = 50 \) kPa
\( \sigma_1 = 100 \) kPa
\( \sigma_1 = 400 \) kPa

---

Fig. 13. Simulated stress, strain and dilatancy relations and skeleton stress paths (\( \sigma_3 = 50 \) kPa)
Simulation of Collapse by Submergence in Triaxial Compression Tests

Another important phenomenon of unsaturated soil during shearing is the so called collapse by submergence. In this section, collapse by submergence happened in three different shearing stages listed below is simulated:

(i) Submergence at post-peak shearing
(ii) Submergence on peak stage
(iii) Submergence at pre-peak shearing stage

Furthermore, in order to investigate the influence of submergence rate on the collapse behavior during shearing, two different submergence rates are considered in the simulation. One is gradual submergence, which is carried out with a gradual reduction of suction accompanied by simultaneous shearing till the end of the shearing test. This load process is called as loading path 1. Another is abrupt submergence, in which submergence is carried out instantly at first with a prescribed suction reduction under the condition that the vertical strain is kept constant and then the strain-controlled shearing continues under the condition that the reduced suction is kept constant. This loading process is called as loading path 2. Loading path 3 is just a normal triaxial compression test without submergence.

Figure 16 shows the simulated results of above loading paths in the case of $\sigma_3 = 50$ kPa, $s = 800$ kPa. The extent of the suction reduction in above submergence processes is 50% of the original suction. It is known from the figure...
that no matter what kind of shearing stage at which submergence is carried out, collapse will definitely occur due to the loss of suction. Meanwhile, submergence always causes extra positive dilatancy no matter what kind of submergence it may be. Abrupt submergence always causes an immediate reduction of strength which is usually called as collapse and extra positive dilatancy. The submergence rate affects the stress-strain-dilatancy relation a lot but at the end of the shearing, they will merge to the same destination, the point at C.S.L.S. or C.S.L., depending on the value of the suction.

Figure 17 shows the skeleton stress paths in the process of the collapse by submergence under different loading paths. In the submergence process at pre-peak shearing stage, if it is carried out abruptly, then stress path will approach the residual line in the region below the line. Otherwise, stress state will always overpass the line and finally reaches the critical state.

**Verification of the Model by Drained and Exhausted Triaxial Compression Tests for a Rockfill with Submergence Process**

Kohgo et al. (2007b) conducted a systematic laboratory test on rockfill materials with drained and exhausted triaxial compression tests including submergence process to investigate the influence of the degree of saturation on mechanical behavior of the rockfill. Detailed description about the tests can be found in the reference. The size of the test specimen is 30 cm in diameter and 60 cm in height. Figure 18 shows the test and simulated moisture characteristics curve of the unsaturated rockfill. The parameters involved in the moisture characteristics curve are listed in Table 5. In the drained and exhausted triaxial compression tests under constant confining pressure,
three kinds of tests were conducted for the rockfill, that is, tests for saturated specimen, tests for unsaturated specimen with constant suction and tests for unsaturated specimen with submergence process. Table 6 listed the

![Figure 17](image1.png)

**Fig. 17.** Influence of submergence on stress path

![Figure 18](image2.png)

**Fig. 18.** Test and simulated moisture characteristics curve of unsaturated rockfill (test data from the work by Kohgo et al, 2007b)

![Figure 18](image3.png)

**Table 5. Parameters of moisture characteristics curve of unsaturated rockfill**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated degrees of saturation $S'_s$</td>
<td>0.85</td>
</tr>
<tr>
<td>Residual degrees of saturation $S'_r$</td>
<td>0.20</td>
</tr>
<tr>
<td>Parameter corresponding to drying AEV (kPa) $S_d$</td>
<td>2.0</td>
</tr>
<tr>
<td>Parameter corresponding to wetting WEV (kPa) $S_w$</td>
<td>0.07</td>
</tr>
<tr>
<td>Initial stiffness of scanning curve (kPa) $k^e_{sp}$</td>
<td>700</td>
</tr>
<tr>
<td>Parameter of shape function $c_1$</td>
<td>0.05</td>
</tr>
<tr>
<td>Parameter of shape function $c_2$</td>
<td>0.05</td>
</tr>
<tr>
<td>Parameter of shape function $c_3$</td>
<td>10.0</td>
</tr>
</tbody>
</table>

**Table 6. Material parameters of rockfill**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression index $\lambda$</td>
<td>0.14</td>
</tr>
<tr>
<td>Swelling index $\kappa$</td>
<td>0.0010</td>
</tr>
<tr>
<td>Critical state parameter $M$</td>
<td>1.7</td>
</tr>
<tr>
<td>Void ratio $N$ ($p' = 98$ kPa on N.C.L.)</td>
<td>0.61</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.30</td>
</tr>
<tr>
<td>Parameter of overconsolidation $a$</td>
<td>5.0</td>
</tr>
<tr>
<td>Parameter of suction $b$</td>
<td>0.50</td>
</tr>
<tr>
<td>Parameter of overconsolidation $\beta$</td>
<td>3.0</td>
</tr>
<tr>
<td>Void ratio $N_r$ ($p' = 98$ kPa on N.C.L.S.)</td>
<td>0.80</td>
</tr>
</tbody>
</table>
material parameters involved in the proposed model, among which, apart from the parameters \(a\), \(b\) and \(\beta\), other parameters are determined based on the laboratory tests. Figure 19 shows the comparison between the test and simulated results of the drained and exhausted triaxial compression tests for the rockfill. The accuracy of the model is quite good comparing with its parameters used. From above discussion, it is concluded that collapse by submergence can be properly described by the proposed model.

**CONCLUSIONS**

In this paper, the authors proposed a new constitutive model for unsaturated soil based on the framework of skeleton stress and degree of saturation. The constitutive model is therefore possible to be able to describe not only the behavior of unsaturated soil but also saturated soil because the skeleton stress can smoothly shift to effective stress if saturation changes from unsaturated condition to saturated condition. The main features of the proposed model can be given as below:

1. The proposed model is very simple and is derived from the base of modified Cam-clay model. It has nine parameters, among which five are the same as those in the Cam-clay model and are familiar to most geotechnical researchers. The other three parameters \(a\), \(b\) and \(\beta\), the parameters that control the losing rate of overconsolidation affected by the state parameter of overconsolidation \(\rho_c\) and saturation \(\rho_s\), have clear physical meanings and can be easily determined based on conventional triaxial compression tests under drained and exhausted conditions. The ninth parameter, \(N\), the void ratio at \(N.C.L.S.\) under \(p=p_r\) and residual unsaturated state \(S_r\), is just a physical state like \(N\) and can be easily determined with the test of moisture characteristics.

2. In order to properly consider the influence of the degree of saturation on stress-strain-dilatancy relationships of unsaturated soils, a theoretical description of moisture characteristics, taking into consideration the moisture hysteresis is proposed, in which the skeleton curves and scanning curves are given in such a way that at any moisture state \((S, s)\), it is possible to obtain an incremental relation between suction and the degree of saturation. Eight parameters are involved in the proposed moisture characteristics curve, among which three parameters \(c_1\), \(c_2\) and \(c_3\) are determined with curve fitting method while other five parameters, \(k_s\), \(S_{r,1}\), \(S_{r,2}\), \(S_d\) and \(S_w\) have definite physical meaning and can be determined by the test of moisture characteristics easily.

3. The model is proposed based on three assumptions, the first is that normally consolidated line in unsaturated state \((N.C.L.S.)\) is parallel to the normally consolidated line in saturated state \((N.C.L.)\) but in a higher position than \(N.C.L.\), which means that under the same mean skeleton stress, unsaturated soil can keep higher void ratio than those of saturated soil. The second is that no matter what kind condition of saturation may be, compression index \(l\) and swelling index \(k\) in void ratio-skeleton stress relation are always kept constant which makes the model very simple and easily understood. The third is that the shear stress ratio \(\eta\) at critical state will be unique no matter what kind of saturation condition may be. All these assumptions can be confirmed with the tests results available in literature.

4. The proposed model can well describe the volumetric change of normally consolidated soil in isotropic consolidation test under different suctions. For instance, in isotropic consolidation test under different suctions, \(\varepsilon_3 = \ln p_{net}\) relation under lower suction behaves like normally consolidated soil while under higher suction it behaves like overconsolidated soil. The volumetric contraction or collapse due to submergence during the
isotropic consolidation test can also be well described.

5. Skeleton stress, strain and dilatancy relations and skeleton stress paths in triaxial compression tests on unsaturated fictional silt under drained and exhausted condition with different confining stresses are also simulated. The overall behavior of unsaturated soil under normally consolidated and overconsolidated states can be described uniquely within the framework of skeleton stress and saturation. The collapse of unsaturated soil due to submergences at the stages of pre-peak, on-peak and post-peak of stress-strain relation is also simulated. It is known from the simulation that whenever submergence happens, collapse will always occur and cause extra positive dilatancy. Furthermore, abrupt submergence always causes an immediate collapse and extra positive dilatancy. The submergence rate affects the stress-strain-dilatancy relation a lot but at the end of the shearing, they will merge to the same destination, that is, the same point at critical state line.

6. In the proposed moisture characteristics curve, if suction is kept constant during shear, then the degree of saturation will also be constant. In reality, however, even if suction is kept constant during shearing, drainage may occur and therefore degree of saturation may change, especially in the case when deformation becomes large enough to reach the range of finite deformation. It is therefore necessary to incorporate this factor into the moisture characteristics curve in future studies.

REFERENCES