CORRELATING CYCLIC STRENGTH WITH FINES CONTENTS THROUGH STATE PARAMETERS

An-Bin Huang* and Sin-Yu Chuang**

ABSTRACT

Attempts have been made in the past few decades to establish an equivalent void ratio indicative of the effective density state of sands that contain fines. The concept of an equivalent void ratio is most useful when the grain size distribution of the sand/silt mixture is similar to that of binary packing. Due to its grain characteristics and the wide range of compressibility when the fines content changes, correlating the cyclic strength to fines content via the equivalent void ratio for Mai Liao Sand (MLS) is not a straightforward procedure. Earlier studies have demonstrated the effectiveness of relating cyclic strength to state parameters for sands. Using a series of cyclic triaxial test results as a database, the authors demonstrated the effectiveness of relating cyclic strength to fines contents \( f_c \) through a state parameter for MLS with \( f_c \) ranging from 0 to 30\% that takes advantage of the equivalent void ratio. In addition to the density and stress states, the state parameter based correlations indirectly reflect the effects of soil grain characteristics. The drawback of requiring a separate critical state line when \( f_c \) changes can be circumvented by using a unified critical state line based on the equivalent void ratio.

Key words: cyclic strength, sand, fines, state parameter, void ratio (IGC: D7)

INTRODUCTION

Studies on the effects of fines (soil particles passing #200 sieve) on the cyclic behavior of granular soils have included a wide spectrum of gradations that span from clean sand, silty sand, clayey sand, sandy silt to pure silt. The unified soil classification of these soils can vary from poorly graded sand (SP), well graded sand to silty sand (SW-SM), silty sand (SM), silty sand to clayey sand (SM-SC) to low plastic silt (ML), depending on the amount and characteristics of the fines. The term silt/sand (M/S) is proposed to serve as an abbreviated term to describe granular soils with various contents of nonplastic and plastic fines with a wide range of gradations, inclusively.

Been and Jefferies (1985) proposed the state parameter \( \Psi \) as a semi-empirical normalizing parameter for sand behavior. The state parameter \( \Psi \) defines the void ratio \( e \) and effective mean normal stress \( p' \) level of a sand relative to a reference state (the steady state) as shown in Fig. 1. The steady state of deformation for any mass of particles is that state in which the mass is continuously deforming at constant volume, constant effective mean normal stress, constant shear stress, and constant velocity (Poulos, 1981). The effective mean normal stress, \( p' \) shown in Fig. 1 is defined as

\[
p' = \frac{(\sigma_v' + 2\sigma_h')}{3}
\]

where \( \sigma_v' \) and \( \sigma_h' \) = effective vertical and horizontal stress, respectively.

The steady state line is the locus of steady state points in the void ratio/stress space. The projection of the steady state line (SSL) on the \( e-p' \) plane is used for data presentation and the definition of the state parameter \( \Psi \).
as shown in Fig. 1. The steady state has traditionally been measured using undrained tests on loose sand samples. Further studies have indicated that the same relationship between \( e_{ss} \) and \( p'_{ca} \) exists if it is determined using drained tests (Been et al., 1991). In this case the \( e_{ss} - p'_{ca} \) relationship is referred to as the critical state line (CSL). It should be noted that the CSL is the same as SSL according to Been et al. (1991). For simplicity, the term critical state line will be used hereinafter.

Studies on M/S soils have mostly been concentrated on laboratory tests using reconstituted specimens. The M/S specimens tend to have been made of mixtures of clean quartz (e.g., Ottawa) sand with crushed silica, kaolin or other types of natural silt. These mixtures of sand and fines, or gap graded artificial soils have been compared to those of coarse and fine spherical grains (Lade et al., 1998), or spherical binary packing. As the diameter ratio of the coarse grains over that of the fine grains exceeded approximately 7, one can expect a bilinear, V shaped correlation depicted in Fig. 2, is affected by the diameter ratio of the coarse grains over that of the fine grains. The V shape becomes less obvious as this ratio approaches 30. The \( e_{min} \) reaches its lowest value as the fines content approaches 30%. The amount of decrease in \( e_{min} \), or depth of the V shaped correlation depicted in Fig. 2, is affected by the diameter ratio of the coarse grains over that of the fine grains. Sand particles may be round in shape, but they are rarely exactly spherical. There can be various degrees of grain size differences or distribution, even for uniformly graded clean sand. For these reasons, natural or artificially mixed M/S soils may not meet the description of binary packing in a strict sense and the threshold fines content may therefore not be 30%. Some of the studies on artificial silty sand specimens, mostly silica in nature, have demonstrated that the threshold fines content generally ranged from 25 to 45% (Koester, 1994; Polito, 1999; Xenaki and Athanassopoulos, 2003; Papadopoulou and Tika, 2008; Rahman et al., 2008).

A series of void ratio indices that relate the active grain contacts (i.e., the soil skeleton) to fines contents have been proposed and evolved in the past few decades (Rahman et al., 2008). These indices are based on the concept of spherical binary packing. The intergranular contact index void ratio, \( e^* \) (Thevanayagam, 2000) or simply the equivalent granular void ratio is a relatively recent development. It is defined as:

\[
    e^* = \frac{e + (1 - b)f_c}{1 - (1 - b)f_c} \tag{2}
\]

where

- \( e = \) void ratio
- \( b = \) parameter reflects the fraction of fines participating in the force structure of the solid skeleton
- \( f_c = \) fines content in decimal

The value of \( b \) ranges from 0 to 1. A higher \( b \) corresponds to an increase in the contribution of fines in the force structure of the soil skeleton. Predicting the \( b \) value was considered to be problematic and controversial by Rahman et al. (2008). Earlier methods generally involved the selection of a case-specific, fines content independent \( b \) value by a back-analysis so that the steady state or cyclic strength for a given sand-fines type can be correlated with an equivalent granular void ratio irrespective of the fines content (Ni et al., 2004). Thevanayagam et al. (2002) suggested that \( b \) depends on \( U_s/U_{t,cr} \), where \( U_s \) is the uniformity coefficient of the coarse matrix, \( U_t \) is the uniformity coefficient of fines and \( \chi \) is the diameter ratio defined as \( D_{50}/d_{50} \) (the ratio of the medium coarse grain size over that of the fines). Ni et al. (2004) reported that \( b \) depends on the grain size ratio between the lower 10% fractile of the coarse \( (D_{10}) \) and \( d_{50} \) of fine particles \( (\chi = D_{10}/d_{50}) \).

Rahman et al. (2008) proposed a semi-empirical equation that defines \( b \) as:

\[
    b = \{1 - \exp \left[-2.5(f_c)^2/(1 - r^{0.25})\right]\}(rf_c/f_{thre})^r \tag{3}
\]

where

- \( r = \chi^{-1} \)
- \( f_{thre} = \) threshold fines content

According to Eq. (3), \( b \) increases with the fines content. Most of the cases reported in the literature involve the use of round shaped coarse particles artificially mixed with fines. These reports showed promising results in the use of an equivalent granular void ratio as an index to reflect the density state of an M/S soil. Rahman et al. (2008) reported that for certain M/S soils, a series of void ratio \( (e) \) based critical state lines that correspond to various

![Fig. 2. A conceptual description of the effects of fines on minimum void ratios of binary packing (after Lade et al., 1998)](image)
finer contents can be collapsed into one by replacing $e$ with $e^*$. This unified $e^*$ based critical state line should be close to or the same as that of zero fines content where $e^* = e$.

The concept of an equivalent granular void ratio covers only the aspect of the density state. The cyclic or undrained strength of sands is generally dependent on the stress and density states. Relating the fines content to the cyclic strength through a state parameter is a more desirable approach as it considers both density and stress states. Jefferies and Been (2006) have demonstrated a clear trend between the state parameter and cyclic strength of clean quartz sands. Huang et al. (2004) and Papadopoulou and Tika (2008) demonstrated consistent relationships between the cyclic strength and state parameter for silty sands based on their laboratory tests.

Mai Liao Sand (MLS) represents a typical M/S soil deposit in Central Western Taiwan. The term Mai Liao Sand is used loosely, and only because most of the tests reported in the paper involve a mixture of sand and silt which came from Mai Liao. Due to the nature of its parental rock, the coarse sand grains of MLS are mostly platy. Because of its mineral content, MLS is significantly more compressible than the typical clean quartz sand reported in the literature (Huang et al., 1999, 2004). The authors have performed a series of monotonic and cyclic triaxial tests on MLS with fines contents of 0, 15 and 30%. These data allow the concept of using the state parameter as a medium for relating the fines content and cyclic strength to be explored. The majority of properties of Mai Liao Sand and the results of laboratory triaxial tests have been reported (Huang et al., 1999, 2004, 2005) earlier. Additional cyclic triaxial tests have been performed to supplement the available data. The physical and mechanical properties of Mai Liao Sand as they relate to the subject of the paper are summarized first. The feasibility of using an equivalent granular void ratio and a state parameter as a means to correlate cyclic strength with fines contents is then evaluated.

**PHYSICAL PROPERTIES OF MAI LIAO SAND/SILT**

Mai Liao Sand originated from the weathering of sedimentary and metamorphic rocks such as sandstone, shale and slate in the Central Mountain Range of Taiwan. The process of transportation ground the fractured rock into sand and silt particles. Figure 3 shows typical grain size distribution curves of MLS samples taken from a land reclamation site. The ratio of $D_{10}$ (=0.08 mm) over $d_{50}$ (=0.044 mm) $(\chi)$ was 1.82. The ratio of medium size grains of the coarse grain ($D_{50}$ = 0.125 mm) over $d_{50}$ $(\chi_T)$ was 2.84. These values are significantly less than the ideal minimum ratio of 7 for binary packing. A few metric tones of MLS was acquired from a land reclamation site. The sand was washed and sieved through a #200 sieve to separate the fines from the sand particles. The specific gravity of clean MLS sand particles had an average value of 2.69 and the fines had an average specific gravity of 2.71. An X-ray diffraction analysis of MLS showed significant amounts of muscovite and chlorite in addition to quartz. The relatively soft muscovite is more prevalent in the fines. The sand particles retained on the #200 sieve were angular and platy as indicated in the scanning electron microscope photograph shown in Fig. 4. The fines which passed through the No. 200 sieve had a liquid limit of 32 and a plasticity index of less than 8. The processed fines and sand particles were mixed according to required proportions to reach the desired fines content for soil specimens. Figure 5 shows the variation of minimum ($e_{\min}$) and maximum ($e_{\max}$) void ratios of MLS as the fines contents changed from 0 to 80%. The $e_{\min}$ (maximum density) was determined according to ASTM D4253 method 1A using a standard 152.4 mm inside diameter compaction mold (total volume = 2830 cm$^3$). The dry sand placed in the compaction mold was subjected to a
surcharge of 14 kPa, and then electromagnetically vibrated under a 60 Hz frequency for 8 minutes. The $e_{\text{max}}$ (minimum density) was obtained according to ASTM D4254 method C, using a 2000 mL glass graduated cylinder. Approximately 1000 g of dry sand was placed in the glass cylinder and then plugged with a stopper. The total volume under the loosest state was determined according to the compressibility of MLS, the relative density could only serve the purpose of initial specimen setup. The analysis of test data used void ratios to represent the soil density state.

Apparently, due to the characteristics of its grain size distribution (i.e., low grain size ratio) and grain shapes (i.e., platy sand grains), the relationship between $e_{\text{min}}$ and fines content for MLS shows little obvious similarity to a V shape as shown in Fig. 2.

**MECHANICAL PROPERTIES OF MAIL LIAO SAND/SILT**

The mechanical property tests included isotropic consolidation, and isotropically consolidated undrained monotonic and cyclic triaxial tests on reconstituted MLS. The consolidation and monotonic triaxial tests were performed on specimens prepared by the moist tamping (MT) method. The MT method was used to create specimens with a wide range of initial densities for the specimens. For cyclic triaxial tests, the specimens were made by water sedimentation (WS) and dry deposition (DD), in addition to the MT method. The additional specimen preparation methods were chosen to evaluate the effects of soil fabric. Readers are referred to Huang et al. (2004) for details on the specimen preparation and triaxial test procedures for this series of laboratory tests. Soil specimens with $f_c$ of 0, 15, 30 and 50% by weight were used in the consolidation and monotonic triaxial tests in this study. No cyclic triaxial tests on specimens with $f_c$ of 50% will be reported herein.

The main purpose of the monotonic triaxial tests was to establish the critical state lines for MLS with various fines contents. The term “critical state line” is used in a general way as it can be a curve or equally referred to as “steady state line”. Figure 6 depicts the consolidation curves and critical state lines resulted from the isotropic consolidation and isotropically consolidated undrained monotonic axial compression triaxial (CIU) tests.

For consolidation tests, $p'$ was the effective confining stress applied during consolidation. For critical state lines, $p'$ represents the effective mean normal stress at the critical state. The initial void ratio ($e_o$) marked in Fig. 6 are back calculated, post-consolidation void ratios determined from the water content of the specimen after the test was completed. Details of obtaining the post-consolidation void ratio for triaxial specimens are given in Huang et al. (2004). The $e_{\text{min}}$ and $e_{\text{max}}$ are marked on the figure. In general, the compressibility increased with fines content and $p'$, and decreased as the specimen became denser. The void ratio, $e$, after application of the initial consolidation pressure of 10 kPa may be significantly less than $e_o$. When the fines contents reached 50%, the application of the initial confining stress and the saturation process apparently caused enough compression that the consolidation curves were almost indistinguishable among specimens with different $e_o$ values. It is thus imperative that the post-consolidation void be used in presenting M/S data that involve density states. Since no meaningful critical state line could be obtained from the monotonic triaxial tests for $f_c=50\%$, none is reported in Fig. 6.

The specimens were sheared monotonically by axial compression to strains well in excess of 20% to reach or be close to critical state (Been et al., 1991). The critical state lines for each of the three types of fines contents were more or less parallel to each other but moved downward as the fines content increased. This phenomenon is similar to the observation reported by Kikumoto et al. (2009). For the densities and confining stresses applied, the specimens rarely demonstrated dilatant behavior, even in the case of dense specimens. The relationships among deviator stress, excess pore pressure and axial strain, as well as the effective stress paths in terms of $q = (\sigma' - \sigma_1)$ and $p'' = (\sigma' + 2\sigma_1)/3$ from all the consolidated undrained triaxial tests are depicted in Figs. 7(a) to 7(c). A combination of deviator stress, $q$ and pore pressure was chosen for each undrained triaxial test according to the definition of critical state where the sand deformed continuously at relatively constant pore pressure and constant $q$. The critical state points are marked in Figs. 7(a) to 7(c). A straight line was fitted to the data set of $(p', q)$ that corresponded to the critical state of each triaxial test, as shown in Fig. 7(c). The slope of this fitted line was referred to as $M_s$. For the tests performed, $M_s=1.24$ for

![Fig. 5. Minimum and maximum void ratios versus fines contents](image-url)
FC = 0 to 30%, and $M_s = 1.21$ as FC reached 50%. These $M_s \left(= \frac{6 \sin \phi'}{3 - \sin \phi} \right)$ values most likely correspond to the interparticle friction angles ($\phi'$) of 30.9° and 30.2°, respectively. The similarity in $\phi'$ regardless of the fines content, indicates that the coarse and fine MLS particles had similar grain to grain frictional behavior.

Table 1 compiles all of the related cyclic triaxial tests performed in the past few years on MLS specimens. These tests provided a database to evaluate the relationships among $e$, $e^*$, cyclic strength and the state parameter ($\Psi$). In the series of cyclic triaxial tests on specimens prepared by the water sedimentation (WS) method, all specimens were consolidated and sheared under a $p'$ of 100 kPa. In cyclic triaxial tests on dry deposition (DD) specimens, $p'$ values of 50, 100 and 200 kPa were used. The advantage of the moist tamping (MT) method is that it creates specimens with a wide range of densities, which is why it has been used extensively in cyclic triaxial tests. In this series of tests, specimens with an over consolidation ratio (OCR) of 4 were used. For over consolidated specimens, the $p'$ values shown in Table 1 are the effective mean normal stress after the unloading stage, just prior to the cyclic triaxial tests. For MT specimens, $p'$ varied from 80 to 200 kPa. Collectively, the cyclic triaxial test series made it possible to consider the effects of density state, stress state, stress history and soil fabric.

The cyclic strength in terms of the cyclic resistance ratio (CRR) was measured using stress controlled cyclic triaxial tests. The soil specimen was consolidated under an isotropic effective confining stress $\sigma_c$ and then subjected to a cyclic deviator stress, $\sigma_d$ in the axial direction. Three to five cyclic triaxial tests were performed with various $\sigma_d/2\sigma_c$ values. The CRR was defined as the $\sigma_d/2\sigma_c$ that produced an axial strain of 5% in double amplitude in 20 cycles ($N_c$) of uniform load application. Figure 8 depicts the CRR versus $N_c$ from selective cyclic triaxial tests on MLS with $f_c$ of 0, 15 and 30% performed under $\sigma_c = 100$ kPa. The specimens used in this series of cyclic triaxial tests were fabricated using the moist tamping (MT) method. According to Fig. 8, the CRR consistently decreased as the fines contents changed from 0 to 15 and 30% for tests conducted under similar void ratios in the

![Fig. 6. Isotropic consolidation curves and critical state lines for MLS with various fines contents (after Huang et al., 2004)](image-url)
Fig. 7a. Stress, excess pore pressure and strain relationships from CIU tests with $f_c = 0$ and 15% (after Huang et al., 2004)

Fig. 7b. Stress, excess pore pressure and strain relationships from CIU tests with $f_c = 30$ and 50% (after Huang et al., 2004)

Fig. 7c. Effective stress paths from triaxial tests (after Huang et al., 2004)

**Table 1. Summary of cyclic triaxial tests**

<table>
<thead>
<tr>
<th>SPM</th>
<th>$f_c$ (%)</th>
<th>OCR</th>
<th>$p'$ (kPa)</th>
<th>CRR</th>
<th>$e$</th>
<th>$\rho^*$</th>
<th>$e^*$</th>
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<td>0.094</td>
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+: SPM = specimen preparation method

The range of 0.7 and 0.8. Note that possible range of void ratios under various fines contents is wide, as shown in Fig. 6, due to differences in compressibilities. Void ratios of approximately 0.82 (close to the maximum applicable $e$ for specimens with $FC = 30\%$) and 0.71 (close to the minimum applicable $e$ for specimens with $FC = 0\%$) represent the maximum and minimum void ratio, respectively, that
could practically be used in fabricating the cyclic triaxial test specimens.

CORRELATING CRR WITH FINES CONTENT THROUGH DENSITY STATES

Comparisons of the CRR obtained from specimens with different fines contents can be useful in evaluating the effects of fines. A valid comparison should be made among specimens prepared by the same method and under a common density state. The database included in Table 1 allows such comparisons to be made. The comparisons are grouped according to the three specimen preparation methods. For specimens made by the WS method, all cyclic triaxial tests were performed under normally consolidated conditions and an effective confining stress ($p'$) of 100 kPa. For DD specimens, $p'$ also included 50 and 200 kPa for certain parts of the cyclic triaxial tests. The MT database included a limited number of cyclic triaxial tests performed on specimens with an overconsolidation ratio (OCR) of 4. The $p'$ values in this case correspond to the unloaded final effective confining stress just prior to the cyclic shearing. The relative density is likely to be a poor index to represent the density state of MLS due to the drastic differences in compressibility and the possible range of post-consolidation void ratios among specimens with different fines contents, as indicated in Fig. 6. The following comparisons are made based on $e$ and $e^*$ to evaluate the effectiveness of equivalent granular void ratio to represent the density state for MLS. As described above, the grain size distribution of MLS deviates significantly from ideal binary packing. There was no clear trough in the $e_{\text{min}} - f_c$ correlation in Fig. 5 that corresponds to typical binary packing. The value of $f_{\text{thre}}$ was assumed as 30% to facilitate computations of $b$ parameters and $e^*$. According to the grain size distribution of MLS shown in Fig. 3, assumed $f_{\text{thre}}$ and Eq. (3), $b = 0.164$ and 0.577 for $f_c = 15$ and 30%, respectively.

Correlations among $e$, $e^*$ and CRR are presented in Figs. 9 to 11. Due to the high compressibility of the fines, a post consolidation void ratio of higher than 0.75 can be assumed to be excessively loose, and it is very difficult to make for specimens with an $f_c$ larger than 15%. On the other hand, specimens with an $f_c$ of less than 15% are significantly less compressible, and a void ratio less than 0.75 would be rather dense. For these reasons, only a narrow band of common void ratios can be found in Figs. 9(a), 10(a) and 11(a) where a comparison of the cyclic strength with different fines contents can be made. According to these figures, void ratios of around 0.75 are best suited for selection as a common ground for cyclic strength comparisons. The results show that regardless of the specimen preparation method, for void ratios around 0.75, CRR decreases with $f_c$. For specimens with the same $f_c$, the lower void ratio corresponds to a denser state and, hence, a higher CRR. For the range of confining stress applied, there was generally a trend of slightly lower CRR from tests performed at higher confining stress. This is to
be expected since granular soil is less dilatant under higher confining stress.

The use of $e^*$ to represent density state shifts all data close to a more unified $e^*$-CRR correlation among three types of fines contents, as shown in Figs. 9(b), 10(b) and 11(b). The results indicate that there appears to be an over adjustment from $e$ to $e^*$ when $f_c = 15\%$, regardless of the specimen preparation method. The fines may play a much more active role ($b$ value needs to be larger) than predicted in Eq. (3) in the force structure of the MLS specimens at $f_c = 15\%$. If the $e^*$ values were computed using Eq. (2) according to the $b$ parameter ($= 0.164$) from Eq. (3), the $e^*$-CRR data of $f_c = 15\%$ would deviate significantly away from a linear $e^*$-CRR correlation established from the remainder of the $e^*$-CRR data within each group of the specimen preparation method. The $e^*$-CRR data would have a coefficient of correlation less than 0.4 with a linear regression, unless the data of $f_c = 15\%$ are excluded. As stated above, $f_{\text{the}}$ had to be assumed when using Eq. (3) due to a lack of clear trough in the $e_{\text{the}}$-$f_c$ correlation for MLS. Other factors such as its platy grain shape and high compressibility mean that MLS behaves significantly differently from typical spherical binary packing. Under these circumstances, it was decided to adjust $b$ empirically to enhance the $e^*$-CRR correlation. This is similar to the approach used by Ni et al. (2004). The $b$ value obtained from Eq. (3) was used as an initial trial value. Following an iterative procedure, it was determined that a revised $b$ value of 0.65 ($b$ remains to be 0.577 for $f_c = 30\%$) for the case of $f_c = 15\%$ would result in a much improved $e^*$-CRR correlation for specimens within each group of preparation method. As shown in Figs. 9(b), 10(b) and 11(b), using the revised $b$ value, the $e^*$-CRR data fits a linear regression with coefficients of correlations ($R^2$) better than 0.734. The same linear regression covers the range of stress conditions applied for each group of specimen preparation method. Less scattering can be expected if all tests of the same specimen
preparation method were conducted under the same
confining stress. For the MT specimens, a separate linear
regression can be made for the normally (OCR = 1) and
over consolidated (OCR = 4) specimens.

It is evident that for MLS and at least for \( f_c \) between 15
and 30\%, \( b \) is close to a constant as suggested by Ni et al.
(2004). The fines play a much more significant role in the
sand-fines force structure than predicted by Eq. (3) in the
case of \( f_c = 15\% \). This relatively high and constant \( b \) value
is believed to be a reflection of the mild differences in
grain size (\( \alpha = 1.82 \)) and mineral contents between sand
and fines in MLS. For \( f_c \) between 15 and 30\%, a first or-
der estimation of \( b \) can be made by linear interpolation.
For practical purposes, a constant \( b \) value selected be-
tween 0.577 and 0.65 may also be assumed for \( f_c \) in the
range of 15 and 30\%, with a less desirable \( e^a\text{-CRR} \)
correlation.

**CORRELATING CRR WITH FINES CONTENT THROUGH THE STATE PARAMETER**

Jefferyes and Been (2006) showed that a clear trend be-
tween the state parameter (\( \Psi \)) and CRR\(_{15} \) (cyclic strength
determined at 15 cycles of loading) can be established as
shown in Fig. 12, for clean quartz sands from different
parts of the world. The state parameter is soil fabric
independent. The cyclic strength, however, is soil fabric de-
pendent. The effects of soil fabric are evident when speci-
mens prepared by different methods results in different
cyclic strength (Huang et al., 2004). The scatter of test
data in Fig. 12 is believed to have been caused by vari-
ations in soil fabrics originated in different specimen
preparation methods involved in the data of Fig. 12
(Jefferyes and Been, 2006). There are clear advantages to
extending the idea of correlating cyclic strength with state
parameter into sands with fines. The strength, including
the cyclic strength, is a function of density and the stress
states which are both covered in state parameter. The
equivalent granular void ratio on the other hand, con-
siders the density state only. The use of the state
parameter does not require a gradation similar to that of
binary packing. For the case of MLS, the state parameter
(\( \Psi \)) in Table 1 was determined following the procedure
shown in Fig. 1. Similar trends between CRR and \( \Psi \) can
be identified for MLS from data of the same specimen
preparation group, as shown in Fig. 13. Only the data
from normally consolidated MT specimens are included
in Fig. 13. The test data generally have coefficients of cor-
relations in excess of 0.75 with their corresponding ex-
ponential curve fits. The state parameters of MLS extend
into the positive side (\( \Psi > 0 \)) much more so than those of
the clean quartz sands reported by Jefferyes and Been
(2006). This is a reflection of the compressive nature of
MLS grains and the mostly contractive behavior during
shear, as shown in Fig. 7. For data of the same specimen
preparation group, those with high fines contents are
clustered further towards the positive side of the state
parameter axis. This is again a reflection of the higher
compressibility associated with M/S mixtures with higher
fines contents. For specimens with the same state
parameter, the CRRs of MT specimens are the highest
and those of the DD specimens are the lowest. This is
consistent with the earlier findings reported by Huang et
al. (2004) which indicated that for specimens with the
same void ratio, the CRRs from MT specimens are the
highest and those from DD specimens are the lowest.
Therefore, in addition to the consideration of the density
state, the use of the state parameter has the additional ad-
vantage of reflecting soil grain characteristics. These are
desirable features that the concept of equivalent granular
void ratio lacks.

To use the state parameter derived from the void ratio,
\( e \) has an important drawback. Ideally, a separate critical
state line is required for M/S mixtures with different fines
contents. For the tests on MLS reported herein, only
three types of fines contents are involved. The three criti-
cal state lines from each of the three types of fines con-

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**Fig. 12.** Correlation between cyclic strength and state parameter for
clean quartz sands (after Jefferyes and Been, 2006)

**Fig. 13.** Correlation between cyclic strength and state parameter, \( \Psi \)
for MLS test data
The state parameter, determined from Eq. (3), was used for the case of the CRR correlation. For specimens prepared by the three methods, a first-order estimation of the parameter can be effectively used to determine these critical state lines. This drawback may be reduced or minimized by invoking the equivalent granular void ratio in the determination of critical state line. The void ratio, $e$, based on the critical state loci that correspond to each of the three types of fines contents included in Fig. 6 along with the those computed based on $e^*$. In establishing a unified $e^*$-$p^*$ critical state line and $e^*$ based state parameters, $e^* = e$ for $f_e = 0\%$, the revised $b$ parameter of 0.65 was used for $f_e = 15\%$, $b = 0.577$ as determined from Eq. (3) was used for the case of $f_e = 30\%$. The $e^*$ based critical state loci shown in Fig. 14 are concentrated in a relatively narrow range. A unified critical state line fitted to these $e^*$ based critical state loci has a marginal $R^2$ value of 0.457. The state parameter determined based on the $e^*$ of the soil specimen and the unified $e^*$-$p^*$ critical state line is referred to as the equivalent granular state parameter ($\Psi^e$). A plot of $\Psi^e$ versus CRR for specimens prepared by the three methods is shown in Fig. 15. Again, only those of normally consolidated MT specimens are included in Fig. 15. A similar trend between the CRR and $\Psi^e$ as those observed in Fig. 13 can be found in Fig. 15. Despite of the marginal $R^2$ associated with the determination of the $e^*$ based critical state line, the CRR-$\Psi^e$ data, show much improved $R^2$ values in fitting with the corresponding exponential curves. Apparently, the determination of $\Psi^e$ based on $e^*$ and a single, unified $e^*$-$p^*$ critical state line is much less noisy than deriving $\Psi$ using $e$ and separate $e$-$p$ critical state lines for each type of fines content.

CONCLUDING REMARKS

A series of monotonic and cyclic triaxial tests have been performed on reconstituted MLS specimens. The cyclic triaxial tests included specimens prepared by water sedimentation (WS), dry deposition (DD) and moist tamping (MT) methods with a wide a variety of confining stresses and densities. The fines contents used included 0, 15 and 30%. Some of the cyclic triaxial tests were performed on over-consolidated specimens. Collectively, the cyclic triaxial test series considered the effects of density state, stress state, stress history and soil fabric. These tests provided a database to evaluate the relationships among $e$, $e^*$, cyclic strength and state parameter ($\Psi$).

According to the grain size distribution of MLS and an assumed $f_{\text{max}}$ of 30%, $b = 0.164$ and 0.577 for $f_e = 15$ and 30%, respectively, following the method by Rahman et al. (2008). However, a $b$ value of 0.65 was required for the case of $f_e = 15\%$ to establish a more consistent $e^*$-CRR correlation, for the combined data of $f_e = 0$, 15 and 30%. The relatively high and constant $b$ value is believed to be a reflection of the mild differences in grain size ($\chi = 1.82$) and mineral contents between sand and fines in MLS. Because of the grain characteristics of MLS, the fines play a much more active role in the granular force structure than most of the cases reported earlier. For $f_e$ between 15 and 30%, a first-order estimation of $b$ can be made by linear interpolation. For practical purposes, a constant $b$ between 0.577 and 0.65 may also be assumed for $f_e$ in the range of 15 and 30%, with a less desirable $e^*$-CRR correlation.

The state parameter considers the combined effects of density and the stress states. The characteristics, notably the compressibility of the sand grains, are reflected in the general range of the state parameters. Because of these advantages, the state parameter is a superior choice over the equivalent granular void ratio as a means of correlating the CRR with fines contents. The drawback and im-

Fig. 14. The $e$ based critical state loci and the critical state line derived using $e^*$

Fig. 15. Correlation between cyclic strength and equivalent granular state parameter, $\Psi^e$ for MLS test data
practicality of requiring a series of critical state lines for each type of fines contents involved can be circumvented by invoking the equivalent granular void ratio in the determination of state parameters. With properly calibrated $b$ parameters or a function that determines $b$ parameters, a unified $e^*$-$p'$ critical state line can be established for the range of fines contents involved. For MLS and the tests performed, the equivalent granular state parameter $\Psi^*$ can be an effective means to correlate cyclic strength with fines contents.

REFERENCES