PHOTOELASTIC STUDIES OF THE STRESS IN POWDER MASS

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SYNOPSIS

Photoelastic method for determination of stress in powder mass is applied to several problems in soil mechanics and foundation engineering.

Stress distribution in the powder mass in a direct single-shear box is observed and discussed; stress distribution caused by two piles and stress distribution under a footing are observed.

From these observations, it is confirmed that this method is very useful for investigation of some problems in soil mechanics and foundation engineering.

INTRODUCTION

In view of the rapid development of soil mechanics, it is considered convenient in many respects to study the distribution of stress in powder mass experimentally in laboratory scale. In this case, ordinary soil pressure cells do not seem to be convenient because their size is not small enough to make two or three dimensional observations possible. Further a pressure cell inserted in the powder mass does not seem to be satisfactory for accurate measurement of stress in the powder mass, because the cell is an elastic body, while powder mass surrounding the cell is, in general, an elastoplastic body of irreversible compressibility.

The author considered it desirable to measure the stress in powder mass without any insertion, and proposed as a practicable means *photoelastic method of determination of stress in powder mass*.¹⁾ This method is only applicable to certain specific powder, however more information will be obtainable as to the stress in powder mass, as far as this method is applied to such specific powder. With no connection with the author, Dr. P. Dantu proposed a similar method in France, on which two reports are available at present.²⁾

By this photoelastic method, the following are possible:

- (1) Observation of the isoclinics similar to those in ordinary two dimensional photoelastic experiment.
- (2) Observation of the distribution of brightness and the equibrightness lines which correspond to the isochromatics in ordinary photoelastic experiment.
- (3) Observation of the isochromatic principal stress lines (named as "isostatics" by Dr. Dantu).
- (4) Measurement of the absolute value of photoelastic effect with Babinet compensator.
- (5) Measurement of the small difference of photoelastic effect by Tank's method.
- (6) Measurement of the displacement of the particles of the powder mass by fuse grid or carborandum particles.

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In short, glass powder mass is quite elastic concerning the photoelastic effect; isoclinics and equibrightness lines correspond to the isoclinics and the isochromatics in usual photoelastic experiments. As to these properties, each of the following experiments is discussed.

OBSERVATION OF STRESS DISTRIBUTION IN THE DIRECT SINGLE-SHEAR BOX

It is already known that direct single-shear test has less reliability due to the non-uniformity of stress distribution on the sheared plane. The following experiments are made to verify such non-uniformity.

Test result :

Figure 1 shows the model used here. Glass powder is filled in this box placed in the vessel, and is sheared under a certain vertical load. Stress distribution in the powder mass is observed at various stages of shearing. Figures 2 to 4 show examples

Fig. 1. Model of Single-Shear Box

Fig. 2. Results Observed by Circular Polarized Light under the Vertical Load of 100 kg and the Shear of 2.8 mm. Exposure is 2 sec. and 1 sec. in (a) and (b), respectively

Fig. 3. Result Observed by Circular Polarized Light under the Vertical Load of 100 kg and the Shear of 0.2 mm

(a)

(b)

Fig. 4. Results Observed by Plane Polarized Light under the Vertical Load of 100 kg and Shear of 1 mm. Azimuths of Plane Polarized Light is 30° and 40° in (a) and (b), respectively

Fig. 5 (a). Isoclinics under the Vertical Load of 100 kg and the Shear of 2.8 mm

Fig. 5 (b). Principal Stress Lines in the Same Case with (a)

Fig. 6. Isoclinics and Principal Stress Lines under the Vertical Load of 100 kg and the Shear of 10.5 mm

Fig. 7. Composite Photograph of the Carborandum Particles under the Various Stages of Shearing

observed by circular polarized light or by plane polarized light. Figures 5 and 6 show examples of the diagrams of isoclinics or principal stress lines. Figure 7 shows composite photograph of the carborandum particles previously scattered in the powder mass.

From these experimental results the following can be concluded :

i) While the shear displacement does not exceed about 0.2 mm., stress is larger at the upper left corner and in the vicinity of point D in Fig. 1 (b). With increase of shear, the stress in the vicinity of D grows larger and extends leftwards, and when the shear displacement becomes about 3 mm., a large stress can be seen in the region around CD in a shape of a slanted lens as in Fig. 2 (b). Further increase of shear does not vary the shape of brightness, and the brightness in the upper left corner comparatively decreases.

ii) From the observation by plane polarized light, the following can be seen. With the increase of shear, directions of principal stress on the plane CD tends to rotate. When the shear displacement is about 0.2 mm., the angle between the direction of major principal stress and the plane CD is about 60 deg. and when the shear is within the range of 0.5 to 1 mm., the angle is about 40 deg. In the latter case, an isolated singular point can be seen. Further increase of shear does not change the inclination of major principal stress lines.

iii) With regard to the position of the singular point, the following can be seen. First, the distance between CD and singular point is not largely affected by the height of powder mass. Second, the singular point has a tendency to move leftwards with the increase of shear, but its height scarcely varies. And when the shear displacement is increased to about 10 mm., the singular point is liable to return slightly rightwards.

iv) Figure 7 shows that the plane CD is really the slip surface.

Considerations :

From the facts ii) and iv), the angle between the direction of major principal stress line and the slip surface is about 10 deg. According to the ordinary theory of soil

mechanics the angle between slip surface and direction of major principal stress line should be $\frac{\pi}{4} - \frac{\phi}{2}$ in plastic equilibrium, when the angle of internal friction is ϕ . Therefore, it may be concluded that the angle ϕ of the glass powder mass is about 10 deg.

According to the present investigations, ϕ of such powder material is presumed to be 20 to 50 deg. With regard to this inconsistency, further investigations should be made as follows.

i) In the above experiments the stress distribution is observed in static state after a certain stage of shear, but in order to discuss the value of ϕ stress distribution should be observed while the shear is in progress. (The maximum speed of shearing obtained by the present apparatus is very small, and in such a case the distribution of stress in static state is confirmed not to differ from that in dynamic state).

ii) The angle of internal friction of this glass powder should be measured with an ordinary testing apparatus and be compared with the results obtained here.

iii) In this apparatus, front and rear glass plates of the box are fixed to the vessel. But they should be fixed to the upper and lower parts of the box separately to make the same condition as in the ordinary apparatus.

DISTRIBUTION OF STRESS AROUND THE TIPS OF TWO PILES

There are many problems not yet solved as to the behaviours of group of piles. It seems to be interesting to observe the distribution of stresses around the tips of piles. Here, stress distribution caused by two piles is observed by photoelastic method, and it is assured that this method can usefully be applied to explain the relation among spacing of piles, wedge angle, magnitude of load and the distribution of stresses.

Test results :

Two piles of 16 mm. in width, 50 mm. in thickness, are pressed down on the surface

Fig. 8

of powder mass in a vessel as shown in Fig. 8. Their tip angles are 50 deg., 70 deg., 90 deg. and 110 deg., respectively. Spacings of piles are 13 mm., 20 mm. and 40 mm. Loads of 100 kg. to 400 kg. are applied on piles. Piles are guided only to make vertical movement. Covering plates are put on the surface of powder mass to avoid the heaving of powder. Piles are pressed down and the distribution of stress is observed. The plates are fixed, but no surcharge is placed.

Figures 9 to 11 are examples observed by circular polarized light. Figures 12 and 13 are examples of isoclinics. Figure 14 is an example of principal stress lines derived from isoclinics.

Fig. 9. Result Observed by Circular Polarized Light under the Load of 300 kg, when $\theta=70^\circ$ and $d=20$ mm

Fig. 10. Result Observed by Circular Polarized Light, when $W=200$ kg, $\theta=70^\circ$, $d=13$ mm

Fig. 11. Result Observed by Circular Polarized Light, when $W=200$ kg, $\theta=70^\circ$, $d=40$ mm

Fig. 12. Result Observed by Plane Polarized Light, when $W=200$ kg, $\theta=70^\circ$, $d=20$ mm and the Azimuth of Polarization is 0°

Fig. 13. Result Observed by Plane Polarized Light, when $W=200$ kg, $\theta=70^\circ$, $d=40$ mm and the Azimuth of Polarization is 60°

Fig. 14. Principal Stress Lines, when $\theta=70^\circ$, $d=20$ mm and $W=200$ kg.

The experiments have revealed the following :

- i) In regard to the observation by circular polarized light,
 - (a) Large stresses are developed under the level of pile tips, while the stress between the sides of piles remains almost unchanged.
 - (b) When the spacing of piles d is large, zero point diffuses ; that is, differential of shear stress is small.
 - (c) When the tip angle θ is large, the same tendency as (b) is noticed.

ii) Positions of zero points are compared by the principal stress lines. If the distance to the zero point from the base line of the triangles at the tip of pile is denoted Z , the relations between Z and d , between Z and θ , and between Z and the load W are shown in Fig. 15. The value of Z is defined as above to discuss the problem referring to the elastic theory of foundation. Figure 15 shows that the position of zero point varies with the spacing, and that it has only a little connection with the tip angle, but has some connection with the load W . These results, however, do not cover general cases. Here, the relation between Z and θ is examined only when d is 3 cm. and W is 200 kg.

Considerations on the results of experiments

At the zero point, major principal stress σ_1 is equal to the minor principal stress σ_2 , and shear stress τ is zero. Therefore, when vertical stress is σ_z and horizontal

Fig. 15. Relations Between Z and α , Z and θ , and Z and W

Fig. 16. The Locus of the Zero Point Based on the Elastic Theory, when Two Uniformly Distributed Pressures are Applied to the Semi-Infinite Elastic Body

stress is σ_h , at zero point $\sigma_z = \sigma_h$. In elastic theory, the appearance of zero point may be understood as follows. In Fig. 16, it is assigned that the uniformly distributed pressure (1) around the origin is applied to the surface XX' of semi-infinite elastic body and the similar pressure (2) is applied at an edge-to-edge spacing PQ . Then $\tau = 0$, on vertical line bb' which bisects PQ . If the stress is developed as in Boussinesq's analysis, locus of points where $\sigma_z = \sigma_h$ is shown as PP' , when only the pressure (1) is applied. Such locus as to the pressure (2) is shown as QQ' . Therefore, the intersecting point B satisfies the condition of zero point. The point B moves on the locus PP' in accordance with the length of PQ . If the width of pressure is 16 mm. and the spacing d is either 13 mm., 20 mm. or 40 mm., the position of zero point is indicated as A , B , and C , respectively. The points A' , B' and C' obtained from the experimental results are corresponding position of zero points when $\theta = 70^\circ$, and $W = 200$ kg. Thus the results of experiments show the same tendency as the theoretical proposition on zero point. (The locus based on Fröhlich's theory is identical with that based on Boussinesq's).

THE OBSERVATION OF THE DISTRIBUTION OF STRESS UNDER A FOOTING

Many reports have been published on the measurement of the vertical component of the stress in soil, but there are few which have gone into the details of the principal stress lines. The principal stress lines can easily be obtained from the present theory. In elastic theory, stresses at any point can be calculated, and in plastic theory, slip lines can be drawn, from which principal stress lines can be obtained. It is of great interest to observe the principal stress lines experimentally and to compare them with theoretical ones. But this experiment has been found, as is expected, to be of great difficulty. Various facts are recognized in this experiment; however, it has become clear that further more precise experiments are required for detailed discussions from the viewpoint of soil mechanics.

Fig. 17. Result Observed by Circular Polarized Light, when the Load is 100 kg, the Width of the Punch is 20 mm, and the Width of the Vessel is 80 mm

Fig. 18. Result Observed by Circular Polarized Light, when the Load is 450 kg, the Width of the Punch is 20 mm, and the Width of the Vessel is 195 mm

Fig. 19. Result Observed by Circular Polarized Light when the Load is Decreased from 450 kg to 100 kg. The Width of the Punch is 20 mm. The Width of the Vessel is 80 mm

Figures 17 to 19 show examples observed by circular polarized light. Figure 20 shows an example of the results observed with Babinet compensator. Figure 21 shows an example of the principal stress lines in comparison with those from Prandtl's theory.

Fig. 20. Result Observed by Babinet Compensator when the Load is 100 kg.

Fig. 21. Principal Stress Lines Obtained from the Experiment are Compared with those Derived from the Prandtl's Theory, Assuming that the Angle of Internal Friction θ is 25°

Considerations on the experimental results

Quite complicated results of observation are obtained as follows:

i) As to the distribution of brightness, the following are noticed. (1) The extreme brightness develops beneath the punch so far as the load is small. With the increase of load the brightness increases but sometimes it is remarkable that a dark area appears beneath the punch. (2) When the load is decreased to 100 kg., large stress in the area remains sometimes between the punch and the inner side of the vessel, and sometimes between the punch and the bottom of the vessel. The distribution of stress is much more complicated when the load is removed.

ii) As to the principal stress lines, it is noticed that the distribution of stress varies not only with the width but also with the condition of cover plates. (In these experiments cover plates are also used to avoid the heaving of powder.)

The following can be concluded in spite of the complicated character of the results:

i) The dark area below the punch is considered to correspond to the elastic zone on the Fröhlich's theory of foundation. (In this respect, it may be interesting to observe the decrease of the dark area, when the failure is about to occur. But this is not yet studied because the capacity of loading apparatus is not enough to cause failure.)

ii) So far as experimental results here obtained are concerned, they are not identical with the results derived from Boussinesq's theory and Terzaghi's theory. But, each of experimental results is similar to those based on Prandtl's theory, if the angle of internal friction is assumed as 10 to 25 deg.

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