RUPTURE LINE IN A TWO-LAYER NON-COHESIVE MEDIUM

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ABSTRACT

The author presents the analytical and the graphical method for determination of the direction and of the stress in the rupture line on one side of the contact between two different non-cohesive media, provided that the following values are known: the shear angles of both media that are assumed to obey Coulomb's failure law, the dilatation angles of both media as well as the stress state and the direction of the failure line on the other side of the contact. The analytical solution has been deduced from the limiting equilibrium conditions for the prismatic contact element whose surfaces are formed by the strain characteristics of both media appearing in the narrow rupture zone in which—according to Bent Hansen—the line rupture develops. The graphical solution can be obtained by constructing the corresponding Mohr's stress circles. The analytical as well as the graphical solution can be used also in order to get the dilation angles provided that the directions of the rupture line at the contact and the magnitude of the normal stress in the contact were determined in appropriate model tests.

INTRODUCTION

Line ruptures are assumed to occur in narrow rupture zones separating rigid domains (Bent Hansen, 1958). As linear strains in the directions of limiting rupture lines are zero, these rupture lines coincide with strain characteristics.

In Mohr's strain circle the strain characteristics are represented, in Fig. 1, by the points a and b , and in Mohr's circle of limiting stress state of a non-cohesive medium the stress characteristics are given, in Fig. 2, by the points m and n .

While along stress characteristics the shear angle is fully mobilized, the inclination of stress vectors towards normals to strain characteristics is less than or equal to the shear angle: $\phi' \leq \phi$. The relationship between the angles ϕ and ϕ' is governed by the dilation angle ν (see Fig. 2) and given by the equation:

$$\sin \phi = \frac{\sin \phi'}{\cos (\phi' - \nu)} \quad (1)$$

When $\nu = \phi'$, the angles ϕ and ϕ' coincide:

$$\phi \equiv \phi' \quad (2)$$

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Fig. 1. Mohr's strain circle

Fig. 2. Mohr's limiting stress circle

ANALYTICAL SOLUTION

In narrow rupture zones of line ruptures the strain characteristics form angles $\frac{\pi}{2} \pm \nu$ (see Fig. 1). Considering different shear angles ϕ_1 and ϕ_2 and different bulk densities γ_1 and γ_2 of two horizontal layers, the rupture lines must be assumed to be broken in the contact of the layers. Fig. 3 represents the corresponding part of the narrow rupture zone for the case of an upward displacement. In the case of a downward displacement of the zone the angles ϕ_1' and ϕ_2' get negative values.

Let us consider the equilibrium conditions for the soil body consisting of two triangular prisms having as common surface the contact surface $c \cdot d$ ($d \rightarrow \infty$) of the layers in the rupture zone and limited by strain characteristics a_1, b_1 and a_2, b_2 as shown in Fig. 3. The body is subjected to volume forces G_1 and G_2 .

and to surface forces

$$\begin{aligned} T_2' &= t_2' \frac{\cos(\nu_2 + \alpha)}{\cos \nu_2} cd, \\ T_2'' &= t_2'' \frac{\sin \alpha}{\cos \nu_2} cd, \end{aligned} \quad (5)$$

$$\begin{aligned} t_1' &= t_1'' = t_1 \\ t_2' &= t_2'' = t_2 \end{aligned} \quad (6)$$

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$$\frac{t_1}{t_2} = \frac{1}{\cos \nu_2 \sin \beta \cos (2\phi_1' - \nu_1)} \{ \cos \nu_1 [\sin \alpha \cos (\beta - \alpha + \phi_1' + \phi_2' - \nu_2) + \cos (\alpha + \nu_2) \sin (\beta - \alpha + \phi_1' - \phi_2')] \} \quad (7)$$

$$\frac{t_1}{t_2} = \cos \nu_1 \sin \alpha \cos (2\phi_2' - \nu_2) \{ 1/\cos \nu_2 [\sin \beta \cos (\alpha - \beta + \phi_1' + \phi_2' - \nu_1) - \cos (\nu_1 + \beta) \sin (\beta - \alpha + \phi_1' - \phi_2')] \} \quad (8)$$

If the strength parameters ϕ_1 , ϕ_2 and ν_1 , ν_2 have been obtained by appropriate testing procedure, and if the values t_1 and β are assumed to be known, Equations (7) and (8) yield the values t_2 and α .

GRAPHICAL SOLUTION

The graphical solution of the problem can be obtained by using Mohr's circles of stress. The solution is governed by the condition that in the contact surface of two layers the same stress vector $\vec{p}_d(\sigma_d, \tau_d)$ must be obtained if computing it from either the stress state of the upper or of the lower triangle shown in Fig. 3. For the case of the upward displacement of the narrow rupture zone the graphical construction is presented in Fig. 4.

The strength parameters ϕ_1 , ϕ_2 and ν_1 , ν_2 as well as the values t_1 and β shall again be assumed to be known. Thus the circle touching, in point 1, the failure envelope ϕ_1 of the upper layer defines the stress state in the triangle above the contact provided that the magnitude of the stress vector $\vec{02}$: $\vec{02} = t_1$. The mirror image of the point 2 about the axis σ is the point 3. The straight line passing through the point 3 intersects the stress circle in the pole P_1 . The horizontal direction drawn through the pole P_1 intersects the Mohr circle at the point 4, and its mirror image 5 defines the stress vector $\vec{05}$ acting in the contact surface of both layers.

The stress state in the lower layer is then given by the stress circle passing through the points 4 and 5 and touching the failure envelope ϕ_2 of the lower layer in the point 6. The angle $\phi_2 - \nu_2 = \angle(6 - 0_1 - 7)$ determines the stress vector $\vec{t_2} = \vec{07}$. The inclination α of the rupture line in the lower layer (see Fig. 3) can be obtained by connecting the mirror image of the point 7, i.e., point 8, with the pole P_2 of the stress circle whose centre is the point 0_2 . The pole P_2 lies on the horizontal drawn through the point 4.

The author has presented a method of determining the direction of strain characteristics and the stress state on one side of the horizontal surface separating two layers of different strengths, provided that the strength characteristics of both layers and the stress state on the other side of the contact are known. According to Bent Hansen, the line rupture was assumed to occur in a narrow rupture zone. The strain characteristics have been allowed to deviate from stress characteristics. The dilatation angle can be obtained by measuring, in plane strain conditions, the strains preceding failure in both principal directions.

If the directions of strain characteristics limiting narrow rupture zones of line ruptures were obtained by model tests made with two media of known shear angles, the procedure

Fig. 4. Graphical solution of the problem

explained in the paper can alternatively be used in order to get the dilatation angles ν provided that the normal stress in the contact surface be measured as well. The solution can be obtained by trial and error.

NOTATIONS

- a, b : strain characteristics
- m, n : stress characteristics
- ϕ : shear angle
- ϕ_1 : inclination of stress vector towards normal to line rupture
- ν : dilatation angle
- γ : bulk density
- G : mass force

- T : surface force
- c, d : sizes of the contact surface of two triangular prisms
- α, β : inclination angles of two triangular prisms
- t : reaction stress
- i : inclination of the contact surface
- $\vec{p}_d$: stress vector acting on the contact surface of two triangular prisms
- σ_d : normal component of the stress vector p_d
- τ_d : tangential component of the stress vector p_d
- P : pole of the stress circle

REFERENCE

- Hansen, B., (1958), Line Ruptures Regarded as Narrow Rupture Zones, Basic Equations Based on Kinematic Considerations, Brussels Conference 85 on Earth Pressure Problems, Proceedings, Vol. I, pp. 39–48.

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