GROUND MOTIONS DURING EARTHQUAKES AND
THE INPUT LOSS OF EARTHQUAKE POWER
TO AN EXCITATION OF BUILDINGS

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INTRODUCTION

At the present time, the method of analysis most commonly used for the earthquake response of structures is based on the assumption that the ground just beneath the foundation vibrates in phase and with the same amplitude everywhere. However, it is very doubtful whether a building having horizontally large size as a school or apartment building vibrates as if it were rested on a shaking table. Such a question has been talked about among earthquake engineers prior to the Tokachioki earthquake of 1968 (Yoshimi and Akagi, 1968) in Japan. The majority of damaged buildings were school buildings having horizontally large size (Nielsen and Nakagawa, 1968).

Many aftershocks were observed in Hachinohe district in order to study the general interrelation between ground motions and building response as well as the cause of damage to individual buildings. It was proved from the observations that there were remarkable phase differences among the movements of different points of building foundations during earthquakes, just as it had been supposed.

The correlation between the movements of a building foundation and the adjacent ground was also made clear. The correlation was found to be mostly affected by the wave length which appeared at the ground surface. It was also found that the maximum amplitude of a building foundation is always smaller than that of the adjacent ground, and that the difference becomes larger as the wave-length becomes shorter.

Up to the present days, many strong earthquakes have been recorded by means of strong motion accelerometers (SMAC) or seismographs. However, it must be borne in mind that those records indicate the movement of individual points where the instruments were installed.

It goes without saying that there are great differences in the frequency and amplitude characteristics of motions simultaneously observed at different points, due to the differences in geological and soil conditions. The detailed descriptions on this subject were presented in the author’s recent paper (Yamahara, 1970).

Another important problem is how to choose an earthquake input for the response calculation of a building, and the subject will be the main point of argument in this paper. For instance, when the wave-length developed on ground surface is relatively short, that

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is to say, the vibration period is relatively short, the effective input power to a building is greatly decreased, because the phases of ground motions at different points beneath the foundation are remarkably disturbed. Therefore, the ground motion having shorter periods does not usually cause severe response of a building as it is shown by the current method of calculation, even if the acceleration of the ground motion is fairly large.

The input loss was numerically estimated under idealized conditions, and the general characteristics of the input loss was given as a function of the ratio of wave-length to the length of a building. Furthermore, the method for eliminating shorter period components from original ground motion in accordance with a given filter characteristic of the input loss is tried in this study. Several sample calculations were carried out with the record of the Tokachioki earthquake, and the effective earthquake inputs to building excitation are presented in this paper.

An effective input motion obtained from analytical means using the earthquake record on ground surface was compared with the observed record at the ground floor of a building, and the two motions were proved to be very similar to each other.

1. THE CORRELATION BETWEEN MOTIONS OF BUILDING FOUNDATIONS AND THE ADJACENT GROUND DURING EARTHQUAKES

Let us first look over some observed records to understand actual phenomena. Many aftershocks were observed simultaneously at seven points, two of them were on the ground surface and five on the ground floor of the building of Hachinohe Technical College as shown in Fig. 1. Because the building has no basement, the motion at ground floor can be regarded equivalent to that at building foundation. Since all of the records obtained during the period can not be copied in this paper for lack of space, several examples of typical records are chosen for discussion in Figs. 2 to 6.

In the first place, a sample record of a small earthquake is shown in Fig. 2. The motions at two points on the ground and one point on the building floor are shown side by side in the figure. This is an example in which comparatively short periods are conspicuous. It can be seen in the record that the movements at two points A and B, spaced 36 m, are almost in phase for longer wave-lengths (longer periods) but out of phase for shorter wave-lengths (shorter periods). The motions of shorter periods on the ground scarcely

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![Fig. 1. Locations of measured points in Hachinohe Technical College](image-url)
Fig. 2. Accelerograms at Hachinohe Technical College at 6:35, Aug. 19, 1968

Fig. 3. Accelerograms at Hachinohe Technical College at 5:50, Aug. 22, 1968

Fig. 4. Accelerograms at Hachinohe Technical College at 9:02, Aug. 25, 1968

appear in the motion on the building floor, and the floor is steadily vibrating with longer period. The period was thought at the beginning to be the natural period of the building, but it was later proved from frequency analyses to be the predominant period of the ground. The natural period of the building for this direction was measured to be 0.28 sec through microtremor observations, but the period in the record is about 0.40 sec. It can not be considered that the period of building would be prolonged more than 40 percent in such a small amplitude. Another interesting point is that the amplitude of the floor is as a whole much smaller than those of ground motions. The maximum
acceleration of the floor is 0.8 gal (cm/sec²), which is less than a half of the maximum acceleration on the ground of 1.8 gal.

Another record in which shorter periods are more predominant than the former example is shown in Fig. 3. The phase difference between the two points A and B is more conspicuous. The periods of the floor response become shorter than the former example due to the shorter period motions of the ground. The maximum acceleration on the floor is 0.7 gal and the value is less than one-third of that of ground motion.

An example in which short periods are most predominantly included in the ground motion is given in Fig. 4. The phase relation between the two points on the ground A and B are remarkably disturbed, though the distance between them is only 36 m. Owing to the extremely short periods on the ground motion, the periods of the floor response
become much shorter than the two examples above. Furthermore, the differences between the amplitudes of floor response and surrounding ground extend still more. In this example, the maximum acceleration on the floor is 1.2 gal which corresponds to only one-fourth of that of the ground motion.

The record presented in Fig. 5 indicates the frequency characteristics essentially different from the preceding three examples. Comparatively long periods are developing in a sinusoidal form in the ground motions. In fact, this type of earthquake was observed most frequently on the ground of Hachinohe Technical College. The predominant period of ground was measured to be about 0.40 sec from microtremor and aftershock observation. Because the ground has a particular characteristic which shows high frequency selectivity (Yamahara, 1970) at the period of 0.4 sec, the ground motions having this period develop most predominantly on the ground surface, and they take a nearly sinusoidal form. This tendency is remarkable when many period components near 0.4 sec are included in an incoming earthquake wave. When the periods of ground motion become comparatively long as seen in this example, not only phases but also amplitudes at two different points A and B are very close to each other. Moreover, the amplitude ratio between the motions of ground and building floor approaches 1.0, and the phase differences between the two motions nearly vanish.

Another earthquake which has similar frequency characteristics is shown in Fig. 6. It is very interesting to see the correlation between the motions of the ground and the building floor in this record. When the two motions on the ground at A and B accord with each other, the movement of floor becomes maximum. Soon after ground motions at the two points become out of phase, the movement of the floor diminishes rapidly. On the contrary, the movement of floor suddenly grows as soon as the phases of ground motions become close. The building may be thought as if it were a ship floating on the sea, rolling and pitching with the motions of waves.

An interesting phenomenon could be found out during the observations. That was the differences between the seismic intensities announced by Hachinohe Meteorological Station and the accelerations observed on the ground of Hachinohe Technical College. When shorter periods were predominant within an original earthquake wave, the seismic intensity announced by Hachinohe Meteorological Station was always larger than the intensity observed at the ground point of Hachinohe Technical College. On the other hand, when the ground motions at the College had a predominant period near 0.4 sec, the seismic intensity announced by the Station was incredibly smaller than that expected from the acceleration at the College. For instance, though the maximum acceleration of the earthquake shown in Fig. 5 was 2.4 gal at the ground surface, the seismic intensity was announced to be 0 (less than 0.8 gal) by the Meteorological Station.

From these facts, it may be concluded that the ground motion in Hachinohe Technical College will be remarkably amplified when the motions having periods near 0.4 sec are included in an incoming earthquake wave. Because the ground in Hachinohe Meteorological Station has no particular amplifying effect at this period, the ground motion will be felt at a lesser intensity.
At the beginning of this investigation, the records of building floor were thought to indicate the response characteristics of building for the input motions of surrounding ground. Such a way of thinking was later proved to be contradictory to the facts described below.

(1) The natural period of the building scarcely appeared in the records of building floor and the periods in the records changed according to the periods included in the ground motion. The period that appeared most frequently in the records was proved to be not the natural period of the building but the predominant period of the adjacent ground.

(2) The amplitudes of the building floor were always smaller than those of the surrounding ground. If the records of building floor had indicated the response characteristics of the building, the amplitudes of building floor would have been larger than those of ground due to elastic deformations of foundation soil.

(3) The moment when the phases of ground motion accorded everywhere within an horizontal extent of the building, as seen in Figs. 5 and 6, the amplitudes of building floor suddenly grew to the level of ground motion accompanied with no phase difference between the two motions. There was no time lag until the response of the building grew.

When an earthquake response of building is calculated, not ground motion but the motion at building foundation should be chosen as an input motion for the building. It is unreasonable that the records of ground motion are directly applied to the response calculation of a structure, especially of a structure having horizontally large size.

The ratios of the maximum acceleration of building floor to that of the adjacent ground surface are plotted in Fig. 7 with respect to the predominant period of ground motion for several earthquakes. It may be understood in Fig. 7 that the effective input of earthquake power decreases to some extent, if shorter period motions are included predominantly in the ground motion.

Whether a period is shorter or longer is determined in this paper not with respect to the natural period of a building but on the basis of a horizontal size of the building. For instance, a ground motion having 100-m wave-length is said to be of longer period for a building having 50-m length, but of shorter period for a building 200 m long.

![Fig. 7. Ratios of the maximum accelerations on building foundation to those on surrounding ground surface](image)
2. MOTIONS OF BUILDING FOUNDATIONS DURING EARTHQUAKES

Building foundations are in general fixed rigidly with girders and slabs. Because of that, a foundation tend to move en masse, even if there are phase differences in the ground motions beneath the foundation.

In order to understand an horizontal behaviors of foundation slab during an earthquake, actual records are shown in Figs. 8 and 9, which were also observed at the two points G and F on the floor of Hachinohe Technical College. The Two points G and F correspond to the center and the end of the building, respectively.

![Fig. 8. Accelerograms at Hachinohe Technical College at 17:02, Aug. 7, 1968](image)

![Fig. 9. Accelerograms at Hachinohe Technical College at 5:05, Aug. 13, 1968](image)

Fig. 8 is an example during an earthquake which included relatively long periods. The movements at the two points are almost in phase for both N–S and E–W directions which correspond to transverse and longitudinal directions, respectively.

An example in which shorter periods are included predominantly in the ground motion is shown in Fig. 9. Considerable phase differences can be seen between the motions at two points in N–S direction. However, the two motions in E–W direction appear to be almost in phase and of the same amplitude. (Note that the two records in E–W direction in this figure are just 180° out of phase.)
Even if there are phase differences among the ground motions at different points beneath a building foundation, the foundation is too rigid to deform with the ground. In this case, the foundation structure is subjected to a tensile or compressive stress in longitudinal direction and a shear or a torsional stress in transverse direction. By the latter loading, the building should be twisted about its longitudinal axis as shown in Fig. 10. A long and narrow building in general does not have enough rigidity against such a torsional deformation. Therefore, it should be kept in mind that a building may be subjected to these kinds of stresses or forced deformations during earthquakes.

Fig. 10. A torsional deformation of long and narrow building

3. AN EFFECTIVE EARTHQUAKE INPUT TO THE EXCITATION OF A BUILDING

As mentioned in the preceding chapters with the aid of actual observation data, the motions at different points of ground surface are by no means in phase during an earthquake. A building seems in general to have enough strength and rigidity against tensile, compressive and shear stresses caused by the phase differences of ground motion. Thus the foundation slab will vibrate in a manner in which the vectors of all ground motions beneath the foundation are integrated. In other words, because the ground motions counteract within the foundation slab and lose part of their power, the input power which is effectively used to vibrate the building is always smaller than the original power of ground motion. The loss of input power resulting from the restraint by the floor slab is called "input loss" for building excitation.

The input loss is given as a function of the wave-length developed on the ground surface and the length of a building. Supposing a simplest model as shown in Fig. 11, let us

Fig. 11. A simplest model for the calculation of the input loss
try to calculate the input loss. The mode of surface motion is supposed to be harmonic and continuous. However, the generality is by no means lost by this assumption, because that is thought to be the case when a component parallel to the length of the building is taken out, and one of the frequency components included in an original ground motion is chosen for this calculation.

The equation of mode developed on the ground surface is expressed as

\[ u = u_0 \cos \frac{2\pi}{T} \left( t - \frac{T}{L} x \right) \]  

(1)

in which \( u \) and \( u_0 \): amplitude of wave-motion, \( T \): period of motion, \( L \): wave-length, \( x \): distance, and \( t \): time.

The amplitude of building foundation, \( U \), can be calculated by integrating amplitudes just beneath the foundation slab. Namely,

\[ U = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} u_0 \cos \frac{2\pi}{T} \left( t - \frac{T}{L} x \right) dx \]  

(2)

The amplitude has the maximum value at \( t = 0 \), thus,

\[ U_{\text{max}} = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} u_0 \cos \frac{2\pi}{L} x dx \]

\[ = u_0 \frac{L}{\pi l} \sin \frac{\frac{l}{L} \pi}{\pi} \]  

(3)

Using the following substitutions,

\[ \eta = \frac{U_{\text{max}}}{u_0}, \quad \xi = \frac{L}{l} \]  

(4)

Eq. (3) is rearranged as

\[ \eta = \frac{\xi}{\pi} \sin \frac{\pi}{\xi} \]  

(5)

in which \( \eta \) denotes the ratio of amplitude of foundation slab to that of ground motion, and \( \xi \) means the ratio of wave-length to the length of the building. From Eq. (5),

\[ \lim_{\xi \to \infty} \frac{\xi}{\pi} \sin \frac{\pi}{\xi} = 1 \]

which means that the motion of foundation slab approaches ground motion when the wave-length becomes very long.

The relationship between \( \eta \) and \( \xi \) is graphically shown in Fig. 12. From the figure, the motion of building foundation is found to be nearly equivalent to the surrounding ground motion when the wave-length is over four times of building length (\( \eta > 0.9 \)). For instance, if the wave-length is counted to be 200 m, the wave-length over four times of
building length corresponds to a building less than 50 m long. Consequently, it can be said that any actual buildings may be more or less affected by the input loss of earthquake.

4. THE EFFECTIVE INPUT OF GROUND MOTION

The actual records of earthquakes which have been observed by various kinds of seismographs indicate no more than the movements at individual points. Even if an earthquake record was obtained at ground surface, it is undesirable to directly use it as an input signal for calculation of building response. A certain degree of input loss should be taken into account so far as the horizontal size of a building is not too small.

It is easily supposed that the effective input to a building will be evaluated by eliminating shorter period components from original ground motion, and the frequency filter may well be thought to have a characteristic similar to that shown in Fig. 12. Thereupon, let us consider a method of evaluating the effective input from an original ground motion by filtering out shorter periods according to the filter characteristic given in the preceding chapter.

Supposing a ground motion $f(t)$, it is expressed in the digital form as follows:

$$f(t) = f(k \Delta t) = f_k$$

in which $\Delta t$ is a digital time increment.

The average value of $f_k$ over a time span of $n \Delta t$ can be given by

$$g_k = \frac{f_k (n-1)/2 + \cdots + f_{k-1} + f_k + f_{k+1} + \cdots + f_{k+(n-1)/2}}{n}$$

in which $n$ must be odd number. In order to find out the general frequency characteristic of series $g_k$ thus obtained, let us assume $f(t)$ to be harmonic motion.

$$f(t) = e^{iwt} = e^{i\omega_k \Delta t}$$
Substituting Eq. (8) into Eq. (7),
\[
g_k = \frac{e^{i\omega dt}}{n} \left\{ e^{\frac{i(n-1)\omega dt}{2}} + \cdots + e^{i\omega dt} + 1 + e^{-i\omega dt} + \cdots + e^{-\frac{n-1}{2}i\omega dt} \right\} \\
= \frac{f_k}{n} \left( 2 \cos \frac{n-1}{2} \omega dt + 2 \cos \frac{n-3}{2} \omega dt + \cdots + 2 \cos \omega dt + 1 \right) \tag{9}
\]

Using the following representation,
\[
H(\omega dt) = \frac{1}{n} \left( 1 + 2 \cos \omega dt + 2 \cos 2\omega dt + \cdots + 2 \cos \frac{n-1}{2} \omega dt \right) \tag{10}
\]
Eq. (9) becomes
\[
g_k = H(\omega dt)f_k \tag{11}
\]

The term of $H(\omega dt)$ has nothing to do with $k$, and it is expressed as a function of frequency; accordingly, it shows the general characteristic of frequency filter. Therefore, the power spectrum ratio of $g_k$ to $f_k$ is given by $H^2(\omega dt)$, which was calculated for several values of $n$, and the results are graphically shown with respect to frequency and period in Figs. 13 and 14, respectively.

On the other hand, it will be convenient for direct comparison to express the characteristic of input loss given in Fig. 12 in terms of the power spectrum ratio ($\gamma^2$), as shown in Fig. 15. Comparing Fig. 15 with Fig. 14, it is found that both are very similar to each other in their frequency characteristics. Therefore, it was proved that the input loss can be practically estimated by filtering out shorter periods from original ground motion by this method. In brief, when the length of a building is equal to the wave-length of period $T$, the effective input for the building is given by the output series $g(t)$ obtained from Eq. (11) by setting

![Graph](image-url)  
Fig. 13. Filter characteristic of the function $H^2(\omega dt)$
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Fig. 14. Filter caracteristic of the function $H^2(\omega \Delta t)$

Fig. 15. Power spectrum characteristic of the effective input determined from Eq. (5)

\[ n = \frac{T}{\Delta t} = \frac{l}{\Delta t \cdot V} \quad (12) \]

in which $V$ is the velocity of wave-mode on ground surface.

For example, the effective input motions were computed for following four cases with the record of Tokachioki earthquake obtained at the Port of Hachinohe (Tsuchida, 1969).

- $n = 5 \quad (T = 0.05 \text{ sec})$
- $n = 9 \quad (T = 0.09 \text{ sec})$
- $n = 19 \quad (T = 0.19 \text{ sec})$
- $n = 39 \quad (T = 0.39 \text{ sec})$

The results are shown in Fig. 16. A tendency that shorter period components are gradually filtered out according to the increase of $n$ value is readily understood in the figure.
5. RESPONSE SPECTRUM CHARACTERISTICS FOR THE EFFECTIVE INPUT MOTIONS

As related in preceding sections, the motions effective to building excitation are always smaller than the original ground motion. It is convenient for understanding the general characteristics of effective input to show the change in response spectra at every stage of $n$ values. Fig. 17 shows the acceleration response spectra for original record of Tokachi-oki earthquake and three motions of its effective input, for damping coefficient of 0.05. The acceleration records of above four motions are shown in Fig. 16.

In Fig. 17, such a tendency is easily understood as the absolute values of response decrease remarkably in a range of shorter periods according to an increase in $n$ value. It is a matter of course that the response levels of all cases coincide in a range of longer periods, because the numerical filter has no effect in eliminating longer period components included in original motion.

The maximum response of original ground motion amounts to 780 gal at 0.22 and 0.35 sec, which corresponds to 3.5 times the maximum input acceleration of 225 gal. In the case of $n = 39$ ($T = 0.39$ sec), the peak responses at those periods diminish to 180 gal or so, which is less than the peak value of original ground motion. In this case, the maximum peak shifts to the period of 0.85 sec, and the response value shows 280 gal which is only 1.24 times the maximum acceleration of the original ground motion.

This tendency leads to a conclusion that a building having horizontally large size is rather advantageous for the earthquake response of the super structure. However, such a building is consequently subjected to some complicated stresses resulting from restraining the phase differences of ground motions just beneath the foundation. Those
Fig. 17. Change in acceleration response spectra of the effective input motions

stresses are not in general accounted for in the structural design of buildings, and they could possibly have considerable influence on the extent of damage to buildings during earthquakes.

6. APPLICATION TO OBSERVED DATA

Many earthquakes had been observed at Hachinohe Technical College simultaneously on the building (ground floor) and on the adjacent ground. In those records, the original ground motion and the effective input motion in this study correspond to the motions at the surrounding ground and at the building, respectively. Therefore, the motion at building (effective input motion) was calculated from the motion at the surrounding ground (original ground motion) by the method presented in Chapter 4, and it was compared with the actual record at the building during the earthquake. The records shown in Fig. 2 of the earthquake at 6.35, Aug. 19, 1968, was chosen for this calculation.

The top curve in Fig. 18 is the record on the surrounding ground at point A, and motions of shorter periods seem to be predominantly included in the record. The amplitude of the record was read digitally with time pitch of 0.02 sec ($\Delta t = 0.02$ sec). The series of the ground motion thus obtained, $f_n$, yielded the new series of $g_n$ by putting $n = 4$ or 5 in Eq. (11), which is nothing but the effective input of the earthquake. The effective input
thus obtained are shown in Fig. 18, in which the second and the third curves are the cases for \( n = 4 \) and \( n = 5 \), respectively. The bottom curve in Fig. 18 is the actual record on the building floor at point C which was obtained simultaneously with the record of the first curve. It can be seen in the figure that the calculated results are very similar to the observed record.

The two records, the top and the bottom curves in Fig. 18, do not seem to have any resemblance at a glance. Nevertheless, the latter curve could be yielded from the former by filtering out shorter periods included therein.

The values of \( n \) in this calculation were determined from the following procedure. The time differences of corresponding wave modes between two points A and B were read to be from 0.03 to 0.04 seconds in this accelerogram. Because the distance between the two points was 36 m, the velocity of wave-mode on the ground surface was estimated to be

\[ V = 900 \sim 1200 \text{ m/sec} \]

On the other hand, the total length of the building for S–N direction is about 100 m. Thus, from Eq. (12),
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\[ n = \frac{l}{\Delta t \cdot V} = 4.2 \sim 5.5 \]

in which \( \Delta t \) was read at a time interval of 0.02 sec \( (\Delta t = 0.02 \text{ sec}) \).

As pointed out in the example above, the velocity of wave-mode developed on the ground surface, \( V \), is in general very faster than the value estimated from experience by assuming the wave to propagate horizontally within the surface layer of the ground. This fact establishes that earthquake waves should be treated to propagate vertically to ground surface.

Phase differences on the ground surface may result from difference in arrival times of wave motions, which in turn may be caused by irregularly inclined soil strata or non-uniform distribution of soil properties.

From these confirmation with observed data, it may be concluded that there should exist the input loss of earthquake power, and the input loss or the effective input could be approximately estimated by the method presented in this paper.

CONCLUSIONS

In the current method of response analysis for buildings during earthquakes, the ground beneath the building is assumed to move in phase at every point. However, it was proved from field observations during moderate earthquakes that there were significant differences in the movements at ground surface.

The effective input power to vibrate a building should be always smaller than that of original ground motion. The effective input motion for response calculation of actual buildings could be approximately estimated by filtering out the shorter periods included in the ground motion. An analytical method to determine the effective input motion is presented in this paper which is shown to give a good agreement with the observed acceleration records.

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NOTATION

- \( f(t) \): Time dependent function of original ground motion
- \( f' \): Original ground motion digitally expressed
- \( g(t) \): Time dependent function of effective input motion
- \( g'e \): Effective input motion digitally expressed
- \( h \): Damping coefficient of building
$H(\omega \Delta t)$: Filter function defined in Eq. (10)

$L$: Wave length

$l$: Length of building

$T$: Period of wave-mode or vibration

$t$: Time variable

$\Delta t$: Digital pitch of time

$U$: Amplitude integrated by ground motions beneath foundation

$U_{max}$: The maximum value of $U$

$u$: Amplitude of wave-mode

$u_0$: Peak amplitude of wave-mode

$x$: Distance

$\xi$: Ratio of wave length to length of building defined as $L/l$

$\eta$: Amplitude ratio defined as $U_{max}/u_0$

$\omega$: Circular frequency

REFERENCES


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