BEARING CAPACITY OF FOOTINGS ON CLAYS

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ABSTRACT

Natural clay deposits exhibit nonhomogeneity and anisotropy in shear strength to a considerable degree. In order to take these into account, an analysis is presented herein for the ultimate bearing capacity of strip footings on two-layered purely cohesive soil, wherein the undrained cohesion is anisotropic and varies linearly with depth, in each layer. Values of the bearing capacity factor $N_u$, are presented for the case of a single layer of clay in which the undrained cohesion decreases linearly with depth in the upper region and thereafter increases linearly with depth, simulating the case of clays subjected to lowering of water table and consequent desiccation.

Key words: anisotropy, bearing capacity, clay

IGC: E3

INTRODUCTION

The ultimate bearing capacity of footings taking nonhomogeneity in shear strength of soil into account, has been studied by several investigators. Two types of nonhomogeneity, viz., stratification and increase of strength with depth, have been considered. The ultimate bearing capacity of footings on two-layered clays was studied by Botten (1953) by applying the `$\phi = 0$' analysis. Variation of strength with depth in the upper layer, was also considered. Bearing capacity of footings on two-layered soils has also been studied by Meyerhof and Chaplin (1953), Tchng (1957) and Yamaguchi (1963). The ultimate bearing capacity of footings considering the variation of cohesion with depth, was studied using `$\phi = 0$' analysis with circular rupture surface, by Nakase (1963), Reddy and Bhargava (1966), Reddy and Srinivasan (1967) and Raymond (1967). In this paper, an analysis is presented for the ultimate bearing capacity of footings on two-layered purely cohesive soil, considering both anisotropy and linear variation of strength with depth in each layer. The types of anisotropy and nonhomogeneity considered herein are explained below.

In natural soil deposits the following cases of nonhomogeneity are normally encountered:

(a) A two-layered soil with the strengths increasing with depth in each layer as in the

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case of recent deposits overlying older deposits (Fig. 1a);
(b) a single layer of soil in which the strength increases with depth, as in the case of normally consolidated or slightly overconsolidated clays (Fig. 1b);
(c) a single layer of soil in which the strength decreases with depth in the upper portion and the strength increases with depth in the lower portion, as in the case of clays subjected to lowering of water table and subsequent desiccation (Fig. 1c).

Case (a) represents the most general case of a two-layered soil, and (b) and (c) are particular cases of Case (a).

The strength anisotropy in soils has been analysed by several investigators (Casagrande and Carrillo, 1944; Lo, 1965; Livneh and Komornik, 1967). All of them have given the variation of strength with inclination of the major principal stress, as:

$$c_i = c_H + (c_V - c_H) \sin^2 i$$

(1)

in which,
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$c_i = \text{the strength corresponding to an inclination, } i, \text{ of the major principal stress with the horizontal direction,}$

$c_H = \text{strength corresponding to horizontal direction of the major principal stress, and } c_V = \text{strength corresponding to a vertical direction of the major principal stress.}$

The strengths corresponding to the horizontal and vertical directions of the major principal stress viz., $c_H$ and $c_V$, are generally referred to as principal strengths. The ratio of the principal strengths, $c_V/c_H$, is approximately constant for any soil deposit and is called the coefficient of anisotropy or anisotropy index, denoted by $k$. Values of $k$ ranging from about 0.75 to about 1.6 have been reported in literature (Lo, 1965). Experiments conducted by Lo (1965) also show that the angle between the failure plane and major principal stress is almost constant irrespective of the direction in which the sample has been taken.

ANALYSIS

Fig. 2 shows the schematic diagram of a continuous footing on the surface of a two-layered cohesive sub-soil. The assumed failure surface is also shown. This makes an angle $2\theta$ at the centre $O$ of the circle and the radius of the circular arc is $r$. The failure surface is symmetrical about the vertical line $OE$ through the point $O$. The failure surface intersects the horizontal interface between the two layers at the points $C$ and $D$. The portion of the failure surface between $C$ and $D$ subtends an angle $2\theta_i$ at centre $O$.

![Rupture surface and definition of parameters used](image)

The angle between the major principal stress and the failure plane is assumed to be constant and is designated by $\phi$. Referring again to Fig. 2, for any point $P$ on the failure surface $AE$, the inclination, $i$, of the major principal stress with the horizontal, is

$$i = \alpha + \phi$$

(2)

in which, $\alpha = \text{angle made by the radius vector OP with OE}$.

Similarly, for the portion $EB$ of the failure surface, the inclination, $i$, of the major principal stress with the horizontal is
\[ i = \alpha - \phi \quad (3) \]

Fig. 3 shows the assumed variation of undrained shear strength with depth in each layer. If \( c_H \) and \( c'_H \) are the strengths, corresponding to the horizontal direction, at depths \( 'h' \) below the surfaces of the upper and lower layers respectively, then

![Undrained Shear Strength Diagram](image)

**Fig. 3. Shear strength versus depth relationship for the general case**

\[ c_H = c_{H_s} + \beta h \quad (4) \]

in the upper layer, and

\[ c'_H = c'_{H_s} + \beta_1 h \quad (5) \]

in the lower layer.

From Fig. 2 it is seen that for a point on the assumed slip circle,

\[ h = r (\cos \alpha - \cos \theta) \quad (6) \]

for the upper layer, and

\[ h = r (\cos \alpha - \cos \theta_i) \quad (7) \]

for the lower layer.

At a depth \( h \) below the surface, the shear strength corresponding to an inclination \( i \) with the horizontal, is written from Eq. 1 as

\[ c_i = c_H + (kc_H - c_H) \sin^2 i \quad (8) \]

for the upper layer, and

\[ c'_i = c'_H + (kc'_H - c'_H) \sin^2 i \quad (9) \]

for the lower layer.

On substituting for \( c_H \) and \( h \) from Eqs. 4 and 6 into Eq. 8,

\[ c_i = [c_{H_s} + \beta r (\cos \alpha - \cos \theta)][1 + (k - 1) \sin^2 i] \quad (10) \]
Similarly, on substituting for \( c'_{H} \) and \( h \) from Eqs. 5 and 7 into Eq. 9
\[
c'_i = [c'_{H} + \beta_2 r (\cos \alpha - \cos \theta_i)][1 + (k - 1) \sin^2 (\alpha + \phi)]
\] (11)

In Eqs. 10 and 11, the corresponding expressions for \( i \) must be substituted from Eq. 2 or 3, depending on the position of the point along the failure surface. The following are the expressions for different parts of the assumed failure surface:
\[
c_i = [c_{H} + \beta_2 r (\cos \alpha - \cos \theta)][1 + (k - 1) \sin^2 (\alpha + \phi)]
\] (12)
for part AC;
\[
c_i = [c_{H} + \beta_2 r (\cos \alpha - \cos \theta)][1 + (k - 1) \sin^2 (\alpha - \phi)]
\] (13)
for part DB;
\[
c'_i = [c'_{H} + \beta_1 r (\cos \alpha - \cos \theta_i)][1 + (k - 1) \sin^2 (\alpha + \phi)]
\] (14)
for part CE; and
\[
c'_i = [c'_{H} + \beta_1 r (\cos \alpha - \cos \theta_i)][1 + (k - 1) \sin^2 (\alpha - \phi)]
\] (15)
for part ED.

For limiting equilibrium of the soil mass above the assumed failure surface, the disturbing moment about \( O \) must be equal to the resisting moment about \( O \). Hence,
\[
q_o B \left( r \sin \theta - \frac{B}{2} \right) = \int_{\theta_1}^{\theta} r^2 d\alpha c_i \bigg|_{AC} + \int_{\theta_1}^{\theta} r^2 d\alpha c_i \bigg|_{DB} + \int_{0}^{\theta_1} r^2 d\alpha c'_{i \bigg|_{OB}} + \int_{0}^{\theta_1} r^2 d\alpha c'_{i \bigg|_{ED}}
\] (16)

On substituting the expressions for \( c_i \) and \( c'_i \) from Eqs. 12 through 15, Eq. 16 reduces to:
\[
q_o B \left( r \sin \theta - \frac{B}{2} \right) = r^2 \left\{ \int_{\theta_1}^{\theta} [c_{H} + \beta r (\cos \alpha - \cos \theta)][1 + (k - 1) \sin^2 (\alpha + \phi)] d\alpha 
+ \int_{\theta_1}^{\theta} [c_{H} + \beta_1 r (\cos \alpha - \cos \theta_i)][1 + (k - 1) \sin^2 (\alpha - \phi)] d\alpha 
+ \int_{0}^{\theta_1} [c'_{H} + \beta_1 r (\cos \alpha - \cos \theta_i)][1 + (k - 1) \sin^2 (\alpha + \phi)] d\alpha 
+ \int_{0}^{\theta_1} [c'_{H} + \beta_1 r (\cos \alpha - \cos \theta_i)][1 + (k - 1) \sin^2 (\alpha - \phi)] d\alpha \right\}
\] (17)

By dividing both sides of Eq. 17 by \( c_{H} \) and introducing the following non-dimensional parameters,
\[
n + 1 = \frac{c'_{H}}{c_{H}},
\] (18)
\[
\nu = \frac{\beta B}{2c_{H}},
\] (19)
and

$$\nu_1 = \frac{\beta_1 B}{2c_{H_s}},$$

(20)

the following expression is obtained for $N_e$:

$$N_e = \frac{q_0}{c_{V_s}} = \frac{r'^2}{2k(r' \sin \theta - 1)} \left\{ 2\theta + 2n\theta_1 + k'\theta + nk'\theta_1 + \frac{k'}{2} \left[ -\sin \frac{2X_1}{2} - \sin \frac{2X_2}{2} \right] \right\}$$

$$+ \frac{nk'}{2} \left[ -\sin \frac{2Y_1}{2} - \sin \frac{2Y_2}{2} \right] + 2\nu r'(\sin \theta - \theta \cos \theta) - 2\nu r'(\sin \theta_1 - \theta_1 \cos \theta)$$

$$+ 2\nu r'(\sin \theta_1 - \theta_1 \cos \theta_1) + \nu r' \frac{k' \cos \phi}{3} (\sin^3 X_1 + \sin^3 X_2)$$

$$+ \frac{r'k' \cos \phi}{3} (\sin^3 Y_1 + \sin^3 Y_2)(\nu_1 - \nu) + \frac{\nu r'k' \sin \phi}{12} (\cos 3X_1 - \cos 3X_2)$$

$$+ \frac{r'k' \sin \phi}{12} (\cos 3Y_1 - \cos 3Y_2)(\nu_1 - \nu) - \frac{\nu r'k'}{4} \cos \theta (2\theta - \sin \frac{2X_1}{2} - \sin \frac{2X_2}{2})$$

$$- \frac{r'k'}{2} \left( 2\theta_1 - \frac{\sin 2Y_1}{2} - \frac{\sin 2Y_2}{2} \right)(\nu_1 \cos \theta_1 - \nu \cos \theta),$$

(21)

in which

$$r' = 2r/B,$$

(22)

$$k' = k - 1,$$

(23)

$$X_1 = \theta + \phi,$$

$$X_2 = \theta - \phi,$$

$$Y_1 = \theta_1 + \phi,$$

$$Y_2 = \theta_1 - \phi.$$ 

(24)

and

Eq. 21 is the general expression for $N_e$ covering all the three types of nonhomogeneity mentioned earlier. Numerical results are obtained herein for the case of single layer with desiccated upper portion. For this case $n$ is related to $d/B$ and $\nu$ as:

$$n = 2\nu \frac{d}{B}$$

(25)

Also in this case $\nu$ is a negative quantity whereas $\nu_1$ is positive.

The angle $\theta_1$ in Eq. 21 can be expressed as

$$\theta_1 = \cos^{-1} \left( \cos \theta + \frac{d}{r} \right)$$

(26)

The above expression for $N_e$ gives values of $N_e$ for assumed values of $r'$ and $\theta$. The minimum $N_e$ is obtained using the conditions:
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\[
\frac{\partial N_c}{\partial \theta} = 0 \tag{27}
\]

and

\[
\frac{\partial N_c}{\partial r'} = 0 \tag{28}
\]

Eqs. 27 and 28 are expressed in terms of the two variables \(\theta\) and \(r'\). For given values of the other parameters, viz., \(k\), \(\nu\), \(\nu_1\), \(d/B\), \(n\) and \(\phi\), the values of \(\theta\) and \(r'\) which satisfy Eqs. 27 and 28 simultaneously are found by using a digital computer and \(N_c\) is evaluated by substituting these values of \(\theta\) and \(r'\) in the corresponding expression for \(N_c\). The ultimate bearing capacity may be obtained from the expression

\[
q_0 = c_{\gamma,s}N_c \tag{29}
\]

Fig. 4. Values of \(N_c\) for \(k = 0.8\) and \(\nu = -0.1\)

Fig. 5. Values of \(N_c\) for \(k = 0.8\) and \(\nu = -0.2\)

Fig. 6. Values of \(N_c\) for \(k = 0.8\) and \(\nu = -0.3\)

Fig. 7. Values of \(N_c\) for \(k = 0.8\) and \(\nu = -0.4\)
RESULTS AND DISCUSSION

Values of $N_c$ as defined by Eq. 29 are obtained for the case of a single layer wherein the cohesion decreases linearly with depth in the upper few feet and then increases linearly with depth. The values used in the numerical work, for the parameters $k$, $\psi$, and $\nu$ cover a range normally encountered in practice. The results of Ward, Samuels and Butler (1959) have shown a value of $k$ around 0.77 for the clay tested by them. On the other hand Lo (1965) obtained values ranging from 1.25 to 1.56 for Welland clay. Hence, in the present numerical work values ranging from 0.8 to 2.0 are used. The shear strength versus depth relationships reported by Bishop (1966) show that the rate of increase in strength with depth can be as much as 120 lbs per sq ft per ft whereas the results pre-
presented by Lumb and Holt (1968) for a soft marine clay show a much smaller value. Therefore, in the numerical work a maximum rate of increase of about 60 lbs per sq ft per ft is assumed and a slightly smaller value is assumed for the rate of decrease in strength in the upper desiccated region. Accordingly, \( \nu \) is varied from \(-0.1\) to \(-0.4\), and \( \nu_1 \) is varied from \(0\) to \(0.8\).

Numerical results are presented in Figs. 4 through 19. In these figures \(d\) is the depth upto which there is a decrease in cohesion. From these figures it is observed that for given values of \(k\) and \(\nu_1\), \(N_e\) increases as \(\nu_1\) increases, the rate of increase being maximum in the case of \(d/B = 0\). For \(d/B = 0.25\), as \(\nu_1\) changes from \(0\) to \(0.6\) the increase in \(N_e\) is 19 per cent for \(\nu = -0.1\) and 22 per cent for \(\nu = -0.4\), when \(k = 0.8\). When \(k = 2\),

**Fig. 12.** Values of \(N_e\) for \(k = 1.4\) and \(\nu = -0.1\)

**Fig. 13.** Values of \(N_e\) for \(k = 1.4\) and \(\nu = -0.2\)

**Fig. 14.** Values of \(N_e\) for \(k = 1.4\) and \(\nu = -0.3\)

**Fig. 15.** Values of \(N_e\) for \(k = 1.4\) and \(\nu = -0.4\)
as \( \nu_1 \) changes from 0 to 0.6 the increase in \( N_e \) is 13 per cent when \( \nu = -0.1 \) and 16 per cent when \( \nu = -0.4 \). The increase in \( N_e \) with \( \nu_1 \) is seen to be practically negligible for values of \( d/B \) equal to 0.5 or higher. In order to show the influence of \( \nu \) on \( N_e \), the basic data presented in Figs. 4 through 15 are replotted in Figs. 20 and 21 for \( k = 0.8, 1 \) and 1.4. It is observed from these figures that as \( \nu \) changes from \(-0.1\) to \(-0.4\) the value of \( N_e \) reduces considerably. The reduction in \( N_e \) is more for \( d/B = 0.5 \). When \( k = 0.8 \) and \( d/B = 0.5 \), \( N_e \) reduces by about 26 per cent as \( \nu \) changes from \(-0.1\) to \(-0.4\). When \( k = 1.4 \) and \( d/B = 0.5 \), the reduction is about 25 per cent. In order to show the influence of \( k \), the numerical results for the extreme values of \( \nu \) and \( \nu_1 \) are replotted in Figs. 22 and 23. These figures show that with increase in \( k \), \( N_e \) decreases considerably. When \( \nu_1 = 0\),

Fig. 16. Values of \( N_e \) for \( k = 2.0 \) and \( \nu = -0.1 \)

Fig. 17. Values of \( N_e \) for \( k = 2.0 \) and \( \nu = -0.2 \)

Fig. 18. Values of \( N_e \) for \( k = 2.0 \) and \( \nu = -0.3 \)

Fig. 19. Values of \( N_e \) for \( k = 2.0 \) and \( \nu = -0.4 \)
the reduction in $N_e$ as $k$ varies from 0.8 to 2, is about 35 per cent, and when $\nu_1 = 0.6$, it is about 38 per cent.

The values of $\theta$ and $r'$ which give the minimum value of $N_e$, are plotted in Figs. 24 through 27. It is observed that for given values of $d/B$, $\nu$ and $\nu_1$, as $k$ increases from 0.8 to 2, $\theta$ decreases. This reduction is about 15 per cent for $\nu_1 = 0$ and about 10 per cent for $\nu_1 = 0.6$. Correspondingly, as $k$ increases from 0.8 to 2, $r'$ increases by about 30 per cent for $\nu_1 = 0$ and by about 15 per cent for $\nu_1 = 0.6$.

CONCLUSIONS

Numerical results show that anisotropy and nonhomogeneity in shear strength have
a considerable influence on the ultimate bearing capacity. The normal practice of finding shearing strength of samples taken in the vertical direction and assuming the same strength for all directions, leads to considerably higher bearing capacity factors when \( k \) is larger than 1 and hence it is unsafe. On the other hand when \( k \) is less than 1, ultimate bearing capacity found assuming the medium to be isotropic is smaller than the actual value and hence will be on the conservative side. Also it has been shown that disregarding the considerable increase in cohesion with depth, as found in many normally consolidated clays, leads to values of \( N_c \) which are too conservative. In a case where \( k \) is greater than 1 and strength increases with depth, the net influence of both depends on the degree of anisotropy and the rate of increase of strength with depth. The values of \( N_c \) have been presented in

Fig. 24. Variation of \( \theta \) with \( k \) for \( \nu_1 = 0 \) and 0.2

Fig. 25. Variation of \( \theta \) with \( k \) for \( \nu_1 = 0.4 \) and 0.6

Fig. 26. Variation of \( r' \) with \( k \) for \( \nu_1 = 0 \) and 0.2

Fig. 27. Variation of \( r' \) with \( k \) for \( \nu_1 = 0.4 \) and 0.6
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the form of graphs for the case of nonhomogeneity that has been studied.

For the case of single layer in which cohesion increases linearly with depth, disregarding this increase in strength with depth leads to considerably low values of ultimate bearing capacity than the actual ones.

When cohesion decreases with depth in the upper portion, the rate of increase of strength in the lower portion has considerable influence on the value of $N_a$ when the depth upto which the shear strength decreases is less than 0.5 times the width of the footing.

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NOTATION

$B =$ width of foundation.
$c_H, c'_H =$ cohesion corresponding to horizontal direction, for the upper and lower layers, respectively.
$c_{Hs}, c'_{Hs} =$ cohesion corresponding to horizontal direction, at ground surface and at surface of lower layer, respectively.
$c_s, c'_s =$ cohesion corresponding to any direction, for upper and lower layers, respectively.
$c_v =$ cohesion corresponding to vertical direction.
$c_{vs} =$ cohesion corresponding to vertical direction, at ground surface.
$d =$ thickness of upper layer or depth of the upper desiccated zone.
$h =$ depth of a point below surface of upper or lower layer.
$i =$ inclination of axis of sample with horizontal.
$k =$ coefficient of anisotropy.
$k' =$ constant ($k' = k - 1$)
$N_a =$ bearing capacity factor.
$n =$ factor representing relative strengths of upper and lower layers.
$q_0 =$ ultimate bearing capacity.
$r =$ radius of the assumed circular arc.
$r' =$ non-dimensional parameter ($r' = 2r/B$).
$X_1 =$ parameter ($X_1 = \theta + \phi$).
$X_2 =$ parameter ($X_2 = \theta - \phi$).
$Y_1 =$ parameter ($Y_1 = \theta_1 + \phi$).
$Y_2 =$ parameter ($Y_2 = \theta_1 - \phi$).
$\alpha =$ angle made with vertical by a line joining the centre of slip circle to any point on the slip circle.
$\beta =$ rate of increase or decrease of cohesion with depth in the upper layer.
$\beta_1 =$ rate of increase of cohesion with depth in the lower layer.
$\theta =$ half the angle subtended by a circular failure arc at its centre.
$\theta_1 =$ half the angle subtended at centre of circular failure arc, by the portion of
failure surface within the lower layer.
\( \nu, \nu_1 \) = non-dimensional parameters representing rate of variation of cohesion with
depth.
\( \phi \) = angle between major principal stress and failure plane.

REFERENCES


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