LATERAL STABILITY OF RIGID POLES SUBJECTED TO AN APPLIED COUPLE

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ABSTRACT

Functional relationships are developed for the load-deflection characteristics of a rigid, vertical pole, embedded in sand and subjected to an applied couple. The study is based on non-dimensional techniques and the physical variables included in the theoretical analysis are the ground-line deflection, depth of embedment, geometry of the pole cross-section, applied couple, time of loading, and soil parameters. The soil parameters used include the density, angle of internal friction, flow characteristics, and relative density of the sand. Non-dimensional techniques in conjunction with small scale model studies are used to determine the explicit form of the interrelationship among the above physical variables for a pole embedded in a very dense, uniform, fine, dune sand of constant properties and subjected to an applied couple. Hyperbolic prediction equations are given from which the load-deflection characteristics of a prototype pole might be estimated. A failure criterion in the form of a stability number relating the ultimate applied couple to the geometry of the pole-soil system is given and correlated with the failure criterion of the same pole-soil system subjected to a horizontal ground-line thrust. Several illustrative examples are worked.

INTRODUCTION

The lateral stability of a pole embedded in soil has not yet succumbed to an altogether satisfactory theoretical analysis. In order to make such an analysis the boundary forces or displacements must be known and an adequate stress-strain-time law must be available for the soil. The assumptions that must be made in order to fulfill the above two requirements seriously reduce the reliability of analytical results.

This paper utilizes the techniques of dimensional analysis and small scale model experiments to study the behavior of a rigid circular pole embedded in a dense, uniform, fine, dune sand of constant properties and subjected to an applied couple. Kondner (1960 a, b, c) (1962 a, b, c) has previously demonstrated the effectiveness of these methods in the field of soil mechanics. These methods allow the development of a prediction equation on a semi-empirical basis without requiring assumptions regarding boundary conditions or stress-strain laws for the soil.

The following factors are reported herein.

- a) The coupled-ground line deflection relationship for a single circular pole embedded in sand.

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- b) The effect of pole diameter and depth of embedment on the couple-ground line deflection relationship.

The majority of published work concerning the lateral stability of rigid poles has been of an analytic nature, involving assumed pressure distributions on the pole and assumed stress-deformation laws for the soil. Y. C. Chang (1960) has compiled a comprehensive bibliography on laterally loaded piles which includes poles as a subsection. S. Prakash (1960) has also presented a rather complete review of partially embedded poles subjected to lateral loads. A further discussion of analytic trends may be found in a recent paper by Kondner and Green (1962 c).

The work of Kondner and Green (1962 c) was the first study to the authors' knowledge employing dimensional analysis and small scale model tests for the purpose of examining the stability of laterally loaded poles. Their investigation dealt with the general formulation of non-dimensional functional relationships for the lateral stability of rigid poles, development of experimental apparatus, and effects of horizontal ground-line thrusts.

The purpose of the present investigation is to formulate an applied couple deflection relation and a stability criterion for the given soil-pole system when the pole is subjected to an applied couple. The problem takes on added significance in view of the fact that preliminary investigation at the Northwestern University soils laboratory has indicated that superposition of ground-line couple-moment and ground-line thrust is not equivalent to a lateral force applied at some distance above ground-line. This illustrates the non-linearity of the general pole problem and hence, each phenomenon must be studied separately. The results reported are not necessarily a stepping stone to a more complete superposition theory but rather a self-contained study of the problem stated.

THEORETICAL ANALYSIS

In this investigation, as in all engineering investigations, the purpose was to obtain a maximum amount of useful data with a minimum expenditure of experimental and computational effort. One of the most effective methods of obtaining this goal for experimental studies is through the use of dimensional analysis. This method can be summarized as follows. If there are m physical quantities containing n fundamental units, which can be related by an equation, then there are $(m-n)$, and only $(m-n)$, independent, non-dimensional parameters (called π terms) such that the π terms are arguments of some indeterminate, homogeneous function κ :

$$\kappa(\pi_1, \pi_2, \pi_3, \dots, \pi_{m-n}) = 0 \quad (1)$$

To properly apply this procedure, a prudent choice of the physical quantities involved must be made. Omission of significant variables may lead to very simplified but erroneous results while the consideration of unimportant variables may greatly reduce the usefulness of the results and considerably increase the expended effort.

Table. 1. Physical Quantities considered in the Dimensional Analysis of a Rigid Pole Embedded in Sand

Physical quantity	Symbol	Fundamental units
1. Deflection at ground-line	y	L
2. Depth of embedment	L	L
3. Cross-sectional area of pole	A	L^2
4. Perimeter of pole	C	L
5. Applied couple	M	FL
6. Dry density of sand	γ	FL^{-3}
7. Angle of internal friction	ϕ	$F^{\circ}L^{\circ}T^{\circ}$
8. Viscosity of sand	η	$FL^{-2}T$
9. Time of loading	t	T
10. Relative density	D_d	$F^{\circ}L^{\circ}T^{\circ}$

The quantities under consideration in this investigation are given in Table 1. A force, length, and time system of fundamental units has been used.

Once the physical quantities involved have been chosen, the π terms contained in the functional formulation can be easily obtained. The form of the functional relationship must then be determined by experiment. Since there are ten physical quantities and three fundamental units, there must be seven independents, non-dimensional π terms. These π terms can be methodically obtained by choosing three physical quantities, which contain all three fundamental units and cannot be formed into a non-dimensional parameter by themselves, such as (M , t and L), and combining them with each of the remaining quantities one at a time. For example, combining them with γ gives :

$$\pi = M^{\alpha} \cdot t^{\beta} \cdot L^{\rho} \cdot \gamma^{\lambda} = F^{\circ} L^{\circ} T^{\circ} \dots\dots\dots (2)$$

Substituting the fundamental units of each of the physical quantities involved into equation (2) gives :

$$(FL)^{\alpha} \cdot (T)^{\beta} \cdot (L)^{\rho} \cdot (FL^{-3})^{\lambda} = F^{\circ} L^{\circ} T^{\circ} \dots\dots\dots (3)$$

Equating exponents of the fundamental units gives :

$$\left. \begin{array}{l} \alpha + \lambda = 0 \\ \beta = 0 \\ \alpha + \rho - 3\lambda = 0 \end{array} \right\} \dots\dots\dots (4)$$

Solving equations (4) by letting $\alpha=1$ gives :

$$\alpha = 1; \quad \beta = 0; \quad \lambda = -1; \quad \rho = -4 \dots\dots\dots (5)$$

Therefore the π term under consideration is: $\pi = \frac{M}{\gamma L^4} \dots\dots\dots (6)$

By successive application of the above procedure, the remaining π terms may be

obtained. The resulting π terms may be algebraically transformed to yield a new set of non-dimensional, independent π terms which have a more significant form with regard to the experimentation.

The following seven non-dimensional, independent π terms were arrived at for this investigation :

$$\pi_1 = \frac{y}{C}, \pi_2 = \frac{M}{\gamma C^3 L}, \pi_3 = \frac{C}{L}, \pi_4 = \frac{C^2}{A}, \pi_5 = \phi, \pi_6 = \frac{\gamma t C}{\eta}, \pi_7 = D_d \dots (7)$$

The π terms of equation (7) may be substituted in equation (1) to obtain the functional relationship which can be rewritten as :

$$\frac{y}{C} = \kappa \left[\frac{M}{\gamma C^3 L}, \frac{C}{L}, \frac{C^2}{A}, \phi, \frac{\gamma t C}{\eta}, D_d \right] \dots \dots \dots (8)$$

In equation (8) and hereafter the symbol κ denotes "some function of", but not necessarily the same function for each equation. This notation is used to avoid the use of numerous subscripts and superscripts as a means of differentiating between the functional forms.

Physical significance can be attached to the π terms appearing in equation (8). The dependent variable $\frac{y}{C}$ is a deflection ratio. Shape effects are given by the term C^2/A .

The term $\frac{L}{C}$ is a relative measure of the embedment and may be thought of as an embedment ratio. The term $\frac{M}{\gamma C^3 L}$ is the ratio of the applied couple to a soil strength

parameter and is called the couple-strength ratio. Creep effects are included in the $\frac{\gamma t C}{\eta}$ term which is the ratio of the time of loading to a characteristic relaxation time of the soil. The significance of the angle of internal friction and relative density of the sand are well known and they are by definition non-dimensional. Effects of repetitional loading could easily be included with the addition of the number of load applications which is by definition non-dimensional. The present paper however is concerned only with a single application of the applied couple. Repetitional effects will be studied in the future. The angle of rotation of the pole might also be considered but it can be easily obtained from the measured data and is also non-dimensional.

The form of the functional relation given in equation (8) can be greatly simplified by restricting the scope of the study. If we confine our attention to poles of circular cross section, the shape factor $\frac{C^2}{A}$ is a constant value equal to 4π , regardless of the diameter. Preliminary work by Kondner and Green (1962) indicates that by proper choice of the loading rate the effect of the term $\frac{\gamma t C}{\eta}$ can be minimized and hence eliminated from equation (8). Because the soil studied was a particular sand, maintained at a constant dense state by a vibratory preparation process, the angle of internal

friction and relative density were constant for the present study. Thus, the functional relation of equation (8) can be written as :

$$\frac{y}{C} = \kappa \left[\frac{M}{\gamma C^3 L}, \frac{C}{L} \right], \dots \dots \dots (9)$$

subject to the restrictions placed on $\frac{C^2}{A}$, ϕ , D_d and loading rate.

EXPERIMENTAL PROCEDURE

Although the functional relationships for the general loading conditions of the rigid pole problem have been formulated for both cohesive and cohesionless soils, Kondner in an unpublished work in 1961, it was considered desirable for this initial phase of investigation to use sand rather than clay as the soil medium for the following reasons. Sand properties are easier to control in the laboratory than those of clay and the time effects are generally considered to be less complex.

Sand

The particular sand used was a uniform fine dune sand from Wolf Lake, Indiana, U.S.A., provided through the courtesy of the Raymond Concrete Pile Company, Chicago. The sand had a gradation curve as shown in Fig. 1 and was air-dried in the

Fig. 1. Sand Gradation Curve

laboratory to a moisture content of about $\frac{1}{4}$ per cent. For a dense state (i.e. density = 108 lb/ft³) this sand had an angle of internal friction ϕ of 37°, as determined from a series of triaxial compression tests. Subsequent to the beginning of each test the sand was vibrated to a constant density of 107 lb/ft³. Thus, the relative density was very close to one.

Poles

The model poles consisted of various lengths of polished aluminum or steel tube, plugged at the lower end. A list of the properties of these poles is given in Table 2.

Table 2. Properties of Model Poles

Pole number	Material	Diameter inches	Area A inches ²	Weight grams	Perimeter C inches
1	Aluminum	0.501	0.197	30	1.573
2	Aluminum	0.626	0.307	62	1.966
3	Steel	0.707	0.393	143	2.221
4	Steel	0.927	0.674	216	2.911
5	Steel	1.248	1.222	229	3.921
6	Steel	1.515	1.801	478	4.755

Fig. 2. Experimental Apparatus

Apparatus

A schematic diagram of the apparatus used in the study is shown in Fig. 2. A cut-down oil-drum was welded to a table-type concrete vibrator and mounted on a sturdy timber base. In order to minimize extraneous vibrations during the test, the timber base was supported on rubber blocks. The tank contained 330 pounds of dry sand and the volume was determined from a calibration curve previously obtained.

The couple was applied to the model pole by hanging weights in the centre of a simple

beam supported by two cords which passed over small ball bearing pulleys and were attached to different points on opposite sides of the pole. Thus, the applied couple was generated by two equal and opposite forces separated by a distance h .

Techniques

In order to eliminate any source of error resulting from loads due to the internal spring forces of the indicator dials, these springs were removed and the gage arms were secured to the pole by elastic bands.

An investigation of the nature of the creep response indicated that after approximately 2 minutes the deflection continues at an extremely slow rate and ultimately tends toward a limiting value for a sufficiently large time. Therefore, it appears very reasonable to neglect the large-time portion of the deflection versus time curve and adopt a couple increment-time interval of 2 minutes. A similar conclusion was reached by Kondner and Green (1962 c) in their analysis of the ground line thrust problem.

To check the effect of load rate on the shape of the couple-deflection curve, a series of tests were conducted where the rate of couple application was varied from 250 in-gms per 2 minutes to 2500 in-gms per 2 minutes. Within this range, the load rate has no significant effect on the resulting couple-deflection curves. Throughout the greater portion of the tests, the rate of couple application used was 500 in-gms per 2 minutes.

Fig. 3. Couple Versus Ground Line Deflection for Variable Moment Arm

For a limited number of tests where the larger poles were embedded very deeply this load rate was doubled. Hence, although the value of the term $\frac{\gamma t C}{\eta}$ did not remain constant, load rate and creep tests indicated its effects were minimized.

The final step of the experimental technique to be investigated was the effect of the change in moment arm on the resulting couple-deflection curve. One of the couple-forces was applied at the ground line and the other at some distance above, h . A series of tests were run where pole number 4 was embedded 4.85 inches, giving $\frac{C}{L} = 0.6$, and the couple was applied at the constant rate of 500 in-gms per 2 minutes while h was assigned values of 2.5, 5 and 10 inches. These results are shown in Fig. 3 and it can be seen that, as would be expected from the laws of statics, there is no appreciable effect due to variation of moment arm. Hence, a moment arm of 5 inches was selected for all results reported herein, where the lower force was applied at the ground line.

EXPERIMENTAL RESULTS

The results analyzed were obtained from tests performed on circular model poles embedded in a dense sand and subjected to a couple applied at a rate of 500 in-gms per 2 minutes with one of the couple forces applied at the ground-line and the other 5 inches above. The only exception to the above was for the deeply embedded large poles when the loading rate was doubled. The purpose of this section is to formulate from the experimental data an explicit expression for the general functional relationship given in equation (9).

The schedule of tests is shown in Table 3 where the required depth of embedment is given for each pole in order to obtain some desired value of $\frac{C}{L}$.

To illustrate the advantage of presenting experimental data in non-dimensional form, tests were conducted on poles 3, 4 and 5 with embedded depths of 2.77, 3.64 and 4.90 inches, respectively, and results are shown in Fig. 4 which is a conventional plot of applied couple versus deflection. Note the seemingly important influence of pole size and depth of embedment. Since the ratio $\frac{C}{L}$ is constant ($\frac{C}{L} = 0.8$) for all three tests,

Table 3. Test Schedule Illustrating Depths of Embedment

Pole number	C (inches)	C/L				
		0.4	0.6	0.8	1.0	1.2
1	1.573	3.92	2.62			
2	1.966	4.93	3.28			
3	2.221	5.55	3.70	2.77		
4	2.911		4.85	3.64	2.91	
5	3.921		6.53	4.90	3.92	3.26
6	4.755			5.94	4.75	3.96

Fig. 4. Applied Couple Versus Ground Line Deflection

the results may be plotted in non-dimensional form in terms of $\frac{M}{\gamma C^3 L}$ versus $\frac{y}{C}$ and reference to Fig. 5 shows that they follow essentially the same curve.

An extensive test program was carried out according to Table 3 for $\frac{C}{L}$ values of 0.4, 0.6, 0.8, 1.0 and 1.2, in which many of the tests were repeated to insure duplicability. The results are shown in Fig. 6. In order to obtain a unique relationship between $\frac{M}{\gamma C^3 L}$, $\frac{y}{C}$ and $\frac{C}{L}$, a constant value of $\frac{\gamma t C}{\eta}$ would have been required for all tests. Theoretically this could have been done by varying the loading rate for each test either by varying the increment of applied couple or by changing the time interval of loading. However, the field of soil mechanics has not yet reached a state of development where such loading rates can be predicted. Thus, the scatter in Fig. 6 is probably due to time effects as well as experimental error, although as previously stated the effects of $\frac{\gamma t C}{\eta}$ are minimized.

The shape of the couple-strength ratio versus deflection ratio curves is definitely

Fig. 5. Non-Dimensional Plot of $\frac{M}{\gamma LC^3}$ versus $\frac{y}{C}$

Fig. 6. Non-Dimensional Plot of $\frac{M}{\gamma LC^3}$ versus $\frac{y}{C}$ for Various Values of $\frac{C}{L}$

that of a non-linear material. These curves tend asymptotically to their respective upper limits as $\frac{y}{C}$ becomes large. The degree of non-linearity remains to be tested but this, in itself, indicates that the principle of superposition and the use of a soil modulus approach are probably not valid.

If the data of Fig. 6, as given by the average curves, are replotted in the form of the reciprocal of the secant modulus, $\left(\frac{y}{C}\right) / \left(\frac{M}{rC^3L}\right) = \frac{y r C^2 L}{M}$, versus $\frac{y}{C}$, the straight lines shown in Fig. 7 are obtained. These straight lines indicate the explicit form of the functional relation between $\frac{M}{rC^3L}$ and $\frac{y}{C}$ to be a two-constant hyperbolic equation and can be written as :

$$\frac{y r C^2 L}{M} = a + b \left(\frac{y}{C} \right) \dots\dots\dots (10)$$

Fig. 7. Hyperbolic Form of $\frac{M}{rLC^3}$ versus $\frac{y}{C}$ for Various Values of $\frac{C}{L}$

or

$$\frac{M}{rC^3L} = \frac{\frac{y}{C}}{a+b\left(\frac{y}{C}\right)} \dots\dots\dots (11)$$

where b and a are the slope and intercept, respectively, of each of the straight lines and are therefore functions of $\frac{C}{L}$. The initial slope of the $\frac{M}{rC^3L}$ versus $\frac{y}{C}$ curve is $\frac{1}{a}$ and the ultimate or limiting value of $\frac{M}{rC^3L}$ is $\frac{1}{b}$.

Fig. 8. Hyperbolic Intercept versus $\frac{C}{L}$

To determine the relation between (a) and $\frac{C}{L}$, the intercept value, a , for each straight line is plotted versus $\frac{C}{L}$ on the arithmetic plot given in Fig. 8. The equation of the straight line obtained is written as :

$$a = 0.00415 \frac{C}{L} \dots\dots\dots (12)$$

In an analogous manner, if the slope b of each straight line is plotted versus $\frac{C}{L}$ on the log graph shown in Fig. 9, the resultant straight line can be written in the form :

$$b = 0.6 \left(\frac{C}{L} \right)^{2.5} \dots\dots\dots (13)$$

Fig. 9. Hyperbolic Slope versus $\frac{C}{L}$

Substitution of equations (12) and (13) into equation (11) gives :

$$\frac{M}{\gamma C^3 L} = \frac{\frac{y}{C}}{0.00415 \left(\frac{C}{L}\right) + 0.6 \left(\frac{C}{L}\right)^{2.5} \left(\frac{y}{C}\right)} \dots\dots\dots (14)$$

Multiplying both sides of equation (14) by $\frac{C}{L}$ gives :

$$\frac{M}{\gamma C^2 L^2} = \frac{\frac{y}{C}}{0.00415 + 0.6 \left(\frac{C}{L}\right)^{1.5} \left(\frac{y}{C}\right)} \dots\dots\dots (15)$$

It is observed that the couple-strength parameter of equation (15) is a function of both the deflection parameter $\frac{y}{C}$ and the embedment parameter $\frac{C}{L}$.

If the functional relationship of equation (9) is rearranged by multiplying $\frac{M}{\gamma C^3 L}$ by $\left(\frac{C}{L}\right)^2$ to get the new term $\frac{M}{\gamma C L^3}$ and then dropping $\frac{M}{\gamma C^3 L}$ to maintain independence, the functional relation takes the form :

$$\frac{y}{C} = \kappa \left[\frac{M}{\gamma C L^3}, \frac{C}{L} \right] \dots\dots\dots (16)$$

Fig. 10. Non-Dimensional Plot of $\frac{M}{\gamma CL^3}$ versus $\frac{y}{C}$

Points from the “average curves” of Fig. 6 are replotted in the form $\frac{M}{\gamma CL^3}$ versus $\frac{y}{C}$ for a range of $\frac{C}{L}$ varying from 0.4 to 1.2 and results are shown in Fig. 10. The scatter reveals no phenomenological pattern and hence the points were fit with the average curve shown. This indicates that the embedment parameter $\frac{C}{L}$ has a very small influence on the results and enables us to write a simpler, but more approximate, equation relating the variables of equation (16). Considering the average curve to be independent of $\frac{C}{L}$, equation (16) may be written :

$$\frac{y}{C} = \kappa \left[\frac{M}{\gamma CL^3} \right] \dots\dots\dots (17)$$

By replotting the average curve of Fig. 10 in the form of the reciprocal of the secant modulus, $(y/c) / \left(\frac{M}{\gamma CL^3} \right)$, versus $\frac{y}{C}$ as shown in Fig. 11, a straight line is obtained indicating the relation between $\frac{M}{\gamma CL^3}$ and $\frac{y}{C}$ is again a two constant hyperbolic

Fig. 11. Hyperbolic Form of $\frac{M}{\gamma CL^3}$ versus $\frac{y}{C}$

equation and may be expressed as :

$$\frac{M}{\gamma CL^3} = \frac{\frac{y}{C}}{0.0048 + 0.57 \frac{y}{C}} \quad \dots\dots\dots (18)$$

where 0.0048 and 0.57 are the intercept and slope, respectively, of the straight line. Note that the term $\frac{C}{L}$ does not appear explicitly in equation (18).

The reciprocal of the slope of the straight line in the hyperbolic test fit, $1/b = 1/0.57 = 1.75$, is equal to the ultimate value of $M/\gamma CL^3$ as given in equation (18). Thus, a stability number for the pole problem under consideration can be written as :

$$\left[\frac{M}{\gamma CL^3} \right]_{\text{ultimate}} = 1.75 \quad \dots\dots\dots (19)$$

This establishes a failure criterion relating the applied couple to the soil and pole parameters. In their analysis of rigid poles in sand subjected to a horizontal ground-line thrust, Kondner and Green (1962 c) obtained the stability number

$$\left[\frac{F}{\gamma CL^2} \right]_{\text{ultimate}} = 1.75 \quad \dots\dots\dots (20)$$

where F is the ground line thrust. Since the applied couple was generated by a ground line thrust F_c plus an equal and opposite force F_c acting at a distance h above the ground line, the applied couple may be expressed as :

$$M = F_c h. \dots\dots\dots (21)$$

Substituting equation (21) into equation (19) and equating the result to equation (20), gives :

$$\left[\frac{F_c}{\gamma C L^2} \frac{h}{L} \right]_{\text{ultimate}} = 1.75 = \left[\frac{F_t}{\gamma C_t L_t^2} \right]_{\text{ultimate}} \dots\dots\dots (22)$$

or

$$\frac{F_{c\text{ultimate}} h}{\gamma C_c L_c^3} = \frac{F_{t\text{ultimate}}}{\gamma C_t L_t^2} \dots\dots\dots (23)$$

where the subscripts c and t refer to the applied couple and horizontal ground-line thrust respectively. For a given sand having constant properties, the factor of safety in the ground line thrust problem, S_t , for an applied thrust F_t is

$$S_t = \frac{F_{t\text{ultimate}}}{F_t}, \dots\dots\dots (24)$$

and the factor of safety in the applied couple problem, S_c , for an applied couple, $F_c h$, is

$$S_c = \frac{F_{c\text{ultimate}}}{F_c}. \dots\dots\dots (25)$$

Therefore

$$F_{t\text{ultimate}} = S_t F_t \dots\dots\dots (26)$$

and

$$F_{c\text{ultimate}} = S_c F_c \dots\dots\dots (27)$$

Substituting equations (26) and (27) into equation (23) gives :

$$S_c = S_t \frac{F_t C_c L_c^3}{F_c h C_t L_t^2} \dots\dots\dots (28)$$

which relates the factor of safety for the applied couple problem to the factor of safety in a horizontal ground line thrust test. If $C_c = C = C_t$ and $L_c = L = L_t$, equation (28) reduces to the form

$$S_c = S_t \left[\frac{F_t}{F_c} \right] \left[\frac{L}{h} \right] \dots\dots\dots (29)$$

which can be further reduced to the form

$$S_c = S_t \frac{L}{h} \dots\dots\dots (30)$$

if $F_t = F_c$.

As h becomes very large or as L becomes very small in equation (30), the factor of

safety becomes small, approaching zero in the limit. Conversely, as the depth of embedment increases or as the forces approach a state of colinearity, the safety factor increases, approaching infinity in the limit.

The relationship between the ground line thrust problem and the applied couple problem is very useful in prototype testing, since it permits a prediction of the stability of one problem from test results of the other. For example, the factor of safety for a pole subjected to an applied couple can be determined directly from the results of an appropriately designed ground-line thrust test, which is relatively simple to conduct. In applying equation (29), the procedure required is to fail a pole by a horizontal ground-line thrust, divide the failure load by the magnitude of one of the couple forces and multiply this quotient by the $\frac{L}{h}$ ratio for the applied couple problem to get the factor of safety for the latter.

INTERPRETATIONS

It should be emphasized at this time that the constants developed in equations (15), (18) and (19) are valid only for the soil conditions reported in this paper. These constants are functions of soil properties, such as density, relative density, ultimate strength, etc.

In order to evaluate these constants for a particular prototype application, some experimental field testing would have to be done. However, the authors feel that the form of the π terms and the equations obtained are quite general and offer considerable insight into the problem of field response of the analogous prototype structure. Hence, by using the π terms as stated and the described procedure, the amount of field testing necessary to determine the appropriate numerical coefficients would be minimized.

For example, the development of the $\frac{M}{rCL^3}$ versus $\frac{y}{C}$ relationship given in equation (18) would theoretically require only one prototype test for a particular soil having constant properties. However, from a practical standpoint it is certainly advisable to conduct more than one test to reduce experimental error. Also, since the authors have no field test data with which to correlate the model study, caution is recommended in trying to use the model results to predict prototype response.

To illustrate the possible potential use, which the general functional forms developed herein may some day be applicable, the following illustrative examples are presented.

ILLUSTRATIVE EXAMPLES

1. *Given:* An 8 inch circular, rigid, steel pole is to be embedded in a very dense uniform fine dune sand, (density 107 lbs per cu. ft. $\phi=370^\circ$) and subjected to an applied couple equal to 25,000 ft-lbs.

Required: The depth of embedment for a safety factor of 2 against failure of the pole-soil system.

Solution: It shall be assumed that soil conditions under consideration are identical

with those reported in this paper, thus establishing the value of the stability number at 1.75.

Therefore :

$$\frac{M}{\gamma CL^3} = 1.75; \quad c = \pi \left(\frac{8}{12} \right) = 2.1 \text{ ft.}$$

and

$$L^3 = \frac{2(25,000)}{1.75 (107) (2.1)}$$

or

$$L = 5.0 \text{ ft.}$$

Alternatively, if the pole was already in place embedded to a depth of 4.0 ft. the safety factor against ultimate failure would be obtained by using equation (19) to give :

$$M_{\text{ultimate}} = 1.75 \gamma CL^3 = 1.75 (107) (2.1) (4)^3 = 25,200 \text{ ft-lbs.}$$

Hence, the factor of safety, S , is

$$S = \frac{25,200}{25,000} = 1.01$$

It is important to note that the stability of a pole subjected to an applied couple is proportional to L^3 . Therefore, small changes in depth of embedment produce large changes in the ultimate couple capacity, as illustrated above.

2. *Given:* A 10 3/4 inch circular, rigid, steel pole is embedded 4.5 ft in a very dense uniform, fine, dune sand, density 107 lb/ft³, $\phi = 37^\circ$, and subjected to a horizontal ground line thrust of 3 tons with an equal and opposite load six feet above the ground line.

Required: The factor of safety against failure of the pole-soil system.

Solution: $M = Fh = 3 (2000) (6) = 36,000 \text{ ft-lb.}$

$$C = \pi (10.75) = 33.8 \text{ in.} = 2.82 \text{ ft.}$$

$$L = 4.5 \text{ ft.}$$

Using the pole stability number of equation (19) :

$$\begin{aligned} \left[\frac{M}{\gamma CL^3} \right]_{\text{ultimate}} &= 1.75 \\ M_{\text{ultimate}} &= 1.75 \gamma CL^3 = 1.75 (107) (2.82) (4.5)^3 \\ &= 48,000 \text{ ft-lb.} \end{aligned}$$

Hence the factor of safety is

$$S_c = \frac{48,000}{36,000} = 1.33$$

Under the same conditions of soil and pole geometry, the factor of safety S_t for a horizontal ground line load of 3 tons was determined by Kondner and Green (1962) to be :

$$S_t = 1.78$$

Using equation (30), the safety factor for the applied couple problem is :

$$S_c = S_t \frac{h}{L} = 1.78 \frac{4.5}{6} = 1.33$$

CONCLUSIONS

As a result of this investigation the following conclusions may be formulated :

- (i) For a rigid pole embedded in a sand and subjected to an applied couple the following theoretical, functional relationship has been developed to describe the phenomena :

$$\frac{y}{C} = \kappa \left[\frac{M}{\gamma C^3 L}, \frac{C}{L}, \frac{C^2}{A}, \phi, \frac{\gamma t C}{\eta}, D_a \right] \dots\dots\dots (8)$$

- (ii) For a rigid, circular model pole embedded vertically in a very dense, uniform, fine, dune sand of density 107 lb per ft³ and angle of internal friction of 37°, subject to an applied couple :

- (a) The explicit form of the functional relation given above takes the form of a two constant hyperbolic equation which can be written as :

$$\frac{M}{\gamma C^3 L} = \frac{\frac{y}{C}}{a + b \left(\frac{y}{C} \right)} \dots\dots\dots (11)$$

where a and b are functions of $\frac{C}{L}$.

- (b) A simplified, but less accurate, form of the functional relationship which is explicitly independent of $\frac{C}{L}$ can be written as

$$\frac{M}{\gamma C L^3} = \frac{\frac{y}{C}}{\bar{a} + \bar{b} \left(\frac{y}{C} \right)}$$

where $\bar{a}$ and $\bar{b}$ are numerical coefficients.

- (c) A failure criterion relating the ultimate applied couple with the geometry of the pole-soil system can be written in the form of a stability number :

$$\left[\frac{M}{\gamma C L^3} \right]_{\text{ultimate}} = 1.75 \dots\dots\dots (19)$$

- (d) The above stability number can be correlated with a similar stability number for the case of an applied horizontal ground-line thrust.

- (iii) Since the authors have no field tests with which to correlate the model study, caution is recommended regarding the use of the model response for prototype

considerations. However, the authors feel that the forms of the non-dimensional parameters and the equations obtained are quite general and offer considerable insight into the actual response of prototype structures.

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