A METHOD OF ESTIMATING THE DEGREE OF COMPACTION OF SANDY SOILS

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SUMMARY

A theoretical as well as an experimental study was made on compaction characteristics of sandy soils, from which a formula has been introduced which makes it possible to estimate the degree of compaction of such soils. As to cohesionless or less cohesive soils such as sandy soils, their compaction is considered to be governed mainly by the configuration of grains or the conditions of void space, and it has been made clear that it is very convenient to adopt such a term as expresses the soil resistance against its compaction.

A theoretical study of a formula based on data of standard compaction test has come to the conclusion that soil resistance against compaction depends on its relative density under consideration, provided the other soil conditions remain constant.

The formula has been proved well applicable to estimation of the degree of compaction of sandy soils not only in tamping compaction, but also in compaction by rolling and by vibration.

INTRODUCTION

Compaction of soil is usually defined as the process of bringing soil particles closer to each other or of reducing the volume of air void in the soil by blows, rollings, vibrations, etc.

Since R. R. Proctor developed a laboratory compaction test and established the idea of optimum moisture content, a number of researches on compaction characteristics have been carried out. Of these, however, only a few researches have dealt with estimation of the degree of compaction by a given number of coverages of a roller.

In our country, G. Sasaki and H. Negishi first derived an empirical formula on settlement of soils by successive coverages of a roller.¹⁾ A further study was conducted on the same type of formula by T. Mogami and G. Kuno.²⁾ S. Murayama and the author found a relation between dry density of compacted soil and number of coverages of rollers.^{3), 4)}

In this paper, the author is going to make clear the physical meaning of our formula and to prove that the formula can be not only extended to the other formulas stated above, but also applied to soil compaction by various methods.

LABORATORY COMPACTION TEST AND ITS RESULTS

A series of laboratory compaction tests were carried out with a well graded sandy soil called "masado". The devices used were those specified by JIS (Japanese Industrial

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Standards) A 1210 which is nearly identical with the AASHO specification on compaction test. The test procedure was, however, modified in stroke of free fall and number of blows of the rammer. The compaction was accomplished by tamping the soil in three layers of nearly equal thickness, each layer being subjected to a combination of 5, 10, 20, 40, 80 or 160 blows with the rammer and 10, 20, 30 or 45 cm in free fall of the rammer.

Examples of the test results are shown in Figs. 1 and 2, which are on the soil samples having moisture contents of 6.9% and 13.0% respectively.

Fig. 1. Relation between Number of Blows by Rammer and Dry Density of Compacted Soil

Fig. 2. Relation between Number of Blows by Rammer and Dry Density of Compacted Soil

From the above, the degree of compaction or dry density of the soil increases with increase in number of blows, rapidly at the beginning and more slowly as compaction develops, and the higher free fall gives the sooner arrival at a final stage of compaction. The other data also show similar characteristics to the above.

ANALYSIS OF THE TEST RESULTS

An analysis of the test results has been made in order to find a formula indicating the relation between dry density of compacted soil and number of blows by a rammer.

If we use the symbols:

γ_d : dry density of soil compacted by N blows,

$\gamma_{d\min}$: initial or minimum dry density of soil

and $\Delta\gamma_d$: increase in dry density of soil by N blows,

and draw relations between $\Delta\gamma_d/N$ and $\Delta\gamma_d$, we get Figs. 3 and 4 corresponding to the data in Figs. 1 and 2 respectively.

Fig. 3. Relation between $\Delta\gamma_d$ and $\Delta\gamma_d/N$ Taken from Fig. 1

Fig. 4. Relation between $\Delta\gamma_d$ and $\Delta\gamma_d/N$ Taken from Fig. 2

According to Figs. 3 and 4, it can be recognized that the relation of $\Delta\gamma_d/N$ and $\Delta\gamma_d$ for each test is approximately linear. Hence, we obtain the following equation:

$$\frac{\Delta\gamma_d}{N} = a - b\Delta\gamma_d, \dots\dots\dots (1)$$

where a and b are positive coefficients independent of N and $\Delta\gamma_d$. When we consider

a test with a certain number of blows and a certain free fall, the coefficient a is the intercept of the corresponding line on the ordinate and b is the gradient of the line in Figs. 3 and 4. Hence, we have

$$\frac{a}{b} = (\Delta\gamma_d)_{\max}, \quad \dots\dots\dots (2)$$

the right-hand side being equal to the intercept of the line on the abscissa. This expression should hold good theoretically for a soil having the same gradation, same moisture content and the same other initial conditions.

Since $\Delta\gamma_d = \gamma_d - \gamma_{d\min}$, Eq. (1) can be transformed as

$$\gamma_d = \gamma_{d\min} + \frac{aN}{1+bN}, \quad \dots\dots\dots (3)$$

which gives the dry density of compacted soil for a given number of N .

PHYSICAL MEANING OF THE DERIVED FORMULA

In tamping compaction of soil, the work done by dropping of a rammer is transformed into compactive energy. If the energy given by a single blow of a rammer is denoted by E_0 , the total energy by N blows of the rammer is $E = NE_0$. Substitution of $N = E/E_0$ into Eq. (1) yields

$$\Delta\gamma_d = \frac{E}{E_0}(a - b\Delta\gamma_d) \quad \dots\dots\dots (+)$$

or

$$E = \frac{\Delta\gamma_d}{\lambda - \mu\Delta\gamma_d}, \quad \dots\dots\dots (5)$$

where

$$\lambda = \frac{a}{E_0}, \quad \mu = \frac{b}{E_0}. \quad \dots\dots\dots (5)'$$

If we have a mental picture of soil structure as shown in Fig. 5, it is easy to derive the following expressions :

Fig. 5. Model Soil Sample, Having Unit Sectional Area

$$\left. \begin{array}{ll} \text{Void ratio:} & e = \frac{H_a + H_w}{H_s} = \frac{H_{a0} + H_w - s}{H_s}, \\ \text{Maximum void ratio:} & e_{\max} = \frac{H_{a0} + H_w}{H_s}, \\ \text{Minimum void ratio:} & e_{\min} = \frac{H_w}{H_s}, \end{array} \right\} \dots\dots\dots (6)$$

$$\left. \begin{array}{ll} \text{Dry density:} & \gamma_d = \frac{G\gamma_w}{1+e}, \\ \text{Maximum dry density:} & \gamma_{d\max} = \frac{G\gamma_w}{1+e_{\min}}, \\ \text{Minimum dry density:} & \gamma_{d\min} = \frac{G\gamma_w}{1+e_{\max}}. \end{array} \right\} \dots\dots\dots (7)$$

and

In the above, the symbols in the right-hand side have the following meanings :

- s : settlement of sample due to compaction
- H : entire depth of sample
- H_s : depth of solids part
- H_w : depth of water part
- H_a : depth of air part in the state under consideration
- H_{a0} : depth of air part in the initial state
- G : specific gravity of solids
- γ_w : unit weight of water

And also, using Eqs. (2), (5)', (6) and (7), we can derive an equation :

$$\begin{aligned} \frac{\lambda}{\mu} &= \frac{a}{b} = (\Delta\gamma_d)_{\max} = \gamma_{d\max} - \gamma_{d\min} \\ &= \frac{G\gamma_w H_s H_{a0}}{H^2} \bigg/ \frac{H - H_{a0}}{H} \dots\dots\dots (8) \end{aligned}$$

Accordingly, we can put

$$\left. \begin{array}{l} \lambda = k \frac{G\gamma_w H_s H_{a0}}{H^2} \\ \mu = k \frac{H - H_{a0}}{H} \end{array} \right\} \dots\dots\dots (9)$$

where k is a coefficient

From Eqs. (6) and (7),

$$\begin{aligned} \Delta\gamma_d &= \gamma_d - \gamma_{d\min} = G\gamma_w \left(\frac{1}{1+e} - \frac{1}{1+e_{\max}} \right) \\ &= G\gamma_w H_s \frac{s}{H(H-s)} \dots\dots\dots (10) \end{aligned}$$

can be derived. Substituting Eq. (10) into Eq. (5), we have

$$E = \frac{G\gamma_w H_s s}{\lambda H^2 - (\lambda H + \mu G\gamma_w H_s)s} \dots\dots\dots (11)$$

Using Eq. (9), we can transform Eq. (11) as follows:

$$E = \frac{1}{k} \cdot \frac{s}{H_{a0} - s} \dots\dots\dots (12)$$

Since $s = H_{a0} - H_s(e - e_{\min})$, Eq. (12) becomes

$$E = \frac{1}{k} \left[\frac{H_{a0}}{H_s(e - e_{\min})} - 1 \right] \dots\dots\dots (13)$$

Differentiation of E in Eq. (13) with respect to e gives

$$\frac{dE}{de} = \frac{1}{k} \cdot \frac{-H_{a0}}{H_s(e - e_{\min})}$$

or

$$\begin{aligned} \frac{dE}{-de} &= \frac{1}{k} \cdot \frac{H_{a0}}{H_s} \cdot \frac{1}{(e_{\max} - e_{\min})^2} \left(\frac{e_{\max} - e_{\min}}{e - e_{\min}} \right) \\ &= \frac{1}{k} \cdot \frac{H_{a0}}{H_s} \cdot \frac{1}{(e_{\max} - e_{\min})^2} \cdot \frac{1}{(1 - D_r)^2} = KH_s \cdot \frac{1}{(1 - D_r)^2}, \dots\dots\dots (14) \end{aligned}$$

where

$$K = \frac{H_{a0}}{kH_s^2} \cdot \frac{1}{(e_{\max} - e_{\min})^2} \dots\dots\dots (14)'$$

and D_r is the relative density.

The left-hand side of Eq. (14) means the energy required to produce a unit amount of decrease in void ratio of soil.

If we can assume such a quantity R as expresses soil resistance against its compaction, differential energy dE required to make a differential settlement ds should take the form:

$$dE = R ds \dots\dots\dots (15)$$

Since ds is given by the relation $ds = -H_s de$, Eq. (15) becomes

$$dE = -RH_s de \dots\dots\dots (16)$$

Comparison of Eq. (16) with Eq. (14) gives

$$R = \frac{K}{(1 - D_r)^2} \dots\dots\dots (17)$$

Eq. (17) implies that soil resistance against compaction is a function of relative density. It increases gradually as relative density approaches its maximum value, i.e. unity.

APPLICABILITY OF THE FORMULA TO ROLLING COMPACTION

The formula Eq. (3), accordingly Eq. (1), obtained from tamping compaction tests, was checked of its applicability to compaction by rolling.

In laboratory rolling compaction tests, a model vibratory roller of 25 cm in drum diameter and 30 cm in drum width was used with a well graded sand of a constant moisture content. The tests were made under various conditions in rolling speed, thickness of lift and vibration. The plot of the dry density of compacted soil against the number of coverages showed similar characteristics to those in Figs. 1 and 2.

Using such data, we get relations between $\Delta\gamma_d$ and $\Delta\gamma_d/N$ as seen in Figs. 6 and 7, where N denotes the number of coverages by the roller and $\Delta\gamma_d$ increase in dry density of samples during N coverages.

Fig. 6. Relation between $\Delta\gamma_d$ and $\Delta\gamma_d/N$ in Compaction by a Model Roller

Fig. 7. Relation between $\Delta\gamma_d$ and $\Delta\gamma_d/N$ in Compaction by a Model Roller

Figure 6 is concerned with one of the tests with varying rolling speeds. From the diagram, it can be recognized that Eq. (1) is well consistent with the data, and the values of $(\Delta\gamma_d)_{\max}$ for all the tests are considered identical. The same statement looks true of Fig. 7, which is obtained from one of the tests by varying the thickness of lift.

Figure 8 is a plot of data obtained by field compaction tests in which various compactive equipments were used.

Fig. 8. Relation between $\Delta\gamma_d$ and $\Delta\gamma_d/N$ in Compaction by Various Compactive Equipments

The approximate linearity of the plots can be seen for each test in Fig. 8. It means that the data is consistent with Eq. (1) or (3). All the lines do not meet at a point on the abscissa in Fig. 8, but it is natural for such tests on different equipments and soil conditions. The disagreement in $(\Delta\gamma_d)_{\max}$ would rather suggest difference in compactability in those tests.

APPLICABILITY OF THE FORMULA TO VIBRATORY COMPACTION

Compaction of sand by vibratory method was conducted with apparatus shown in Fig. 9. In this test, soil samples with surcharge load on them were subjected to vertical vibration, and vibration acceleration of the container and settlement of the load were measured. Thus, relations of settlement and time for given vibration conditions were obtained.

Figures 10 and 11 are examples of such experimental data, the former concerning a test of 1,000 cpm in frequency of vibration, 0.8 g (g : acceleration of gravity) in vibration acceleration and 0.1 kg/cm^2 in surcharge pressure and the latter a test of 1,500 cpm, 1.2 g and 0.1 kg/cm^2 respectively.

Fig. 9. Devices for Vibratory Compaction Tests

Fig. 10. Relation between Time and Settlement in Vibratory Compaction

Fig. 11. Relation between Time and Settlement in Vibratory Compaction

In investigating the applicability of the formula to this case, it is convenient to use Eq. (12) which is derived from Eq. (1). If E_0 denotes energy given to the sample during one cycle of vibration,

$$E = E_0 f t \dots\dots\dots (18)$$

is the total energy given by the vibration of f cpm for t minutes. Then, Eq. (12) is written as

$$E_0 f t = \frac{1}{k} \cdot \frac{s}{H_{a0} - s} \dots\dots\dots (19)$$

or

$$\frac{s}{t} = k E_0 f (H_{a0} - s) = \alpha - \beta s, \dots\dots\dots (20)$$

where

$$\alpha = k E_0 f H_{a0}, \quad \beta = k E_0 f. \dots\dots\dots (20)'$$

Eq. (20) is the formula expressed by the term of settlement, being considered equivalent to Eq. (1). Accordingly, we can check Eq. (20) instead of Eq. (1) by using experimental data shown in Figs. 10 and 11. From the data, $(s/t) - s$ relations can be obtained as seen in Figs. 12 and 13.

According to these diagrams, the relations between s and s/t look approximately linear except only at a final stage of settlement, and the same is true of the other data. Hence, it can be said that Eq. (1) or (3) is applicable to vibratory compaction.

Fig. 12. Relation between S and S/t Taken from Fig. 10

Fig. 13. Relation between S and S/t Taken from Fig. 11

ESTIMATION OF THE DEGREE OF COMPACTION OF SANDY SOILS

It has been made clear so far in the above that the formula Eq. (1) or Eq. (20) is in good agreement with experimental data.

If the formula is written down again, it is

$$\frac{\Delta\gamma_d}{N} = a - b \Delta\gamma_d \dots\dots\dots (1)$$

in the terms of dry density and the number of blows, and

$$\frac{s}{t} = \alpha - \beta s \dots\dots\dots (20)$$

in the terms of settlement and time of operation, one being equivalent to the other. Since there are some correspondences of $\Delta\gamma_d$ to s and of N to t , we have only to deal with Eq. (1) for instance as in the following.

Using Eqs. (5)', (6), (9) and (14)', the coefficients a and b are given by

$$\begin{aligned} a &= \lambda E_0 = k E_0 \frac{G\gamma_w H_s H_{a0}}{H^2} = \frac{H_{a0}}{KH_s^2} \cdot \frac{E_0}{(e_{\max} - e_{\min})^2} \cdot \frac{G\gamma_w H_s H_{a0}}{H^2} \\ &= \frac{E_0}{K} \cdot \frac{H_{a0}}{H_s} \cdot \frac{H_s^2}{H_{a0}^2} \cdot \frac{G\gamma_w H_s H_{a0}}{H^2} = \frac{E_0}{KH} \gamma_s n_s \dots\dots\dots (21) \end{aligned}$$

and

$$\begin{aligned} b &= \mu E_0 = k E_0 (1 - n_{a0}) = \frac{H_{a0}}{KH_s^2} \cdot \frac{E_0}{(e_{\max} - e_{\min})^2} (1 - n_{a0}) \\ &= \frac{E_0}{K} \cdot \frac{H_{a0}}{H_s^2} \cdot \frac{H_s^2}{H_{a0}^2} (1 - n_{a0}) = \frac{E_0}{KH} \cdot \frac{1 - n_{a0}}{n_{a0}}, \dots\dots\dots (22) \end{aligned}$$

where

$$n_{a0} = H_{a0}/H, \quad n_s = H_s/H, \quad \gamma_s = G\gamma_w \dots\dots\dots (23)$$

Equation (1) gives a set of straight lines, each of them being determined by compactive energy for one blow, as long as soil conditions are kept constant. It is necessary to get some test data in order to sketch such lines. For example, the line UT in Fig. 14 is drawn through two points P_1 and Q_1 which correspond to data for N_1 and N_2 blows respectively in a test of E_{01} in compactive effort. This line gives the value of $(\Delta\gamma_d)_{\max}$ indicated by $\overline{OT}$.

For cases of different compactive effort, only one point such as P_2 or P_3 is enough to determine the corresponding line. Such lines make it possible to estimate the degree of compaction for various numbers of blows, and Eq. (3) is conveniently used in this process.

The formula includes something unreasonable with the term of energy, which may be caused by neglect of energy loss in actual compaction works, but it does not prevent applying the formula to the estimation described in the above.

Fig. 14. Method of Drawing Relation between $\Delta\gamma_d/N$ and $\Delta\gamma_d$

CONCLUSION

The formula presented here is successfully applicable to the estimation of the degree of compaction or the settlement of sandy soils subjected to a certain number of blows by a compactive rammer. This comes true not only of compaction by tamping, but also that by rolling and by vibration, based on many test data including some described in this paper. The minimum dry density of a sandy soils is very important in this problem, and another paper⁵⁾ by the author has dealt with a method of measuring it, which turns out good and efficient.

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