A STUDY ON THE CONSOLIDATION PROCESS AFFECTED BY WELL RESISTANCE IN THE VERTICAL DRAIN METHOD

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SYNOPSIS

The present study has treated a theoretical consideration on the behavior of the loss of head in a vertical drain well and has made clear several factors affecting the rate of consolidation in the vertical drain method. Numerical solutions of this problem have been obtained with the aid of an electric lumped-parameter model and these results have shown fairly good agreement with those obtained from a large scale laboratory experiment. The quantitative data given in graph forms will be of great use in the three-dimensional analysis of the stabilization of soft soils by means of drain wells.

1. INTRODUCTION

In recent years, it has become a common practice to use vertical drain method for the stabilization of soft clayey strata. Two-dimensional consideration based on Barron's theory is usually employed for the design purpose of the vertical drains. However remarkable differences in shear strength through depth have been found between the predicted values based on Barron's theory and the observed values in the field. These differences may be caused by such factors as considerable flow resistance in drain wells, peripheral smear around drain wells and stress-concentration. Here, the authors conclude: three-dimensional treatment is inevitable in case where the length of drain is so long that the loss of head in the drain well should not be ignored.

Even though the conventional method—two-dimensional treatment—is applicable for the problem of smear, it is helpless for the prediction of the consolidation process affected by well resistance.

2. THEORETICAL CONSIDERATION FOR THE HEAD LOSS IN DRAIN WELLS

Theoretically, some well resistance should be considered in all drain wells. The greater the rate of flow is, which is affected by the length, the area of drain wells and the compressibility of clay stratum, the greater is the modification of consolidation due is to well resistance. That is to say, the effect of well resistance becomes remarkable in the case of deep and very soft deposit. Weak grounds in Japan are generally very much soft and deep. Accordingly, it will be impossible to ignore the effect of well resistance.

For the purpose of this study the following basic assumptions are made:

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1) K. Terzaghi's assumptions for the basic theory of consolidation and R. A. Barron's assumptions for the consolidation by radial flow toward drain wells are all admitted.

2) There is no vertical flow in the consolidating layer except the flow due to normal one-dimensional consolidation.

3) The discharge due to one-dimensional consolidation will have some influence on the behavior of the loss of head in drain wells, but this effect is neglected in the present study.

4) The applied load is initially carried by uniform pore-water pressure in the consolidating layer and the drain well.

These assumptions are not completely satisfied in the real phenomenon of consolidation. However, it may be said that the discrepancy between theory and phenomenon is usually small in the engineering sense.

In order to derive the basic differential equation for the head loss in drain wells, we consider consolidation by radial flow toward drain wells, as shown in Fig. 1.

![Fig. 1. Fundamental concept of vertical drain method affected by well resistance](image)

The total pressure, \( p_0 \), exerted on the consolidating soil equals the sum of the effective pressure, \( p \), and the neutral pressure, \( \gamma_w h \), (the lost head) in drain wells, and is assumed to be kept constant:

\[
\rho_0 = p + \gamma_w h = \text{const.} \tag{1}
\]

Also, these values of \( p \) and \( \gamma_w h \) are function of two variables; the depth coordinate, \( z \), below the consolidating surface and time, \( t \). If the effective pressure in drain wells is kept constant, the total discharge, \( q(t) \), toward drain wells from the annular space
having elementary length, $dz$, at time, $t$, is

$$q(t) = \frac{1}{4} \pi m_c (n^2 - 1) d_w^2 U(t) p \cdot dz$$ ...........................................(2)

where $n = d_e / d_w$,

$m_c =$ compression modulus of consolidating soil,

d_e = diameter of zone of influence,

$d_w =$ diameter of drain well, and

$U(t) =$ average consolidation due to radial flow to drain wells without smear and resistance.

According to Yanai and others, 2)

$$U(t) = 1 - \frac{\bar{u}}{u_0}$$

where

$$\bar{u} = -\frac{2r_w}{(r_e^2 - r_w^2)} \sum_k \frac{A_k}{k} \left[ \frac{J_1(kr_w)}{Y_1(kr_e)} - \frac{J_1(kr_e)}{Y_1(kr_w)} \right] e^{-k \rho c_h k}$$

$$A_k = \frac{2u_0 r_w}{r_e^2 V^2(kr_e) - r_w^2 V^2(kr_w)}$$

In the case where the effective pressure in drain wells is not constant with time, from eq. (1),

$$dp = \frac{\partial p}{\partial t} dt = -\gamma_w \frac{\partial h}{\partial t} dt$$ .................................................................(3)

and eq. (2) can be written as

$$q(t, z) = \frac{1}{4} \pi m_c (n^2 - 1) d_w^2 \sum_{t} U(t - \tau) dp \cdot dz$$ ...........................................(4)

where

$$\tau = \tau_{0}, \tau_{1}, \tau_{2}, \ldots \ldots \ldots \ldots \tau = t$$

Developing eq. (4) to the form of integration, we obtain

$$q(t, z) = \frac{1}{4} \pi m_c (n^2 - 1) d_w^2 \int_{0}^{t} U(t - \tau) dp \cdot dz$$ ...........................................(5)

Substituting eq. (3) in eq. (5), we obtain

$$q(t, z) = \frac{1}{4} \pi \gamma_w m_c (n^2 - 1) d_w^2 \int_{0}^{t} U(t - \tau) \frac{\partial h}{\partial \tau} dt \cdot dz$$ ...........................................(6)

The discharge due to consolidation from the interval between depth, $z$, and $z + dz$ per unit time is

$$dq = \frac{\partial q}{\partial t} dt$$ .................................................................(7)
and substituting eq. (6) in eq. (7), we obtain

\[ dq = \frac{1}{4} \pi r_m^2 \nu (n^2 - 1) d^2 \frac{\partial}{\partial t} \int_0^t U(t - \tau) \frac{\partial h}{\partial \tau} d\tau \cdot dz \cdot dt \] ..........................(8)

On the other hand, according to Darcy's law, the filtration velocity, \( v \), in drain wells at depth, \( z \), is

\[ v = k_w \cdot i(t, z) = -k_w \left( \frac{\partial h}{\partial z} \right)_{t, z} \] ..........................(9)

where \( k_w \) = coefficient of permeability of drain wells

\( i \) = hydraulic gradient in drain wells

The general discharge equation for water through the cross-sectional area of a drain well is

\[ q = vA_w dt = -\frac{1}{4} \pi d_w^2 k_w \frac{\partial h}{\partial z} dt \] ..........................(10)

where \( q \) = total discharge of water and \( A_w \) = the cross-sectional area of a drain well. Consequently, the total change in flow, \( dq \), from the entrance face to the exit face of the elementary column of a drain well over a distance, \( dz \), is

\[ dq = \frac{\partial q}{\partial z} dz = -\frac{1}{4} \pi d_w^2 k_w \frac{\partial^2 h}{\partial z^2} dt \cdot dz \] ..........................(11)

Obviously, the discharge difference between both faces of the elementary column ought to be equal to the amount of water which flows out of the consolidating soil and into the elementary column. Then, from eq. (3) and eq. (11),

\[ \frac{k_w}{m_v \gamma_w (n^2 - 1)} \frac{\partial^2 h}{\partial z^2} = \frac{\partial}{\partial t} \int_0^t U(t - \tau) \frac{\partial h}{\partial \tau} d\tau \] ..........................(12)

This is the fundamental equation for the dissipation process of the head loss in drain wells with resistance.

Setting

\[ \varphi = \int_0^t \frac{d\tau}{U(t - \tau)} \] ..........................(13)

and substituting eq. (13) in eq. (12), we obtain finally

\[ \frac{\partial h}{\partial \varphi} = \frac{k_w}{m_v \gamma_w (n^2 - 1)} \frac{\partial^2 h}{\partial z^2} \] ..........................(14)

This is analogous to the differential equation for one-dimensional consolidation.

Now, we consider the following set of boundary and initial conditions to solve eq. (14).
SOIL AND FOUNDATION

1) \( t = 0 \quad \varphi = 0 \) and \( 0 \leq z \leq L \quad h = h_0 \)
2) \( t = t \quad \varphi = \varphi \) and \( z = L \quad \frac{\partial h}{\partial z} = 0 \)
3) \( t = t \quad \varphi = \varphi \) and \( z = 0 \quad h = 0 \)
4) \( t = \infty \quad \varphi = \infty \) and \( 0 \leq z \leq L \quad h = 0 \)

Consequently, the solution for the head loss in drain wells with resistance is obtained as follows,

\[
h(\varphi, z) = \frac{4h_0}{\pi} \sum_{m=1,3,5,...} \frac{1}{m} \sin \frac{m\pi z}{2L} e^{-m^2 \frac{4m^2}{\gamma_0} e^{k_0} L^2 (\varphi - 1)} \cdot \frac{h_0}{k_c} \ 
\]

Substituting eq. (13) in eq. (16), we obtain

\[
h(t, z) = \frac{4h_0}{\pi} \sum_{m=1,3,5,...} \frac{1}{m} \sin \frac{m\pi z}{2L} e^{-m^2 \frac{4m^2}{\gamma_0} e^{k_0} L^2 (\varphi - 1)} \int_0^t \frac{dv}{U(t-v)} \ 
\]

Setting \( t = (d_e^3/c_h) T \) and \( c_h = k_e/(\gamma_0 m_e) \), and substituting these relations in eq. (17), we have

\[
h(T, z) = \frac{4h_0}{\pi} \sum_{m=1,3,5,...} \frac{1}{m} \sin \frac{m\pi z}{2L} e^{-m^2 \frac{4m^2}{\gamma_0} e^{k_0} L^2 (\varphi - 1)} \cdot \frac{d_e^3}{k_e} \left( \frac{T}{L} \right)^2 \int_0^T \frac{dv}{U(T-v)} \ 
\]

where, \( T \) is the time factor of consolidation by radial flow toward drain wells and \( k_e \) is the coefficient of permeability of consolidating soil.

The obtained solution for the head loss in drain wells contains the integral term as follows,

\[
\varphi = \int_0^T \frac{dv}{U(T-v)}
\]

This becomes a singular integral equation at the time \( \tau_1 = T \), and the judgement of possibility of the integration is not so easy on account of nature of the function \( U(t) \). If we assume the following approximation for the purpose of facilitating numerical treatment the integration becomes possible:

when \( 0 \leq T \leq \Delta T \), \( 0 < U(T) = \text{const} \).

However, although the values of function \( U(T) \) in the early stages of time are needed for numerical analyses, it is difficult to carry out the integration, because the Fourier series in the function \( U(T) \) does not converge at satisfying rate in the early stage of time. Then, in the present study, the numerical solutions of the problem are carried out with the aid of the electric lumped-parameter model described in Chapter 5. From the theoretical consideration, these results suffice to discuss the factor affecting the consolidation process with well resistance.

The factor affecting the dissipation process of the head loss in drain wells will be identical with that affecting the process of consolidation by radial flow to drain wells
with resistance, because the dissipation of the head loss corresponds to the variation of consolidation load and the consolidation has the characteristics of superposition in general. For this reason, it will be found from eq. (18) that two factors modify the consolidation process with well resistance, the one being $R''$ and the ratio $n$ included in function $U(t)$, in which

$$
R'' = \frac{n^2}{n^2-1} \frac{k_w}{k_e} \left( \frac{d_w}{L} \right)^2.
$$

According to R. A. Barron, the equal-strain consolidation without well resistance and peripheral smear is approximately shown as

$$
U(t) = 1 - e^{-\frac{8}{F(n)} \frac{\epsilon_n}{d_e} t} = 1 - e^{-\frac{8}{F(n)} \frac{1}{d_e^2} \frac{k_e}{\tau w n_0} t}
$$

where

$$
F(n) = \frac{n^2}{n^2-1} \log_e n - \frac{3n^2-1}{4n^2}
$$

It is well-known that eq. (19) closely approximates the theoretical solution except for the stages of small value of $t$ and $n$. In eq. (19), the variation in the coefficient of permeability, $k$, of consolidating soils corresponds to the reciprocal variation of $F(n)$. Therefore, it is concluded that the rate of consolidation with well resistance is modified by the following parameter:

$$
R' = \frac{F(n)n^2}{(n^2-1)} \frac{k_w}{k_e} \left( \frac{d_w}{L} \right)^2
$$

The dimensionless parameter, $R$, which is reciprocal of $R'$, will be used in the present study as a parameter for analyses by means of the electric simulate model, because the time required for consolidation is in proportion to the value of $R$.

3. THE SIMILARITY BETWEEN CONSOLIDATION AND ELECTRIC DIFFUSION

Prior to programming of an electric simulate model, the analogy and the corresponding relation of parameters between consolidation and electric diffusion will be ascertained.

We consider now the medium layer, the specific electro-resistance and the electric capacitance of which is $\rho_e \cdot \Omega \cdot \text{cm}$ and $c$ farad/cm$^3$, respectively, and assume that Ohm's law is applicable in the layer.

$$
q = \frac{1}{\rho_e} \frac{dE}{dz}
$$

and

$$
Q = E \cdot c \cdot V
$$

where $q$=electric flux
$\rho_e$=specific electro-resistance of the layer
$E$=electric potential
$Q$=electric quantity
\( c = \text{electric capacitance of medium of unit volume} \)
\( z = \text{a coordinate in rectangular system} \)
\( V = \text{volume of medium} \)

In order to simplify the mathematical analysis of the problem, we assume that electrical current takes place in \( z \) direction only. And for the purpose of analysis we imagine an elementary, rectangular prism cut out of the medium layer, as shown in Fig. 2. The base area of this elementary medium prism is unity, and its height is \( dz \). Electrical current would flow vertically downward from the top (the entrance face) and to and through the bottom face of the medium prism.

![Elementary medium prism](image)

**Fig. 2. Elementary medium prism**

From eq. (21), the total change in current from the entrance face to the face of exit over a distance \( dz \) is

\[
dQ = \frac{1}{\rho_e} \left[ \left( \frac{\partial E}{\partial z} \right)_z - \left( \frac{\partial E}{\partial z} \right)_{z+dz} \right] dt = -\frac{1}{\rho_e} \frac{\partial^2 E}{\partial z^2} dz \cdot dt \quad \text{..................................(23)}
\]

On the other hand, from eq. (22), the electric discharge of the medium prism is

\[
dQ = -\left( \frac{\partial E}{\partial t} \right) c \cdot dt \cdot dz \quad \text{................................................(24)}
\]

Because the change of current through the both faces over a distance \( dz \) of the elementary medium prism is equal to the electric discharge of the prism, equating eq. (23) and eq. (24) and canceling \( dz \) and \( dt \), we get.

\[
\frac{\partial E}{\partial t} = \frac{1}{\rho_e c} \frac{\partial^2 E}{\partial z^2} \quad \text{................................................(25)}
\]

Replacing \( 1/\rho_e \cdot c \) and \( E \) by \( C_e \) and \( u \), respectively, eq. (25) turns into the differential equation for one-dimensional consolidation. Therefore the problem of consolidation will be simulated by an electric model. The corresponding relations of parameters.
between consolidation and electric diffusion are shown in Table 1.

<table>
<thead>
<tr>
<th>consolidation</th>
<th>electric diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>excess hydraulic pressure</td>
<td>: ( u )</td>
</tr>
<tr>
<td>time</td>
<td>: ( t )</td>
</tr>
<tr>
<td>coefficient of permeability</td>
<td>: ( k_e )</td>
</tr>
<tr>
<td>coefficient of volume change</td>
<td>: ( m_v = \frac{\Delta V}{V_o \Delta p} )</td>
</tr>
<tr>
<td>coefficient of consolidation</td>
<td>: ( C_v )</td>
</tr>
<tr>
<td>unit weight of water</td>
<td>: ( \gamma_w )</td>
</tr>
<tr>
<td>electric potential</td>
<td>: ( E )</td>
</tr>
<tr>
<td>time</td>
<td>: ( t )</td>
</tr>
<tr>
<td>coefficient of electric conductivity</td>
<td>: ( \frac{1}{\rho_e} )</td>
</tr>
<tr>
<td>electric capacity times unit volume</td>
<td>: ( \frac{\Delta Q}{Q_e \Delta E} )</td>
</tr>
<tr>
<td>diffusivity</td>
<td>: ( \frac{1}{\varepsilon \rho_e} )</td>
</tr>
<tr>
<td>electric quantity per unit volume</td>
<td>: ( \frac{Q_s}{V} )</td>
</tr>
</tbody>
</table>

4. LUMPING APPROXIMATION AND SCALING OF THE CONSOLIDATION PROBLEM

In this simulate model test, the whole vertical drain system is divided into \( M \) pieces of slice and each slice is radially subdivided into \( N \) elements in the manner which their volume may all be the same, as shown in Fig. 3. And considering that water included in each element is concentrated to its center, the simulate model is constructed. The value of electrical elements, such as condenser and resistance, in the simulate model are calculated as below. The electric resistance, \( R_{ct} \) corresponding to hydraulic resistance in consolidating soils, between two adjacent lumps in Fig. 3 will be seen to be:

\[
R_{ct} = \frac{\rho_e \ell_t}{A_t} = \frac{d_i-d_{i-1}}{2\pi(d_i+d_{i-1})\Delta L} \cdot \rho_e
\]

and the electric capacitance \( C \), corresponding to hydraulic capacitance of consolidating soils is:

\[
C = \frac{A_e \Delta L}{N} = \frac{\pi \Delta L}{4N} \cdot (n^2-1)d_w^2 \varepsilon
\]

and the electrical resistance, \( R_w \), corresponding to well resistance is:

\[
R_w = \frac{\rho_w \Delta L}{A_w} = \frac{4\Delta L}{\pi d_w^2 \rho_w}
\]

where

- \( \ell_t \) = distance between center of two adjacent lumps
- \( d_i \) = outer diameter of \( i \)-th lump
- \( A_i \) = average area between the two drainage faces of \( i \)-th lump
- \( A_e \) = cross sectional area of clay cylinder
- \( \Delta L \) = height of elementary slice of column

\[
= \frac{1}{2} \left( \frac{1}{2}(d_{i-1}+d_{i-2}) - \frac{1}{2}(d_{i-1}+d_{i+2}) \right)
\]

\[
d_i = \pi(d_i+d_{i-1}) \Delta L
\]

\[
\frac{1}{2} \pi(d_i+d_{i-1}) \Delta L
\]

\[
= L_i/M
\]
Fig. 3. Corresponding relationship between clay cylinder and lumped-parameter model
The electrical analogy of consolidation by radial flow to drain well with resistance will therefore be set up as shown in Fig. 3c. In the present study, the whole drain well system is divided into 110 lumps, in which \( M = 10 \) and \( N = 10 \), and the analyses by means of a simulate model are made in the cases where \( n = 5 \) and \( n = 30 \). The procedure of this model test is made in following manner: whole system is entirely charged under some convenient constant potential at first, and the point representing the top of a vertical drain well is suddenly connected to ground (zero voltage). The transient electric-flow process takes place as each capacitor discharges to ground through the resistance network. The electrical current passing through at arbitrary points of the network is measured by an electromagnetic oscillograph.

For the purpose of ascertaining the accuracy of the model, the obtained consolidation curve in the ideal case where both peripheral smear and well resistance do not exist is compared with the theoretical consolidation curve given by S. Takagi,\(^3\) as shown in Fig. 4. There is a slight discrepancy between the two curves in the later stage of consolidation, but it may be considered that the expected purpose of this model is accomplished.

![Fig. 4. Comparison between the consolidation process by theoretical analysis and the one by lumped model analogue method](image)

5. THE RESULT OF THE LUMPED-PARAMETER MODEL TEST

Fig. 5 shows the distribution of the head loss in a vertical drain well, in which

\[
n = \frac{d_e}{d_w} = 5 \quad \text{and} \quad R = \frac{n^2 - 1}{n^2 F(n)} \frac{k_e}{k_w} \left( \frac{L}{d_w} \right)^2 = 0.29.
\]

The distribution curve of head loss in depth is approximately parabola. In Fig. 6, the dissipation processes of head loss at the bottom of a drain well having various values of \( R \) are presented in the case of \( n = 5 \), and these processes are compared with the
consolidation process by radial flow to an ideal drain well. From this comparison, it is supposed that the consolidation due to vertical drain wells will be remarkably affected by the head loss under the circumstances where the value of $R$ becomes greater than about 0.1.

Though the previous descriptions are connected with the head loss in drain wells, the direct purpose in engineering is to study the characteristics of the consolidation process subjected by the resistance of drain wells. In Figs. 7 and 8 are shown the consolidation in two cases, one of them being the effect of well resistance on average consolidation process over entire clay cylinder, the other being that at the bottom of a drain well, where $n=5$ in both cases. In these results, the retardation of the consolidation affected by well resistance is especially remarkable in its early stages and somewhat recovered in its latter stage when the rate of discharge becomes small. Thus, it will be found that the consolidation process affected by well resistance is not so simple and can not be obtained by means of using the modified coefficient of consolidation—for instance, which is half or one third of a coefficient of consolidation. Since
these graphical expressions of the consolidation affected by well resistance are for the special case \( n = 5 \), those for arbitrary value of \( n \) are presented in a more convenient form as shown in Figs. 9 to 12: Fig. 9 presents the effect of well resistance on a overall average consolidation and Figs. 10 to 12 show the retardation of average consolidation at the depth of \( L/10 \), \( L/2 \) and the bottom of a drain well having various value of \( R \), respectively.

**Fig. 6.** Head loss at the bottom of drain wells having various value of \( R \) in case of \( n = d_i/d_w = 5 \)

**Fig. 7.** Effect of well resistance on average consolidation process over entire clay cylinder in which \( n = 5 \)
Fig. 8. Effect of well resistance on average consolidation process at the bottom of drain wells in which $n=5$

Fig. 9. Effect of well resistance on overall average consolidation rate
Fig. 10. Effect of well resistance on consolidation rate at depth of 1/10 length of drain wells having various value of $R$

Fig. 11. Effect of well resistance on consolidation rate at depth of 1/2 length of drain wells having various value of $R$
Barron has developed a solution for the consolidation by radial flow to drain wells with or without peripheral smear and well resistance. According to him, the average excess pressure, \( u_x \) between \( r_e \) and \( r_i \) at depth \( z \) is

\[
\frac{u_x}{u_0} e^{f(x)} \quad \text{(26)}
\]

where

\[
f(x) = \frac{e^{\beta(x - 2h)} + e^{-\beta L}}{1 + e^{-2\beta H}} \quad \text{(27)}
\]

\[
\xi = -\frac{8T_h}{\nu}
\]
\[ \beta = \left[ \frac{2k_e(n^2 - s^2)}{k_w r_w^2 \nu} \right]^{\frac{1}{2}} \]  
\[ s = \frac{r_e}{r_w} \]  
and  
\[ \nu = F(n, s, k_e, k_i) = \frac{n^2}{n^2 - s^2} \log_e \left( \frac{n}{s} \right) \frac{3}{4} + \frac{s^2}{4n^2} + k_e \left( \frac{n^2 - s^2}{n^2} \right) \log_e (s) \]  
in which \( k_i \) is the coefficient of permeability of smear zone and \( H \) is the length of drain wells. For the purpose of comparison between Barron's solution and the result of the simulate model test, we consider the consolidation at the bottom of drain wells in the case of no smear. Substituting \( s = 1 \), \( z = H = L \) and \( r_e = 1/2 \cdot n \cdot d_w \) in eqs. (27), (28) and (29), we obtain  
\[ f(L) = \frac{2e^{-\beta L}}{1 + e^{-2\beta L}} \]  
\[ \beta = \left[ \frac{8k_e(n^2 - 1)}{k_w n^2 d_w^2 F(n)} \right]^{\frac{1}{2}} \]  
\[ \nu = F(n) = \frac{n^2}{n^2 - 1} \log_e n - \frac{3n^2 - 1}{4n^2} \]  

Fig. 13. Comparison between the effect of well resistance by Barron's theory and by the lumped model analogue method.
and

\[
\beta \cdot L = \sqrt{\frac{8}{\frac{h_v(n^2-1)L^2}{k_w n^2 F(n)d_w^2}}}^{\frac{1}{2}}.
\]

Substituting \( f(x) = 1 \) in eq. 26, the solution for the average excess pressure in the case without smear and well resistance is obtained. Thus, the equation \( f(x) \) governs the process of consolidation with smear and well resistance, and also it is a function of the only multiplication \( \beta L \), as shown in eq. (30). Consequently, the factor modifying the consolidation by drain wells with resistance is the group of terms in the bracket in eq. (33). This factor is identical with \( R \) found in the above mentioned theoretical study.

In comparison between Barron's solution and the result by means of the simulate model, there are differences between them on two matters: (1) the amount of retardation according to Barron's solution is somewhat smaller than that by means of the simulate model in the range of \( R \) usually used in practice; (2) Barron's solution is independent of consolidation degree, but the result obtained by the model changes in each consolidation stage. Fig. 13 shows the relationship between the retardation

\[\text{Consolidation Degree (\%)}\]

\[\text{Depth} \]

\[\text{Fig. 14. Distribution of consolidation in depth at the time when 80\% consolidation at the top of drain wells has taken place} \]
of average consolidation at the bottom of drain wells and the value of $R$, in comparison with Barron's solution. There will be an appreciable difference in the distribution of consolidation degree in depth between them, because the effect of well resistance according to Barron's solution is constant in whole process of consolidation although the one in this study changes. But they coincide with each other only near the final stage of consolidation. In Fig. 14, the distribution of the average consolidation degree in depth are shown for the several values of $R$ at the time when the 80% consolidation at the top of drain wells has taken place. The distribution curve is approximately parabola and there is a considerable retardation even in a near part of the surface.

6. LARGE SCALE MODEL TEST

To ascertain the results obtained by the electric simulate model, the large scale model test on vertical drain well was made using the arrangement as shown in Fig. 15. The

![Cross Section](image)

**Fig. 15. Large scale consolidation model**
dimensions of the model were respectively \( L = 8.0 \text{ m}, d_e = 20.0 \text{ cm} \) and \( n = 6.8 \) and the Toyoura standard sand were used as a backfill material of drain well. From permeability test and one-dimensional consolidation test, the permeability coefficient of the backfill material and that of the consolidating soil are estimated to be \( k_w = 3.0 \times 10^{-2} \text{ cm/sec} \) and \( k_c = 2.6 \times 10^{-7} \text{ cm/sec} \), respectively. Thus, the value of \( R \) in the large scale model is calculated as \( R = 5 \times 10^{-1} \).

The observed curves of average consolidation \( U(t, z) \) at arbitrary depth versus time factor \( T \) are shown in Fig. 16. In Fig. 16 the curves estimated from the result of the electric simulate model test are also given. Between them at the end of drain wells, though there is some difference caused by the frictional resistance between the consolidating soil and the container, it can be supposed to be obtained a fairly good agreement as a whole.

\[
\text{Fig. 16. Comparison of experimental result and estimated value in which } n = 6.8 \text{ and } R = 0.5
\]

7. EXAMPLE

In order to study how much the consolidation in practice is affected by well resistance, the following values in sand drain method are chosen: \( n = 5 \), \( L = 20.0 \text{ m} \), \( d_e = 2.0 \text{ m} \), \( d_w = 40.0 \text{ cm} \), \( k_e = 10^{-7} \text{ cm/sec} \) and \( k_w = 3.0 \times 10^{-8} \text{ cm/sec} \). Thus, we obtain

\[
R = \frac{n^2 - 1}{n^2 F(n)} \frac{k_e}{k_w} \left( \frac{L}{d_w} \right)^2 = 8.46 \times 10^{-2}.
\]
Under this condition, the time required for 80% over-all average consolidation over the entire clay cylinder between $z=0$ and $z=L$ is about 1.4 times in comparison with the time in an ideal case. Also, when 80% consolidation has progressed in an ideal case, the average consolidations between $r_e$ and $r_e$ at the depth of half length and the bottom of drain wells take place are only at 68% and 63%, respectively.

Thus, it will be assumed that the effect of well resistance on the consolidation by radial flow is considerably large even in usual cases. Especially, the retardation of consolidation in the depth below the half length of drain wells is evident and this has been frequently pointed out by engineers with their field experiences.

8. CONCLUSIONS

As a result of the present study for the consolidation affected by well resistance, it is concluded as follows:

1) The effect of well resistance on the consolidation by radial flow to drain wells cannot be ignored in the case where consolidating stratum is as soft and deep as clays often encountered in Japan.

2) The factor modifying the consolidation process with well resistance is

$$R = \frac{n^2-1}{n^2 P(n)} \frac{k_e}{k_w} \left( \frac{L}{d_w} \right)^2$$

3) Several diagrams are given for the quantitative evaluation of the consolidation process affected by well resistance, and these diagrams can be used directly for design or analysis and also show the limitation of the size and permeability of drain wells for practical purposes.

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APPENDIX — NOTATION

The following symbols have been adopted for use in this paper:

- $A_c =$ cross sectional area of clay cylinder
- $A_w =$ cross sectional area of a drain well
- $C_h =$ coefficient of consolidation in the horizontal direction
- $C =$ electric capacitance
- $c =$ electric capacitance of medium of unit volume
- $d_e =$ diameter of zone of influence
- $d_w =$ diameter of drain well
- $E =$ electric potential
- $h =$ loss head in drain well
\[ i = \text{hydraulic gradient in drain well} \]
\[ k_e = \text{coefficient of permeability of consolidating soil} \]
\[ k_w = \text{coefficient of permeability of well backfill} \]
\[ L = \text{length of drain well} \]
\[ l = \text{distance between center of two adjacent lumps} \]
\[ m_v = \text{coefficient of volume change} \]
\[ n = \text{a ratio} = \frac{d_v}{d_w} \]
\[ p = \text{effective pressure in drain well} \]
\[ p_0 = \text{total pressure exerted on consolidating soil} \]
\[ Q = \text{electric quantity} \]
\[ q = \text{total discharge of water} \]
\[ R = \text{a factor defined by Eq. 34} \]
\[ R_e = \text{electric resistance corresponding to hydraulic resistance in consolidating soil} \]
\[ R_w = \text{electric resistance corresponding to well resistance} \]
\[ r_e = \text{radius of zone of influence} \]
\[ r_w = \text{radius of drain well} \]
\[ s = \text{a ratio} = \frac{r_s}{r_w} \]
\[ T = \text{time factor for radial flow in case with well resistance} = C_h \frac{t}{d_w^2} \]
\[ T_h = \text{time factor for radial flow in case of no well resistance} = C_h \frac{t}{d_w^2} \]
\[ t = \text{time} \]
\[ U(t) = \text{average consolidation due to radial flow to ideal drain wells} \]
\[ \bar{u} = \text{average excess pore-water pressure} \]
\[ u_0 = \text{initial excess pore-water pressure} \]
\[ u_z = \text{average excess pore-water pressure between} r_e \text{ and } r_s \text{ at depth } z \]
\[ V = \text{volume of medium} \]
\[ v = \text{filtration velocity in drain well} \]
\[ z = \text{a coordinate in both the rectangular and the cylindrical systems} \]
\[ \rho_e = \text{specific electro-resistance of medium layer} \]

**REFERENCE**


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