EFFECT OF SURFACE ROUGHNESS ON THE SHEAR CHARACTERISTICS OF GRANULAR MATERIALS

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SYNOPSIS

In this paper the effect of surface texture on the shear characteristics of granular materials is studied from the results of drained triaxial compression tests. The materials tested are uniform spheres of steel having different surface roughnesses. Test results indicate that for the granular materials the development of failure zone in the triaxial specimen is affected by surface roughness in the same manner as in Mogami's test results on one-layer assembly of uniform spheres, and that shearing strength and dilatancy increase correspondingly with an increase in surface roughness. The expression for the physical components of shear strength in granular materials is proposed, and the relationship between surface roughness and " k value" in Mogami's theory of mechanics of granular materials is also described concerning the test results.

INTRODUCTION

The shear characteristics of such materials as sands are significantly influenced not only by the drainage condition and stress condition, but by the properties of individual grains, namely, size, toughness, shape, and surface roughness, and the properties of grain mass, namely, grading, density, arrangement of grains, and moisture content. These properties are related to the packing of grains. In view of the fact that sands consist of individual grains, these properties constitute the basic problem of the shear characteristics of sands.

The author has previously presented his work on the effect of shape and surface texture of particles on the shearing strength of sands (Yamanouchi and Haruyama, 1967). The present paper contains the results of drained triaxial compression tests on uniform spheres of steel, and detailed discussions on the effect of surface roughness of grains on above-mentioned factors which influence the shear characteristics of granular materials.

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MATERIALS TESTED, SPECIMENS, AND TESTING PROCEDURE

The materials used in this investigation were uniform spheres of steel of 1.56 mm in diameter. The microphotographs of typical steel spheres are shown in Fig. 1. Smooth grains were given polish, and rough grains were eroded in water. Tests were carried out in six series in the range of spherical surface roughness from smooth to rough texture. The specimens of smooth grains were designated as Ba, and those of rough grains as Bb, Bc, Bd, Be, and Bf in ascending order of roughness. The specific gravities of spheres of Ba and Bf were 7.83 and 7.75, respectively.

The specimens were initially about 5.0 cm in diameter and 13.1 cm in height. Tests were preformed on the specimens prepared to initial void ratio of the closed packing corresponding to the surface roughness. The initial void ratio was 0.616 for Ba, 0.632 for Bb, and approximately 0.65 for the others. There is no significant difference between these void ratios and the void ratio of 0.65 (porosity, 39.4 per cent) for the hexagonal loose-packed structure of uniform spheres. Therefore, it is implied that the number of the points of

Fig. 1. Microphotographs of steel spheres

contact which a sphere makes with its neighbors is eight on an average. It has previously been known that such packing occurs most frequently in irregular packings of uniform spheres. The surface texture of grain is related to the degree of packing, or the density of specimen, i.e., the rougher the texture is the lower the density of specimen becomes.

The method to prepare the specimen is illustrated in Fig. 2, in which a split forming mold is set to pour and compact the spheres. The spheres were carefully placed into the rubber membrane which was sealed against the side of the pedestal by rubber bands and supported by the mold, and then compacted by tapping the side of mold with a wooden bar. The filter paper and the cap was then placed on the specimen, and the membrane was sealed against the cap by rubber bands. The pedestal and cap were connected to drainage lines. After the specimen was formed, it was held up with a slight negative pressure of less than 15 cmHg applied to the top of specimen through the drainage line, the mold was removed, and the diameter and the height of specimen was then measured. The void ratio of a specimen in this condition is the initial void ratio. A specimen thus formed is shown in Fig. 3.

The specimen was saturated by introducing water by suction through the bottom drainage line. The suction during supplying water was limited to the above-mentioned 15 cm Hg. The triaxial cell was completely filled with water, a confining pressure was applied by a hydraulic system, and the suc-

Fig. 2. Preparation of specimen

Fig. 3. Specimen formed

tion was then released when the confining pressure was increased to 0.2 kg per sq cm. Drained triaxial compression tests were performed by increasing axial load at a constant confining pressure. The specimens were fully consolidated in the cell under the pressure ranging from 0.5 to 3.0 kg per sq cm prior to compression, and then the specimens were subjected to deviator stress at a constant rate of strain of 0.8 per cent per minute. Measured stresses are equal to the effective stresses and the cavitation of pore water does not occur under such shear rate and drained condition.

Mercury manometer was used to measure the confining pressures less than 2.0 kg per sq cm, and Bourdon-tube pressure gauge was used for higher pressures. Axial loads, axial strains and volume changes during shear were measured. Volume changes were measured by observing the changes of water level in burette connected to the specimen. The influence of the friction of a loading piston, membrane resistance, and constraint at both ends of specimen on the deviator stress and on the rate of volume change was insignificant, and therefore was neglected in this study.

GENERAL MODE OF FAILURE OF SPECIMEN

The characteristics of specimen after failure in triaxial tests are shown in Fig. 4. None of the specimens developed a failure plane, but a failure

(a) Specimen of smooth spheres

(b) Specimen of rough spheres

Fig. 4. Cylindrical specimen of uniform spheres of steel after failure in compression at confining pressure of 3.0 kg/cm²

zone was observed, the extent of which became progressively larger with an increase in intergranular friction. Therefore, it will be confirmed that an increase in friction causes the range of influence of the failure to widen. Mogami and Imai (1969) performed experiments on one-layer assemblage of uniform spheres and observed the same mode of deformation as mentioned above.

Careful observations during all tests indicated that no failure zone was ever visible before the peak strength was reached, and that the specimens began to develop the failure zone at some appreciable strain after the peak was passed. The deformation was much less uniform with the increase in axial strain after the early development of the failure zone. Therefore, it is probable that up to and past the peak the deformation was nearly uniform. Deviator stresses were calculated by using the conventional area correction for a right circular cylinder. This suggests that, until failure zone was formed after the peak, deviator stresses and volume changes of specimen calculated from overall deformation will not be greatly in error, provided no marked barrel shape develops.

PROPERTIES OF STRESS, STRAIN AND VOLUME CHANGE CURVE

The relationships between deviator stress, axial strain, and volume change for drained triaxial compression tests on the uniform spheres of steel are shown in Fig. 5. These curves demonstrate the effect of increasing the surface roughness on the deformation and the strength characteristics of granular materials. Reference to the data shown in Fig. 5 would indicate that the

Fig. 5-a.

Fig. 5-b.

Fig. 5-c.

Fig. 5. Typical deviator stress-axial strain-volume change curves for drained triaxial compression tests on the uniform spheres of steel.

shear properties of sands should be affected by the roughness of individual grains. Deviator stress decreases progressively with an increase in axial strain and appears to have a tendency finally to become constant after the peak was passed. In all tests the associated volume changes of specimen during shear show dilation, but it may be expected that the specimen is finally sheared under a constant volume. Volume change-axial strain curves show the steepest rise at the strain corresponding to the peak strength. The peak of stress-strain curves is more clearly defined in the assemblage of rough spheres than in that of smooth ones. The amount of volume change in the specimen of rough spheres is larger than that of smooth ones. Reference to the properties of peak strength and volume change will indicate the effect of the volume of failure zone due to the increase in surface roughness.

The curves shown in Fig. 5 were obtained by plotting deviator stresses

Fig. 6. Serrated behaviour during drained shear

and volume changes against given strains. In practice, however, deviator stresses and volume changes resulted in the serrated behaviour as shown in Fig. 6. Newland and Allely (1957) called such serrated behaviour accompanying the shearing deformation the "bridge collapses". Careful observations during all tests indicated that, for the specimens of the rougher spheres, no serrated behaviours were clearly observed before the peak strength and the serrated changes began to occur at the time when the peak was reached. The serrated changes in the specimens of smooth spheres, however, appeared at early strains before the peak.

In the path *a* of deviator stress-axial strain-volume change curve shown in Fig. 6, a slight increase in axial strain and little volume change were observed, but deviator stress increased considerably. In the path *b*, a quick progression in axial strain and expansion in volume occurred; consequently, a considerable decrease in deviator stress was accompanied. The amount of variation in the deviator stress, namely, the difference between deviator stress at the top *d* and that at the bottom *c* was slight in early stages of shear, increased gradually with the shear, passed through a maximum, and finally fell off or became constant. The difference in deviator stresses between the top *d* and the bottom *c* increased with an increase in pressure. For example, it was about 0.3 kg per sq cm on the average and 0.6 kg per sq cm in the maximum variation at the confining pressure of 3.0 kg per sq cm, and about 0.1 kg per sq cm on the average and 0.18 kg per sq cm in the maximum at the confining pressure of 1.0 kg per sq cm.

The amount of axial strain required for a cycle consisting of the path *a* and *b* changed during all stages of shear, and increased with an increase in confining pressure. However, it was only 0.15 per cent of the specimen height or the axial compression of 0.2 mm in height, at its maximum value at the confining pressure of 2.0 kg per sq cm. The serrated behaviour in volume change was most remarkable at the intermediate stages of shear, and was little recognized at ultimate shear.

It is considered that the cause for such serrated behaviour will be dependent on the interlocking, which is generally known to be a part of the shearing strength. The rolling and sliding between the grains in shear zone will not occur in the case where interlocking contributes to a large portion of shearing resistance; consequently, deviator stress increases considerably while the axial strain and volume change are of little significance. Such characteristics are represented by the path *a* in Fig. 6. Because interlocking becomes looser when a part of shear stress overcomes interlocking, the deviator stress decreases, and an increase in axial strain and the expansion in volume seem to occur. Such properties are given by the path *b* in Fig. 6. It may be generally expected that granular materials such as sand are sheared in the

mode mentioned above. The serrated behaviours in this investigation would occur on account of the equal spheres and the rigidity of steel.

The various properties obtained from the relationships between deviator

Table 1. Summary of Test Results

Specimen	e_0 (n%)	e_f	σ_3 ($\frac{\text{kg}}{\text{cm}^2}$)	ε_f (%)	$(\frac{dv}{d\varepsilon})_f$	$\sigma_3(\frac{dv}{d\varepsilon})_f$ ($\frac{\text{kg}}{\text{cm}^2}$)	$(\sigma_1 - \sigma_3)_f$ ($\frac{\text{kg}}{\text{cm}^2}$)	$(\sigma_1 - \sigma_3)_{rf}$ ($\frac{\text{kg}}{\text{cm}^2}$)	$(\sigma_1 - \sigma_3)_u$ ($\frac{\text{kg}}{\text{cm}^2}$)	$(\sigma_1 - \sigma_3)_R$ ($\frac{\text{kg}}{\text{cm}^2}$)
Ba	0.616 (38.1)	0.616	0.5	2.69	0.21	0.10	0.64	0.54	0.50	0.44
		0.609	1.0	3.85	0.17	0.17	1.18	1.01	0.93	0.86
		0.601	1.5	1.54	0.26	0.39	1.71	1.32	1.38	1.05
		0.601	2.0	6.15	0.09	0.18	2.04	1.86	1.72	1.71
		0.600	3.0	4.62	0.21	0.63	3.24	2.61	2.52	2.16
		Cohesion int. (kg/cm ²) Angle of friction					0.046 19°43'	0.053 17°03'	0.047 16°39'	0.045 15°05'
Bb	0.632 (38.7)	0.650	0.5	5.34	0.39	0.19	0.86	0.67	0.67	0.48
		0.640	1.0	4.58	0.37	0.37	1.88	1.51	1.27	1.10
		0.635	1.5	4.58	0.36	0.54	2.43	1.89	1.80	1.39
		0.623	2.0	6.87	0.24	0.48	3.17	2.69	2.73	2.17
		0.633	3.0	6.11	0.29	0.87	4.94	4.07	3.53	3.16
		Cohesion int. (kg/cm ²) Angle of friction					0.037 26°15'	0.009 23.36'	0.039 21°45'	-0.019 20°25'
Bc	0.652 (39.5)	0.656	0.5	1.91	0.46	0.23	0.90	0.67	0.68	0.46
		0.648	1.0	2.29	0.42	0.42	1.87	1.45	1.45	1.02
		0.642	2.0	3.05	0.39	0.78	3.48	2.70	2.92	1.94
		0.629	3.0	1.91	0.47	1.41	5.17	3.76	4.06	2.56
		Cohesion int. (kg/cm ²) Angle of friction					0.033 27°15'	0.051 22°21'	0.027 22°52'	0.049 17°11'
Bd	0.645 (39.2)	0.651	0.5	1.54	0.57	0.28	0.94	0.66	0.68	0.42
		0.659	1.0	3.08	0.51	0.51	2.07	1.56	1.42	1.03
		0.633	2.0	1.54	0.57	1.14	3.61	2.47	2.75	1.57
		0.627	3.0	1.15	0.62	1.86	5.70	3.84	3.38	2.37
		Cohesion int. (kg/cm ²) Angle of friction					0.019 28°46'	0.052 21°50'	0.098 20°41'	0.028 16°50'
Be	0.643 (39.1)	0.648	0.5	1.54	0.69	0.34	1.05	0.71	0.68	0.42
		0.640	1.0	1.54	0.66	0.66	2.07	1.41	1.21	0.85
		0.630	2.0	1.92	0.62	1.24	4.04	2.80	2.58	1.73
		0.627	3.0	2.31	0.62	1.86	6.01	4.15	3.69	2.56
		Cohesion int. (kg/cm ²) Angle of friction					0.021 29°50'	0.010 24°03'	0.019 22°20'	-0.002 17°29'
Bf	0.651 (39.4)	0.660	0.5	1.92	0.71	0.35	1.09	0.74	0.73	0.43
		0.647	1.0	1.54	0.71	0.71	2.20	1.49	1.40	0.87
		0.641	2.0	1.92	0.68	1.36	4.22	2.86	3.19	1.70
		5.632	3.0	1.54	0.73	2.19	6.44	4.25	4.32	2.46
		Cohesion int. (kg/cm ²) Angle of friction					0.010 31°00'	0.021 24°18'	0.040 25°07'	0.019 16°44'

stresses, axial strains, and volume changes for drained triaxial compression tests on the specimens of uniform spheres with varying surface roughnesses are summarized in Table 1.

DEFORMATION CHARACTERISTICS

The characteristics of deformation can be clarified from the relationship between deviator stress, axial strain, and volume change during shear. In this paper, failure is defined as the peak strength at which deviator stress, $(\sigma_1 - \sigma_3)$, reaches a maximum. As shown in Fig. 5 and Table 1, the strain, ϵ_f , at which failure occurs is 2 to 7 percent for Ba and Bb, but only 2 percent at most for the others. Generally speaking, failure strain tends to decrease with an increase in the roughness of surface texture up to a certain roughness, and remains approximately constant for greater roughnesses. Therefore, the slope of tangent to deviator stress-axial strain curves at the origin, which corresponds to the initial tangent modulus of deformation, is steeper for the specimens of rough spheres than that of smooth ones, and increases with an increase in the confining pressure.

The granular materials subjected to drained shear produce a volume change due to the changes in the arrangement of grains. The measured total volume change during triaxial test is constituted of both the volume change which occurs as a result of shear stress and the volume change due to the change in mean principal stress. The former is called dilatancy. All specimens except Ba and Bb continued to increase in volume from the initial increments of axial strain. Some specimens of Ba and Bb having small intergranular friction decreased slightly in volume at initial shear stage, but the decrease was insignificant.

In the volume change-axial strain curves shown in Fig. 5, the characteristics of volume change during shear is indicated by dilatancy rate, which is expressed by the ratio of the increment of the rate of volume change, $d(\Delta V/V)$, to the increment of axial strain, $d\epsilon$. This also represents the inclination of the tangent to volume change-axial strain curve (Bishop and Eldin, 1953). At failure the dilatancy rate is denoted by $(dv/d\epsilon)_f$, which is defined as positive for expansion, where v = rate of volume change given by $\Delta V/V$, ϵ = axial strain, f = suffix pertaining to failure, V = volume of specimen, and ΔV = volume change.

As shown in Table 1, $(dv/d\epsilon)_f$ is far more sensitive to change in roughness and rises rapidly with the roughness. The value of $(dv/d\epsilon)_f$ for the assemblage of the roughest spheres, Bf, is more than 3 times as great as that of the smoothest ones, Ba. The increase in $(dv/d\epsilon)_f$ will indicate that the rougher the surface is, the tighter the interlocking is; therefore interlocking has to become looser for the grains to move during shear. Consequently

the large dilation in volume is necessary for the shear of specimens of rough spheres. The amount of dilation which occurs is related to interlocking. That $(dv/d\varepsilon)_f$ does not decrease with an increase in the confining pressure but remains nearly constant differs from the fact that dilatancy in any sands decreases generally with an increase in the confining pressure. The void ratio at failure also decreases slightly with the magnitude of confining pressure.

STRENGTH CHARACTERISTICS

Different deviator stresses and different parameters of shearing strength which are obtained by the least square method are shown in Table 1. The typical Mohr-Coulomb diagrams are shown in Fig. 7. In the present paper,

Fig. 7. Typical Mohr-Coulomb diagrams for drained triaxial compression tests on the uniform spheres having different textures

$(\sigma_1 - \sigma_3)_f$ and $(\sigma_1 - \sigma_3)_u$ are the measured deviator stresses at failure and at ultimate shear, respectively, and $(\sigma_1 - \sigma_3)_{rf}$ and $(\sigma_1 - \sigma_3)_R$ are the deviator stresses corrected by applying Bishop's and Rowe's expressions for volume change, respectively.

The associated volume change during shear based on interlocking affects the shearing strength. In this paper the effect of volume change for the strength of granular materials is denoted "structural effect." Many comprehensive investigations have been made to express the effect of volume change on the strength obtained by triaxial test (Bishop and Eldin, 1953; Bishop, 1954; Newland and Allely, 1957; Akai, 1958; Kirkpatrick, 1961; Ladanyi, 1961; Roscoe et al., 1963; Wroth and Bassett, 1965; Rowe, 1962).

According to Bishop (1954) the relationship between the measured deviator stress and the corrected one is given by

$$(\sigma_1 - \sigma_3)_f - \sigma_3 \left(\frac{dv}{d\varepsilon} \right)_f = (\sigma_1 - \sigma_3)_{rf} \quad (1)$$

and using the stress-dilatancy equation by Rowe (1962), corrected deviator stress is expressed by

$$\frac{\sigma_1}{1 + \left(\frac{dv}{d\varepsilon} \right)_f} - \sigma_3 = (\sigma_1 - \sigma_3)_R \quad (2)$$

where σ_1 = axial strain and

σ_3 = confining pressure.

As shown in Table 1, $(\sigma_1 - \sigma_3)_f$ increased with an increase in roughness for each confining pressure. The value of $(\sigma_1 - \sigma_3)_f$ for Bb having slightly rough surface texture was about 1.5 times as large as that for Ba having the smoothest texture, and $(\sigma_1 - \sigma_3)_f$ for Bf having the roughest texture of all the specimens showed the value of about 2.0 times as large as that for Ba.

Now, considering the parameters of strength, the apparent cohesions in Table 1 can be ignored in the assemblage of uniform spheres because of insignificant amount. The parameters of friction strength corresponding to $(\sigma_1 - \sigma_3)_f$, $(\sigma_1 - \sigma_3)_{rf}$, $(\sigma_1 - \sigma_3)_u$, and $(\sigma_1 - \sigma_3)_R$ are here denoted by ϕ_d , ϕ_{rf} , ϕ_u , and ϕ_R , respectively. While the angle of shearing resistance, ϕ_d , increased to 26.2° for Bb and 31.0° for Bf from the initial value of 19.7° for Ba, the angle of friction corrected by Rowe's formula (2), ϕ_R , is 15.1° , 20.4° , and 16.7° for Ba, Bb, and Bf, respectively. The value of ϕ_d varied about 10° for the variation in roughness from Ba to Bf, but the difference in ϕ_R between Ba and Bf is only less than 2° . Therefore, these test results show that even though the increase in the intergranular friction is slight, shear-

ing strength increases remarkably and that the angle of shearing resistance, ϕ_d , is much affected by the surface texture.

Then, the value of $(\sigma_1 - \sigma_3)_f$ was reasonably the largest value of all deviator stresses, and $(\sigma_1 - \sigma_3)_{rf}$ was larger than $(\sigma_1 - \sigma_3)_R$, being approximately equal to $(\sigma_1 - \sigma_3)_u$. The values of $(\sigma_1 - \sigma_3)_u$, $(\sigma_1 - \sigma_3)_{rf}$, and $(\sigma_1 - \sigma_3)_R$ and the parameters of friction component corresponding to these deviator stresses increased rapidly in the range of roughness from Ba to Bb, but did not show a stable trend in the range of roughness from Bc to Bf. There seemed to be general tendencies that $(\sigma_1 - \sigma_3)_u$ and $(\sigma_1 - \sigma_3)_{rf}$ increased slightly and $(\sigma_1 - \sigma_3)_R$ was approximately constant with respect to the increase in roughness.

The physical significance in which shearing resistance increases or decreases will have to be considered from the viewpoint of the structural effects. As mentioned in the previous section, volume change during shear depends on interlocking. According to the considerations on the direct shear test by Taylor (1948) and on the triaxial compression test by Bishop (1954), it is known that the portion of strength contributed by interlocking of the total shear strength is equal to the work required to dilate the specimen, which is expressed by $\sigma_3(dv/d\varepsilon)_f$ for the drained triaxial test, and that the residual strength computed by subtracting $\sigma_3(dv/d\varepsilon)_f$ from the total strength is the value which is contributed by only intergranular friction. However, the author has previously stated that from the test results on the granular soil such as "Shirasu" (volcanic deposits) the contribution of interlocking to strength could not fully be expressed by $\sigma_3(dv/d\varepsilon)_f$ (Haruyama and Yamanoichi, 1966). Because dilation occurs after interlocking is overcome by shear stress, $\sigma_3(dv/d\varepsilon)_f$ does not describe the interlocking effect but expresses only the effect of surface energy required to dilate the specimen.

The structural effect which contributes to the shear strength of granular materials will consist of the surface energy effect, which is expressed by $\sigma_3(dv/d\varepsilon)_f$, and the interlocking effect, which is given by the difference between deviator stress due to intergranular friction and that after the surface energy effect is deducted from the measured maximum deviator stress. As seen in Table 1, the surface energy effect increases with increasing roughness as the consequence of the increase in the rate of volume change. For example, when the smooth spheres, Ba, are compared with the roughest ones, Bf, the surface energy effect for the latter is 3 to 4 times as large as that for the former. From Rowe's stress-dilatancy theory for uniform spheres, it is generally assumed that $(\sigma_1 - \sigma_3)_R$ and ϕ_R depend on the intergranular friction. Therefore, they are approximately constant. Then, structural effect is assumed to be represented by the difference between $(\sigma_1 - \sigma_3)_f$ and $(\sigma_1 - \sigma_3)_R$. For each specimen the structural effect versus the confining pressure are plotted in Fig. 8.

Fig. 8. Structural effect versus confining pressure for uniform spheres of steel with varying surface texture

Reference to Fig. 8 or Table 1 indicates that the effect of structure increases with an increase in roughness and confining pressure. Therefore, it is concluded that the physical cause by which deviator stress at failure increases with roughness perhaps consists of the increase in the structural effect in view of the fact that $(\sigma_1 - \sigma_3)_R$ is nearly constant. The residual deviator stress computed by subtracting the surface energy effect from the structural effect is termed as the interlocking effect.

Now, the ratios of $(\sigma_1 - \sigma_3)_f$, $(\sigma_1 - \sigma_3)_u$, and $(\sigma_1 - \sigma_3)_R$ to $(\sigma_1 - \sigma_3)_{rf}$, and the ratios of $(\sigma_1 - \sigma_3)_f$, $(\sigma_1 - \sigma_3)_{rf}$, and $(\sigma_1 - \sigma_3)_u$ to $(\sigma_1 - \sigma_3)_R$ are shown in Table 2. This Table means that, of the total shearing energy, the rate occupied by the extra energy which is required to overcome interlocking of the grains in shear zone and to expand the specimen against the confining pressure increases with tightening of interlocking. Then, the ratios of $(\sigma_1 - \sigma_3)_u$ to $(\sigma_1 - \sigma_3)_{rf}$ is very close to unity.

Since $(\sigma_1 - \sigma_3)_u$ is the deviator stress at ultimate shear at which little volume change occurs during shear, it is reasonable for $(\sigma_1 - \sigma_3)_u$ to be approximately equal to the deviator stress, $(\sigma_1 - \sigma_3)_{rf}$, which is corrected for the surface energy effect by $\sigma_3(dv/d\varepsilon)_f$. Also, considering the nature of an assembly of uniform spheres, at ultimate shear the individual grains cannot move along a single plane having a constant inclination, but a typical sphere resting in the hollow formed by the lower spheres will move by sliding or rolling along the grooves between grains and over the top of lower grains. That is, some spheres slide directly over the top of one of the bottom spheres, and

Table 2. Ratios between Various Deviator Stresses

Specimen	$\sigma_3 \left(\frac{\text{kg}}{\text{cm}^2} \right)$	$\frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{rf}}$	$\frac{(\sigma_1 - \sigma_3)_u}{(\sigma_1 - \sigma_3)_{rf}}$	$\frac{(\sigma_1 - \sigma_3)_R}{(\sigma_1 - \sigma_3)_{rf}}$	$\frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_R}$	$\frac{(\sigma_1 - \sigma_3)_{rf}}{(\sigma_1 - \sigma_3)_R}$	$\frac{(\sigma_1 - \sigma_3)_u}{(\sigma_1 - \sigma_3)_R}$
Ba	0.5	1.19	0.93	0.81	1.45	1.21	1.14
	1.0	1.17	0.92	0.85	1.37	1.17	1.08
	1.5	1.30	1.05	0.80	1.63	1.26	1.32
	2.0	1.10	0.92	0.92	1.19	1.09	1.01
	3.0	1.24	0.97	0.83	1.50	1.21	1.17
Bb	0.5	1.28	1.00	0.72	1.79	1.39	1.40
	1.0	1.24	0.84	0.73	1.71	1.37	1.15
	1.5	1.29	0.95	0.74	1.75	1.36	1.29
	2.0	1.18	1.01	0.81	1.46	1.24	1.26
	3.0	1.21	0.87	0.78	1.56	1.29	1.13
Bc	0.5	1.34	1.02	0.69	1.96	1.46	1.48
	1.0	1.29	1.00	0.70	1.83	1.42	1.42
	2.0	1.29	1.08	0.72	1.79	1.39	1.51
	3.0	1.38	1.08	0.68	2.03	1.47	1.58
Bd	0.5	1.42	1.03	0.64	2.24	1.57	1.62
	1.0	1.33	0.91	0.66	2.01	1.51	1.38
	2.0	1.46	1.11	0.64	2.30	1.57	1.75
	3.0	1.48	0.88	0.62	2.22	1.62	1.31
Be	0.5	1.48	0.96	0.59	2.50	1.69	1.62
	1.0	1.47	0.86	0.60	2.44	1.66	1.42
	2.0	1.44	0.92	0.62	2.33	1.62	1.49
	3.0	1.45	0.89	0.62	2.35	1.62	1.44
Bf	0.5	1.47	0.98	0.58	2.54	1.71	1.70
	1.0	1.48	0.94	0.58	2.53	1.71	1.61
	2.0	1.48	1.12	0.59	2.48	1.68	1.87
	3.0	1.52	1.02	0.58	2.62	1.73	1.75

other spheres slide through the cleavage between two of the lower spheres. Therefore, it may be suggested that $(\sigma_1 - \sigma_3)_u$ still contains the portion of strength due to interlocking. That is also proved from both the failure mode of specimens as shown in Fig. 4 and the serrated behaviour in deviator stress at ultimate shear as shown in Fig. 6.

Therefore, it is clarified that $(\sigma_1 - \sigma_3)_{rf}$ which is approximately equal to $(\sigma_1 - \sigma_3)_u$ contains the contribution to shearing strength due to interlocking, and that for the assembly of uniform spheres $(\sigma_1 - \sigma_3)_u$ is consequently larger than $(\sigma_1 - \sigma_3)_R$. However, if granular materials are not uniform spheres but are well graded, ultimate shear deformation will occur along a single plane.

Then, if the ultimate shear strength is not influenced by remolding, rearranging, and crushing of grain, $(\sigma_1 - \sigma_3)_u$ will become equal to $(\sigma_1 - \sigma_3)_R$.

EXPRESSION FOR PHYSICAL COMPONENTS OF SHEARING STRENGTH

The expressions for the structural effect in granular materials can be derived from the relationships between different deviator stresses. From equations (1) and (2),

$$\frac{(\sigma_1 - \sigma_3)_{rf}}{(\sigma_1 - \sigma_3)_R} = 1 + \left(\frac{dv}{d\varepsilon} \right)_f \quad (3)$$

As the interlocking effect can be given by the difference between $(\sigma_1 - \sigma_3)_{rf}$ and $(\sigma_1 - \sigma_3)_R$ from the foregoing experimental study, using equation (3),

$$\begin{aligned} & (\sigma_1 - \sigma_3)_{rf} - (\sigma_1 - \sigma_3)_R \\ &= \left\{ 1 + \left(\frac{dv}{d\varepsilon} \right)_f \right\} (\sigma_1 - \sigma_3)_R - (\sigma_1 - \sigma_3)_R \\ &= (\sigma_1 - \sigma_3)_R \left(\frac{dv}{d\varepsilon} \right)_f \end{aligned} \quad (4)$$

whence, structural effect is expressed by

$$\left. \begin{aligned} \text{surface energy effect} &= \sigma_3 \left(\frac{dv}{d\varepsilon} \right)_f \\ \text{interlocking effect} &= (\sigma_1 - \sigma_3)_R \left(\frac{dv}{d\varepsilon} \right)_f \end{aligned} \right\} \quad (5)$$

Then, the measured deviator stress, $(\sigma_1 - \sigma_3)_f$, is described by the expression,

$$(\sigma_1 - \sigma_3)_f = (\sigma_1 - \sigma_3)_R + (\sigma_1 - \sigma_3)_R \left(\frac{dv}{d\varepsilon} \right)_f + \sigma_3 \left(\frac{dv}{d\varepsilon} \right)_f \quad (6)$$

There was a complete agreement between equations (3) to (6) and the test results shown in Table 1. When $(\sigma_1 - \sigma_3)_R$ is equal to deviator stress, $(\sigma_1 - \sigma_3)_\mu$, contributed by only intergranular friction, equation (6) will be expressed as follows:

$$(\sigma_1 - \sigma_3)_f = (\sigma_1 - \sigma_3)_\mu + (\sigma_1 - \sigma_3)_\mu \left(\frac{dv}{d\varepsilon} \right)_f + \sigma_3 \left(\frac{dv}{d\varepsilon} \right)_f \quad (7)$$

In all the test results, since the confining pressure, σ_3 , was greater than $(\sigma_1 - \sigma_3)_R$, surface energy effect was greater than interlocking effect from equation (5). However, according to unpublished test results on flat lead shots made by the author, $(\sigma_1 - \sigma_3)_R$ was greater than σ_3 , and interlocking effect was consequently greater than surface energy effect. Therefore, it

is suggested that the grain shape has close relation with the interlocking among grains rather than with the accompanied dilation of specimen during shear, and that the surface roughness has close relation with the dilation.

ON THE " k VALUE" IN MOGAMI'S THEORY

Mogami (1965, 1966) has developed a theory of mechanics of granular materials on the basis that the mechanical behaviour of granular materials is characterized by the mean void ratio and the distribution of voids in the materials. In his theory the angle of internal friction (i.e., the angle of shearing resistance) is expressed by the following equation in the case of axial symmetry:

Table 3. k Values in Mogami's Theory of Mechanics of Granular Materials

Specimen	σ_3 (kg/cm ²)	$1+e_f$	$\sin \phi$	k	$\bar{k}$
Ba	0.5	1.616	0.3902	0.483	0.441
	1.0	1.609	0.3711	0.454	
	1.5	1.601	0.3631	0.441	
	2.0	1.601	0.3377	0.406	
	3.0	1.600	0.3506	0.423	
Bb	0.5	1.650	0.4624	0.601	0.589
	1.0	1.640	0.4845	0.632	
	1.5	1.635	0.4475	0.573	
	2.0	1.623	0.4421	0.561	
	3.0	1.633	0.4516	0.579	
Bc	0.5	1.656	0.4737	0.621	0.613
	1.0	1.648	0.4832	0.633	
	2.0	1.642	0.4652	0.603	
	3.0	1.629	0.4628	0.594	
Bd	0.5	1.651	0.4845	0.636	0.639
	1.0	1.659	0.5086	0.677	
	2.0	1.633	0.4744	0.613	
	3.0	1.627	0.4872	0.631	
Be	0.5	1.648	0.5122	0.679	0.665
	1.0	1.640	0.5086	0.670	
	2.0	1.630	0.5025	0.659	
	3.0	1.627	0.5004	0.651	
Bf	0.5	1.660	0.5215	0.698	0.688
	1.0	1.647	0.5238	0.697	
	2.0	1.641	0.5134	0.678	
	3.0	1.632	0.5177	0.681	

$$\sin \phi = \frac{3k}{2(1+e)+k}, \quad (8)$$

in which coefficient k is defined by the deviation of void ratio, s , mean void ratio, e , and shear strain, γ , as follows:

$$s = 2k\gamma e(1+e) \log \frac{1+e}{e}. \quad (9)$$

It is described by the writer that the validity of the theory is given by inserting the experimental data of void ratios and the angle of internal friction in the equation (8) and by confirming the constancy of the value k . He presented also that the k value has close relation with the kind of materials and the gradation of materials (1968).

The k values which were obtained by applying Mogami's approach to the test data for the triaxial specimens of uniform spheres of different surface roughnesses are shown in Table 3. In this Table, the values of $\sin \phi$ for each confining pressure were given by the following equation:

$$\sin \phi = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 + \sigma_3)_f}. \quad (10)$$

The k values were, as shown in Table 3, approximately constant for the specimens of equal roughness, increased with an increase in roughness, and seemed to be affected by the confining pressure.

CONCLUSIONS

A series of drained triaxial compression tests were performed on the assemblages of uniform spheres with varying surface texture. The main results obtained are as follows:

- (1) The triaxial specimens do not develop a failure plane but develop a failure zone. The volume of the failure zone increases with an increase in surface roughness. All the specimens begin to develop the failure zone at some appreciable strain after the peak strength.
- (2) The relationships between deviator stresses, axial strains, and volume changes for the specimens of rigid uniform spheres such as steel balls show a serrated behaviour due, probably, to interlocking among grains.
- (3) Failure strain decreases and the amount of volume expansion of specimens increases with an increase in surface roughness of the spheres.
- (4) The shearing strength of granular materials consists of both the intergranular friction and the structural effect, and the structural effect consists mainly of both the surface energy effect and the interlocking effect.

The physical cause by which shearing strength increases with an increase in the roughness consists of an increase in the structural effect.

(5) The portion of shearing strength which is contributed by surface energy and that which is contributed by interlocking are expressed respectively by the confining pressure multiplied by the dilatancy rate, and the deviator stress which depends on intergranular friction multiplied by the dilatancy rate. Structural effect increases with an increase in the roughness. It varies according to the shape of grains irrespective of whether the ratio between surface energy effect and interlocking effect exceeds unity. For the assemblages of uniform spheres such as steel balls, surface energy effect is greater than interlocking effect.

(6) For the assemblages of uniform spheres, the deviator stress corrected for surface energy effect is approximately equal to the ultimate deviator stress and contains the effect of interlocking. Then, the ratios of deviator stress corrected for surface energy effect to deviator stress which depends on intergranular friction is given by $1 + (dv/d\varepsilon)_f$.

(7) The “ k values” in Mogami’s theory of mechanics of granular materials are approximately constant for the same roughness of grain surface, and increase with an increase in the roughness.

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NOTATIONS

- e : void ratio,
- e_0 : initial void ratio,
- e_f : void ratio at failure,
- k : coefficient in Mogami’s theory
- s : deviation of void ratio,
- V : volume of specimen,
- v : volume change rate given by $(\Delta V/V) \times 100$,
- ΔV : volume change,
- $(dv/d\varepsilon)_f$: dilatancy rate, i.e. ratio of increment of volume change rate to increment of axial strain,
- γ : shear strain,
- ε : axial strain,
- ε_f : axial strain at failure,
- σ_1 : axial stress,

- σ_3 : confining pressure,
- $(\sigma_1 - \sigma_3)_f$: measured maximum deviator stress,
- $(\sigma_1 - \sigma_3)_R$: deviator stress corrected by applying Rowe's formula for volume change,
- $(\sigma_1 - \sigma_3)_{rf}$: deviator stress corrected by applying Bishop's formula for surface energy effect,
- $(\sigma_1 - \sigma_3)_u$: measured deviator stress at ultimate shear,
- ϕ : angle of internal friction, i.e. angle of shearing resistance,
- ϕ_d : angle of shearing resistance for drained shear test,
- ϕ_R : angle of friction corrected for structural effect,
- ϕ_{rf} : angle of friction corrected for surface energy effect, and
- ϕ_u : angle of friction at ultimate shear.

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