EFFECT OF SUBSURFACE LIQUEFACTION ON THE STRENGTH OF SURFACE SOIL

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ABSTRACT

After a subsurface layer of soil is liquefied completely, the excess pore water pressure therein is dissipated through the overlying layer of soil. The Terzaghi consolidation theory is applied to the problem to determine the excess pore water pressure, and numerical solutions are obtained using the finite element method for a variety of initial and boundary conditions. If the compressibility of the surface soil is one order of magnitude smaller than that of the initially liquefied soil, the maximum pore pressure in the surface soil is not significantly affected by the initial excess pore pressure in the surface soil, the relative thickness of the surface soil with respect to the initially liquefied soil, the drainage condition at the lower boundary of the initially liquefied soil, or by the depth of the water table. The smaller the permeability and compressibility of the surface soil, the smaller becomes the shear strength of the soil due to the upward seepage. The surface soil can even be liquefied if its permeability and compressibility are sufficiently low as compared to those of the initially liquefied soil.

Key words: consolidation, earthquake, finite element method, liquefaction, pore pressure, sand, seepage

IGC: E7/E8

INTRODUCTION

Analytical and experimental studies for the last several years have indicated that liquefaction of saturated sand during earthquakes could be initiated at some depth below the ground surface where the seismic shear stress reached the shear stress required to cause liquefaction (Seed, 1967). Suppose the soil of Fig. 1 is completely liquefied in Stratum II. The probable distribution of the excess pore water pressure, u_0 , at the instant of the complete liquefaction is as shown in solid lines. Note that the excess pore water pressure in Stratum II is equal to the initial effective stress, σ'_{v0} , as a result of complete liquefaction. The linear distribution of the excess pore water pressure in Strata I and III are inferred from the results of laboratory shear tests on saturated sands on the assumption that the relative density is nearly constant within each of the strata (see Fig. 13).

Because of dissipation of the excess pore water pressure away from Stratum II, the pore water pressures in Strata I and III first rise causing rebound, and then fall causing recompression, whereas the pore pressure in Stratum II will decrease nearly monotonically as in the case of the conventional consolidation problem. Ignoring possible ef-

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Fig. 1. Distribution of excess pore water pressure at the instant of complete liquefaction of a subsurface stratum

Fig. 2. Simplified initial distribution of excess pore water pressure employed by Ambraseys and Sarma (1969)

fects of earthquake ground motions after the liquefaction, the prediction of the pore water pressures may be made by treating the problem as a transient flow problem.

Ambraseys and Sarma (1969) made a theoretical analysis of a simplified model as shown in Fig. 2, in which the excess pore water pressure in the initially liquefied zone, Stratum II, was assumed constant. Their results show that for the effective stress in the upper layer to vanish, the upper layer must be much less permeable and compressible than the lower layer. It was suspected, however, that their model might lead to an underestimation of the excess pore water pressure in the upper layer. This has prompted the present authors to undertake analytical studies on the basis of more realistic initial and boundary conditions as shown in Fig. 3.

Fig. 3. Initial excess pore water pressure distributions and boundary conditions

STANDARD PROBLEM

The initial and boundary conditions of the standard problem are as shown in Fig. 4. The soil consists of two horizontal strata of an equal thickness underlain by an im-

Fig. 4. Initial and boundary conditions for the standard problem

permeable boundary, with the water table at or above the ground surface. Initially, the lower stratum is completely liquefied with the pore water pressure, u_{w0} , equal to the total vertical stress, σ_v , while in the upper stratum the pore water pressure shows a hydrostatic distribution.

It is assumed that the subsequent flow of pore water takes place in the vertical direction only, and that the vertical total stress does not vary with time. Thus, the problem is reduced to a one-dimensional transient flow problem which can be described by the following equation:

$$m_v \gamma_w \frac{\partial u}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial u}{\partial z} \right) \quad (1)$$

in which k is the coefficient of permeability in the vertical direction, γ_w the unit weight of water, and m_v denotes the coefficient of volume change defined by

$$m_v = - \frac{dn}{d\sigma'_v} \quad (2)$$

in which n is the porosity and σ'_v is the vertical effective stress. Note that with m_v thus defined, Eq. (1) is applicable to the consolidation problem in which the increase in the effective stress is due to the self-weight of the soil.

The finite element method (Zienkiewicz and Cheung, 1967) is employed to solve the problem in two-dimensional plane-strain formulation as shown in Fig. 5. The boundary elements at the top of the assembly serve to give either impermeable or permeable boundary at the surface by assigning them a very small (10^{-18} cm/sec) or very large (10^{22} cm/sec) coefficient of permeability, respectively.

The forward difference in the time domain is used, in which the time increment is successively increased by a factor of $\sqrt{2}$ beginning with 0.01 sec.

Fig. 5. Finite element representation of the standard problem

Coefficient of Volume Change

Fig. 6 shows the relationship between the void ratio and the confining stress during isotropic compression, before and after liquefaction which was caused by static cyclic shear under undrained conditions (Tatsuoka, 1972). The chain-dotted curve is a virgin compression curve of a different specimen prepared by pouring sand in a mold. The close resemblance between the compression curves following liquefaction and the virgin compression curves was probably caused by a complete breakdown of the soil structure as a result of the liquefaction. It seems reasonable to assume, therefore, that the compression following the liquefaction of Stratum II follows the path of a virgin compression.

Where no liquefaction is involved, on the other hand, the

Fig. 7. Typical void ratio vs. stress relationship

Fig. 6. Void ratio vs. stress relationship following liquefaction (Tatsuoka, 1972)

void ratio vs. effective stress relationship of sands takes the form of the solid curves of Fig. 7, in which the rebound curve BC and the recompression curve CD usually have slopes considerably smaller than that of the virgin compression curve AB . If the effective stress at B represents the existing overburden stress at a certain depth within Stratum I, it seems reasonable to assume that the path BCD will be traced during the rise and fall of the pore water pressure following the liquefaction of Stratum II. There may remain a possibility, however, that if complete breakdown of the soil structure takes place in Stratum I as a result of the upward seepage, the recompression may proceed along a much steeper path $C'D'$ as in the case of Fig. 6.

In the following analysis, linear relationship between the porosity and the vertical effective stress

Fig. 8. Linear porosity-stress relationship assumed in the analysis

will generally be assumed as shown in full lines in Fig. 8, i.e., the coefficient of volume change in each stratum is assumed constant. It is conceivable that Stratum I is likely to be considerably less compressible—probably one order of magnitude less so—than Stratum II, not only because the latter undergoes virgin compression while the former rebound and recompression, but also because the former is likely to be initially denser than the latter.

Darcy's Law and Coefficient of Permeability

For a given sand the coefficient of permeability, k , has been found to be nearly proportional to $e^3/(1+e)$ (Taylor, 1948), i.e.,

$$k \propto \frac{e^3}{1+e} \quad (3)$$

By differentiating Eq. (3), one can obtain the following expression for the relative change in k :

$$\frac{dk}{k} = \frac{(3+2e)de}{(1+e)e} = \frac{(3+2e)dn}{e} = -\frac{3+2e}{e} m_v d\sigma' \quad (4)$$

Or, approximately

$$\frac{\Delta k}{k} = -\frac{3+2\bar{e}}{\bar{e}} m_v \Delta\sigma' \quad (5)$$

in which $\bar{e}$ denotes the average void ratio. Because the type of soil which is conducive to complete liquefaction during an earthquake has more or less limited range in the void ratio, compressibility, and in the stress increment, a typical value of the relative change in the coefficient of permeability, $\Delta k/k$, can be estimated at approximately 11 per cent by letting $\bar{e}=0.8$, $m_v=0.02 \text{ cm}^2/\text{kg}$, and $\Delta\sigma'=2 \text{ kg}/\text{cm}^2$.

It will be assumed in the subsequent analysis that Darcy's law is valid, and that the coefficient of permeability within each stratum is constant for virgin consolidation, rebound, and recompression.

Summary of Assumptions

The assumptions and limitations involved in the analysis may be summarized as follows:

- 1) The problem is one-dimensional, i.e., all physical changes occur in the vertical direction only.
- 2) Stratum II is completely liquefied at time $t=0$ throughout its depth.
- 3) The seismic shear stress vanishes throughout the entire system immediately after the liquefaction of Stratum II.
- 4) Darcy's law is valid with a constant coefficient of permeability within each stratum.
- 5) Stratum I undergoes rebound and recompression with a constant coefficient of volume change.
- 6) Stratum II undergoes virgin compression with a constant coefficient of volume change.
- 7) No ground-breaks develop within the soil.

RESULTS AND DISCUSSIONS

The computer was applied to three problems for which theoretical solutions are available in series form (Terzaghi and Froehlich, 1936; Schiffman and Gibson, 1964; Ambraseys and Sarma, 1969), and was found to give satisfactory stability and accuracy.

Standard Problem

The results of the numerical solution to the standard problem are shown in Fig. 9 in the form of excess pore water pressure isochrones. Because the distribution of the maxi-

Fig. 9. Isochrones of excess pore water pressure for the standard problem

mum pore pressure in Stratum I is nearly linear, the maximum hydraulic gradient in Stratum I is nearly constant. Evidently, the value of the maximum hydraulic gradient in Stratum I increases as k_1/k_2 or m_{v1}/m_{v2} decreases, as pointed out previously (Ambraseys and Sarma, 1969). Before presenting quantitative results on the effect of the permeability, the effect of the initial and boundary conditions will be investigated.

Effect of the Initial Hydraulic Gradient in Stratum II

The results of the standard problem in which the initial hydraulic gradient in Stratum II is at the critical value and those of Ambraseys and Sarma's analysis for which the initial hydraulic gradient in Stratum II is assumed zero are compared in Fig. 10 for the case in which $k_1/k_2=1$ and $m_{v1}/m_{v2}=0.1$. As shown in Fig. 10(a), the present

Fig. 10. Effect of initial hydraulic gradient in Stratum II on the effective stress at mid-depth of Stratum I

analysis gives considerably higher pore pressures which are sustained for longer duration than the solution given by Ambraseys and Sarma for their simplified model. The difference appears even more pronounced when the data are replotted in terms of the ratio of the vertical effective stress, as shown in Fig. 10(b). The fact that Ambraseys and Sarma's model resulted in an overestimation of the effective stress, or the strength, of Stratum I justifies the attempt by the present authors to include the initial hydraulic gradient in Stratum II.

Effect of the Relative Thicknesses of Strata I and II

In order to investigate the effect of the relative thicknesses of Strata I and II on the development of excess pore water pressure in Stratum I, a series of computations were made on the problem of Fig. 3(a) with various thickness ratios. Fig. 11 shows for the case of $k_1=k_2$ the ratio of the maximum excess pore water pressure to the initial effective stress in Stratum I plotted against the relative thickness. Because the distribution of the maximum pore pressure in Stratum I is nearly linear, the ordinate is equivalent to the ratio of the maximum hydraulic gradient in Stratum I to the critical hydraulic gradient.

Fig. 11. Effect of relative thickness of Stratum I on maximum pore pressure in Stratum I

It is evident in Fig. 11 that the effect of the relative thickness of Stratum I on the maximum pore pressure depends a great deal on the compressibility ratio, m_{v1}/m_{v2} . For $m_{v1}/m_{v2}=1$, the maximum pore pressure decreases linearly with an increase in the thickness ratio. On the other hand, if $m_{v1}/m_{v2}=0.1$ in a more likely situation, a high level of pore pressure is maintained until the thickness ratio reaches approximately 0.5.

Fig. 12. Effect of drainage conditions at bottom of Stratum II (see Fig. 3(b))

Effect of Permeable Boundary at the Bottom of Stratum II

A three-layer problem as shown in Fig. 3(b) was solved to study the effect of having a permeable boundary at the bottom of the initially liquefied region, Stratum II. Fig. 12(a) shows a set of pore pressure isochrones for $k_1=k_2=k_3=0.05$ cm/sec and $m_{v1}=m_{v3}=0.1m_{v2}=0.002$ cm²/kg.

The maximum pore pressure in Stratum I for the three-stratum problem is compared with the standard two-stratum problem as shown in Fig. 12(b). Evidently the presence of the permeable boundary at the bottom of the Stratum II has negligible effect on the maximum pore pressure in Stratum I. It is also shown in the figure that the maximum pore pressure at middepth of Stratum III reaches approximately one half of the initial effective stress.

Effect of Initial Pore Pressure in Stratum I

According to cyclic shear tests on saturated sands in the laboratory, the pore water pressure increment during each stress cycle, Δu , prior to liquefaction may be related to the octahedral shear stress amplitude, τ_{oct} , and the mean principal stress, σ_m , as shown in Fig. 13 (Shibata, et al. 1972). Thus, it is conceivable that the pore water pressure in Stratum I may have been increased to some extent when Stratum II is completely liquefied as a result of a certain number of seismic shear stress. If Stratum I is uniform with respect to the gradation and relative density, the excess pore pressure distribution within

Fig. 13. Pore pressure increment per shear stress cycle during cyclic tri-axial tests on saturated sand (Shibata et al., 1972)

Fig. 14. Effect of initial excess pore pressure in Stratum I (see Fig. 3(c))

Stratum I may be nearly linear because the maximum shear stress during an earthquake at shallow depths is expected to increase nearly linearly with depth.

In order to investigate the effect of the initial pore pressure in Stratum I, the problem of Fig. 3(c) was solved. In Fig. 14 the time histories of the excess pore pressure in Stratum I for three values of the initial hydraulic gradient are compared with the result of the standard problem. Evidently, the initial hydraulic gradient in Stratum I results in relatively small changes in the magnitude and the time of the peak pore pressure, and practically no effect on the post-peak time histories.

Effect of Water Table

At building sites the water table is usually at some depth below the ground surface. The degree of saturation of the soil above the water table depends on the grain size and on the history of saturation and drainage. Because of the difficulty of treating a problem of partially saturated soil, only the case of full saturation will be analyzed here. If the soil above the water table is medium sand or finer, it may be fully saturated to a height of one meter or so above the water table (Lambe and Whitman, 1969).

Fig. 15. Initial and final pore pressure distribution for the case in which water table is below ground surface

In reference to Fig. 15, AEB gives the hydrostatic pore pressure distribution before the earthquake, and $AEFC$ the distribution immediately after the complete liquefaction of Stratum II. If the lower boundary of Stratum II is impermeable, it is reasonable to assume that the pore pressure distribution at the end of the transient flow following the liquefaction is represented by OD rather than by AB . The excess pore pressure, u , may be conveniently defined with respect to OD as shown in Fig. 15(b), with a constant negative value in Stratum I.

Because drainage cannot take place at the ground surface until the pore water pressure becomes positive, the boundary condition at the ground surface is considered impermeable if $u < 0$ and permeable if $u > 0$. Fig. 16 shows the excess pore pressure isochrones for $k_1/k_2=1$ and $m_{v1}/m_{v2}=0.1$. The maximum value of the excess pore pressure in Stratum I are plotted in Fig. 17 for several values of the depth of the water table, D_w . It can be seen in the figure that the effect of the depth of water table is small when $m_{v1}/m_{v2}=0.1$, but significant when $m_{v1}/m_{v2}=1$.

STABILITY OF STRATUM I

Fig. 18 shows the time histories of the excess pore pressure at mid-depth of Stratum I for the four cases of Fig. 9. Because the total vertical stress at a given depth is

Fig. 16. Excess pore pressure isochrones for problem of Fig. 15

Fig. 17. Maximum excess pore pressure for problem of Fig. 15

constant, the effective vertical stress decreases as pore pressure increases. Although the effective horizontal stress does not vary in proportion to the vertical effective stress, it can be shown that the ratio of the vertical effective stress to its initial value, σ'_v/σ'_{v0} , is approximately equal to the corresponding ratio of the mean principal stress, σ'_m/σ'_{m0} , which in turn is equal to the ratio of the shear strength for a cohesionless soil. Therefore,

$$\frac{s}{s_0} = \frac{\sigma'_m}{\sigma'_{m0}} \div \frac{\sigma'_v}{\sigma'_{v0}} = 1 - \frac{u}{\sigma'_{v0}} \quad (6)$$

in which s is the shear strength at any time and s_0 the initial value. The strength ratio, s/s_0 , is shown on the right side of Fig. 18.

Because the maximum pore pressure varies nearly linearly with depth in Stratum I, the minimum strength ratio, $s_{\min}/s_0$, corresponding to the maximum pore pressure may be considered a constant throughout Stratum I, and expressed in terms of the hydraulic gradient, as follows:

$$\frac{s_{\min}}{s_0} \div 1 - \frac{u_{\max}}{\sigma'_{v0}} = 1 - \frac{i_{\max}}{i_{cr}} \quad (7)$$

in which $i_{\max}$ and i_{cr} are the maximum and critical hydraulic gradients, respectively.

Fig. 18. Time histories of excess pore pressure at mid-depth of Stratum I for the cases of Fig. 9

Fig. 19. Maximum pore water pressure or minimum strength in Stratum I

Fig. 20. Minimum strength ratio in Stratum I

As shown in Fig. 19, the maximum pore pressure in Stratum I tends to reach greater values when Stratum I is less compressible or permeable than Stratum II, although the maximum pore pressure is nearly independent of the coefficient of permeability if k_1/k_2 is less than 0.5. The results of Fig. 19 are replotted in Fig. 20 showing contours of equal values of the minimum strength ratio. It can be seen in the figure that Stratum I undergoes liquefaction if a combination of k_1/k_2 and m_{v1}/m_{v2} falls in the shaded area, or if $k_1 m_{v1}/k_2 m_{v2}$ is smaller than approximately 0.05. Such combinations of the compressibility and permeability may be considered probable if one accepts the assumption that Stratum II undergoes virgin compression, making the ratio m_{v1}/m_{v2} fairly small.

In terms of the field situation, however, one must realize that the critical hydraulic gradient corresponding to the condition of $u_{max}/\sigma'_{v0}=1$ cannot probably be maintained without causing fissures and local eruption of sand and water (Housner, 1958).

Although the presence of a foundation in Stratum I will affect the state of stress and seepage conditions, the strength ratio of Eq. (7) may still give crude indication of the bearing capacity of the foundation which is influenced by a partial loss of the soil strength (Ambraseys and Sarma, 1969). As shown in Fig. 20, the possibilities for a partial reduction of the soil strength are expanded considerably compared to those for a complete loss of strength. The curves (a), (b), and (d) in Fig. 18 for which the minimum strength ratio, s_{min}/s_0 , is less than $1/3$, the time at the peak pore pressures range between 40 sec and 3 min. Although the actual time depends on the coefficient of permeability, compressibility, and thickness of the soil, the order of magnitude may be considered representative of typical field situations in which both Stratum I and II consist of fine sands. In this respect, the order of magnitude of the time to peak pore pressure does not seem to contradict the observations during the Niigata earthquake of 1964 regarding the time at which the settlement of the buildings and the sand blow from the ground were taking place most actively.

Even if Stratum II is too deep to be effectively improved to prevent liquefaction, one may still wish to minimize damage to foundations placed in Stratum I by modifying the properties of that soil. Densification will cause a reduction in k_1 and m_{v1} resulting in

a reduction in s/s_0 as shown in Fig. 20. On the other hand, densification will cause an increase in the initial shear strength s_0 and a reduction in the pore pressure during seismic shear, which should be beneficial. Therefore, the net effect of densification of Stratum I may or may not be advantageous depending on the initial soil properties and the degree of densification. On the contrary, increasing the permeability of the soil of Stratum I has a positive effect on improving stability, as shown in Fig. 20. In this respect, the vibroflotation or the sand compaction piles using coarse backfill should be more effective than other densification methods in which density alone is increased.

CONCLUSIONS

A method of numerical analysis has been presented to study quantitatively the pore water pressure development in the surface soil overlying a completely liquefied stratum. The method is capable of handling a realistic situation in which the water table is initially below the ground surface and the pore water pressure in the surface soil has increased somewhat when the subsurface stratum undergoes complete liquefaction. The following conclusions may be made on the basis of a number of numerical solutions covering a variety of initial and boundary conditions:

- 1) If the compressibility of the surface soil is one order of magnitude smaller than that of the initially liquefied soil—a likely situation because the surface soil undergoes rebound while the liquefied soil virgin compression—the maximum pore pressure in the surface soil is not significantly affected by the initial excess pore pressure in the surface soil, the relative thickness of the surface soil with respect to the initially liquefied soil, the drainage condition at the lower boundary of the initially liquefied soil, or by the depth of the water table.
- 2) The smaller the permeability and compressibility of the surface soil, the smaller becomes the shear strength of the soil as a result of the upward seepage. The surface soil can even be liquefied if its permeability and compressibility are sufficiently low as compared to those of the initially liquefied soil.
- 3) The loss of strength of the surface soil due to the upward seepage can be minimized by a substantial increase in the permeability as well as an increase in the initial strength of the soil, e.g., by means of the vibroflotation or the sand compaction pile using coarse backfill.

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NOTATION

c =coefficient of consolidation	n =porosity
D_w =depth of groundwater table	s =shear strength
e =void ratio	T =time factor ($c_v t/H_s^2$)
G_s =specific gravity of solids	u =excess pore water pressure
H =thickness of soil stratum	u_w =pore water pressure
i =hydraulic gradient	z =vertical coordinate
i_{cr} =critical hydraulic gradient	γ =unit weight of soil
k =coefficient of permeability	γ' =submerged unit weight of soil
m_v =coefficient of volume change	γ_w =unit weight of water

σ =normal stress	0 (subscript)=initial state
σ' =effective normal stress	1 (subscript)=Stratum I
τ_{oct} =octahedral shear stress	2 (subscript)=Stratum II

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