TECHNICAL NOTES

INFLUENCE OF SOIL INHOMOGENEITY ON
RAFT BEHAVIOUR

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ABSTRACT

A half-space of elastic material, the Young's modulus of which increases linearly with depth, has been shown, as the degree of inhomogeneity increases, to respond to surface loading more as predicted by Winkler's subgrade reaction theory and less like a homogeneous isotropic elastic material. The present paper discusses the relationships between degree of inhomogeneity and Poisson's ratio and the corresponding bending moments and displacements in rafts. Detailed consideration is given to the variation of maximum bending moment in a uniformly loaded circular raft of any flexibility, variation of differential displacement in a uniformly loaded very flexible circular raft, and displacement of a rigid raft.

Key words: bending moments, deflection, elasticity, footings, inhomogeneous, rafts, soil mechanics, theoretical

IGC: E2

INTRODUCTION

In an earlier paper (Gibson, 1967) it is shown that a half-space of incompressible isotropic elastic material, the Young's modulus of which increases linearly with depth from zero at the surface, responds to surface loading exactly according to the assumptions made in Winkler's subgrade reaction theory. However in a later paper (Brown and Gibson, 1972) it is shown that the effect of removing the conditions of incompressibility or of zero surface modulus is to cause the material behaviour to become more like the behaviour of a homogeneous material. When the soil modulus increases slowly, the behaviour is almost indistinguishable from that of a homogeneous material.

Since zero surface modulus is likely to be associated with zero strength, this extreme case appears to be of less interest from a practical viewpoint than the intermediate cases which will be considered in the present paper.

Because the purpose of this paper is only to describe general trends, a circular raft is the only form considered. Although a wide range of stiffnesses is considered with regard to bending moments, only the two extremes of stiffness, namely rigid and very flexible, are considered in connection with displacements. Forms of loading in which the load is distributed only over a small region near the centre of the raft are not considered since soil behaviour has little effect on bending moments in such cases, and differential displacements in a very flexible raft appear to be of little interest since such a raft is unsuitable for this form of loading. Hence the only case of loading considered is that of uniform loading over

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the whole of the raft.

The value of Poisson's ratio of the raft is assumed to be 0.3. A lower value of Poisson's ratio would lead to slightly lower values of bending moment, hence the assumption is conservative for reinforced concrete, and values of differential displacement in a very flexible raft and displacements of a rigid raft are independent of Poisson's ratio of the raft.

The form of elastic inhomogeneity considered in this paper is that in which the shear modulus G and the Young's modulus E increase linearly with depth, thus the Poisson's ratio of the soil is a constant. This form of inhomogeneity may be described by either of the equations

$$G(z) = G(o) + mz$$

or

$$G(z) = G(o) \left[1 + \frac{z}{\beta} \right]$$

where m and β are constants and $\beta = G(o)/m$. For the present purpose it is convenient to describe the degree of inhomogeneity by the parameter $G(o)/G(a)$ (where a is the radius of the raft). For a homogeneous material $G(o)/G(a) = 1$ and as $G(o)/G(a)$ becomes smaller the material becomes more inhomogeneous and its behaviour approaches the behaviour of a Winkler material if Poisson's ratio of the soil, $\nu_s = 0.5$, or a behaviour close to that of a Winkler material if ν_s is other than 0.5.

It has been shown (Brown and Gibson, 1972) that as Poisson's ratio of the soil changes from $1/3$ to 0 , the changes in the form of the surface displacement profiles are insignificant. Hence the values of bending moment for $\nu_s = 1/3$ may be taken to apply when $\nu_s = 0$ to $1/3$.

METHOD OF ANALYSIS

The basis of the results presented in this paper is a set of values of surface displacement produced by the techniques described in an earlier paper (Brown and Gibson, 1972). In the case of very flexible rafts these values of surface displacement constitute the desired results.

In the case of stiffer rafts, a set of values of surface displacement for uniformly loaded circles whose radii increase in equal steps are used. These values are shown in Tables 1 and 2 as deflections $\times G(o)/qb$, where q is the load intensity, b is the radius of the loaded area, and r is the radial coordinate of the point at which the deflection is given. Differences between values at the same position for different load radii provide values of displacement due to annular loads.

The raft is considered as being divided into nine annuli and a central disc, the width of each annulus being equal to the radius of the central disc. The reaction acting on each of these zones is assumed to be uniform and vertical. These ten values of reaction pressure are determined by solving a set of equations consisting of one equation of vertical equilibrium and nine equations obtained by equating the displacements of the soil and the raft relative to the centre of the raft. Details of the formation of these equations are given in the Appendix.

Having obtained the values of reaction pressure the displacements of the raft can be determined from the data relating annular loads and displacements. Determination of bending moments is carried out using the following equations which arise from thin plate small deflection theory

$$\frac{d^2\phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} - \frac{\phi}{r^2} = -\frac{Q}{D} \quad (1)$$

Table 1. Displacement $\times G(o)/qb$ (for $\nu_s=1/2$) due to circular loaded areas

BETA= 10		NUS= 0.5									
B	R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95
0.1		.4821	.1637	.0897	.0609	.0455	.0360	.0295	.0248	.0213	.0185
0.2		.4698	.3915	.1971	.1270	.0933	.0731	.0597	.0500	.0428	.0372
0.3		.4592	.4276	.3592	.2096	.1469	.1131	.0914	.0763	.0650	.0564
0.4		.4497	.4326	.3993	.3375	.2148	.1585	.1260	.1041	.0883	.0764
0.5		.4411	.4305	.4104	.3769	.3212	.2166	.1656	.1348	.1133	.0974
0.6		.4332	.4260	.4125	.3910	.3587	.3080	.2168	.1701	.1409	.1201
0.7		.4259	.4206	.4110	.3959	.3742	.3434	.2971	.2159	.1729	.1453
0.8		.4190	.4150	.4078	.3967	.3809	.3595	.3304	.2877	.2144	.1745
0.9		.4125	.4094	.4038	.3952	.3833	.3674	.3466	.3191	.2794	.2125
1.0		.4063	.4039	.3994	.3926	.3832	.3709	.3552	.3351	.3090	.2720

BETA= 1		NUS= 0.5									
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95	
0.1	.4060	.1136	.0526	.0314	.0210	.0151	.0113	.0088	.0071	.0058	
0.2	.3577	.2937	.1220	.0674	.0440	.0311	.0233	.0180	.0144	.0117	
0.3	.3230	.2998	.2467	.1188	.0719	.0495	.0364	.0279	.0221	.0179	
0.4	.2961	.2846	.2615	.2155	.1132	.0726	.0518	.0391	.0307	.0247	
0.5	.2742	.2676	.2549	.2327	.1926	.1073	.0716	.0526	.0405	.0322	
0.6	.2559	.2517	.2439	.2308	.2102	.1748	.1018	.0699	.0524	.0410	
0.7	.2403	.2375	.2323	.2238	.2112	.1922	.1606	.0966	.0679	.0517	
0.8	.2267	.2247	.2211	.2153	.2068	.1948	.1773	.1488	.0919	.0658	
0.9	.2148	.2134	.2107	.2065	.2006	.1923	.1810	.1649	.1389	.0876	
1.0	.2043	.2032	.2012	.1981	.1937	.1877	.1798	.1691	.1542	.1304	

BETA= 0.1		NUS= 0.5									
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95	
0.1	.2042	.0304	.0091	.0042	.0023	.0014	.0009	.0006	.0005	.0003	
0.2	.1393	.1177	.0263	.0099	.0051	.0030	.0020	.0014	.0010	.0007	
0.3	.1070	.1016	.0859	.0223	.0094	.0052	.0033	.0022	.0016	.0012	
0.4	.0872	.0852	.0805	.0680	.0192	.0087	.0051	.0033	.0023	.0016	
0.5	.0738	.0728	.0708	.0666	.0563	.0169	.0081	.0048	.0032	.0023	
0.6	.0641	.0635	.0625	.0605	.0569	.0481	.0152	.0074	.0046	.0031	
0.7	.0566	.0563	.0557	.0546	.0529	.0497	.0421	.0137	.0069	.0043	
0.8	.0508	.0505	.0502	.0495	.0485	.0469	.0441	.0374	.0125	.0064	
0.9	.0460	.0459	.0456	.0452	.0446	.0437	.0422	.0396	.0337	.0115	
1.0	.0421	.0420	.0418	.0415	.0411	.0405	.0397	.0383	.0360	.0306	

BETA= 0.01		NUS= 0.5									
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95	
0.1	.0420	.0017	.0003	.0001	.0001	.0000	.0000	.0000	.0000	.0000	
0.2	.0228	.0214	.0013	.0003	.0001	.0001	.0000	.0000	.0000	.0000	
0.3	.0156	.0154	.0145	.0010	.0003	.0001	.0001	.0000	.0000	.0000	
0.4	.0119	.0118	.0117	.0110	.0008	.0002	.0001	.0001	.0000	.0000	
0.5	.0096	.0096	.0095	.0094	.0089	.0007	.0002	.0001	.0001	.0000	
0.6	.0081	.0081	.0080	.0080	.0078	.0074	.0006	.0002	.0001	.0001	
0.7	.0069	.0069	.0069	.0069	.0069	.0067	.0063	.0005	.0002	.0001	
0.8	.0061	.0061	.0061	.0061	.0060	.0060	.0059	.0056	.0005	.0001	
0.9	.0054	.0054	.0054	.0054	.0054	.0054	.0053	.0053	.0049	.0004	
1.0	.0049	.0049	.0049	.0049	.0049	.0049	.0049	.0048	.0048	.0045	

$$M_r = D \left(\frac{d\phi}{dr} + \nu_r \frac{\phi}{r} \right) \quad (2)$$

$$M_t = D \left(\frac{\phi}{r} + \nu_r \frac{d\phi}{dr} \right) \quad (3)$$

where ϕ is the raft slope, r is the radial coordinate, D is the raft stiffness, ν_r is the Poisson's ratio of the raft and Q , M_r and M_t are respectively the shearing force, radial moment and tangential moment per unit length.

In the case of a rigid raft ϕ is zero and D is infinite, however it may be shown that the product ϕD is finite, and for the rigid case this product ($\psi = \phi D$) is the dependent

Table 2. Displacements $\times G(o)/qb$ (for $\nu_s=1/3$) due to circular loaded areas

BETA= 10		NUS= 0.3333								
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95
0.1	.8383	.2208	.1219	.0832	.0625	.0497	.0409	.0345	.0298	.0260
0.2	.6331	.5282	.2673	.1735	.1282	.1010	.0827	.0697	.0599	.0523
0.3	.6205	.5779	.4863	.2855	.2013	.1559	.1267	.1062	.0910	.0792
0.4	.6093	.5861	.5414	.4585	.2938	.2181	.1743	.1449	.1235	.1072
0.5	.5993	.5847	.5577	.5126	.4378	.2974	.2288	.1872	.1582	.1366
0.6	.5900	.5800	.5618	.5328	.4893	.4213	.2985	.2358	.1964	.1682
0.7	.5814	.5740	.5611	.5407	.5113	.4699	.4076	.2982	.2404	.2030
0.8	.5733	.5677	.5580	.5428	.5215	.4927	.4533	.3958	.2970	.2434
0.9	.5657	.5612	.5537	.5420	.5258	.5043	.4762	.4389	.3855	.2953
1.0	.5585	.5548	.5488	.5396	.5268	.5101	.4888	.4615	.4261	.3763

BETA= 1 NUS= 0.3333											
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95	
0.1	.5585	.1619	.0776	.0476	.0327	.0240	.0185	.0146	.0120	.0099	
0.2	.5008	.4120	.1778	.1016	.0681	.0494	.0377	.0298	.0242	.0201	
0.3	.4589	.4260	.3517	.1763	.1103	.0780	.0587	.0460	.0372	.0307	
0.4	.4260	.4093	.3763	.3114	.1706	.1130	.0828	.0640	.0513	.0421	
0.5	.3990	.3891	.3706	.3386	.2817	.1639	.1130	.0851	.0671	.0546	
0.6	.3761	.3697	.3581	.3390	.3091	.2586	.1572	.1116	.0858	.0688	
0.7	.3565	.3520	.3442	.3316	.3129	.2851	.2399	.1508	.1096	.0857	
0.8	.3393	.3360	.3305	.3216	.3090	.2911	.2653	.2244	.1449	.1073	
0.9	.3241	.3215	.3175	.3111	.3019	.2894	.2725	.2486	.2112	.1394	
1.0	.3105	.3085	.3054	.3005	.2937	.2846	.2725	.2564	.2342	.1998	

BETA= 0.1		NUS= 0.3333									
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95	
0.1	.3105	.0547	.0192	.0098	.0060	.0041	.0029	.0022	.0017	.0014	
0.2	.2245	.1880	.0508	.0224	.0130	.0085	.0060	.0045	.0035	.0028	
0.3	.1798	.1696	.1422	.0454	.0225	.0140	.0096	.0071	.0054	.0043	
0.4	.1516	.1472	.1379	.1156	.0409	.0218	.0142	.0101	.0077	.0060	
0.5	.1319	.1295	.1252	.1167	.0980	.0372	.0209	.0141	.0103	.0079	
0.6	.1172	.1158	.1133	.1091	.1015	.0855	.0342	.0199	.0138	.0103	
0.7	.1058	.1049	.1034	.1009	.0968	.0901	.0760	.0317	.0190	.0135	
0.8	.0967	.0960	.0950	.0934	.0909	.0872	.0811	.0686	.0295	.0182	
0.9	.0892	.0886	.0880	.0869	.0852	.0828	.0794	.0738	.0626	.0277	
1.0	.0828	.0824	.0819	.0811	.0800	.0784	.0762	.0729	.0678	.0576	

BETA= 0.01		NUS= 0.3333									
B \ R	0	.15	.25	.35	.45	.55	.65	.75	.85	.95	
0.1	.0828	.0072	.0022	.0011	.0006	.0004	.0003	.0002	.0002	.0001	
0.2	.0500	.0449	.0063	.0025	.0014	.0009	.0006	.0005	.0004	.0003	
0.3	.0367	.0355	.0318	.0055	.0024	.0015	.0010	.0007	.0006	.0004	
0.4	.0294	.0288	.0278	.0248	.0048	.0023	.0015	.0010	.0008	.0006	
0.5	.0246	.0243	.0238	.0229	.0204	.0043	.0022	.0015	.0011	.0008	
0.6	.0213	.0211	.0208	.0204	.0195	.0174	.0039	.0021	.0014	.0011	
0.7	.0188	.0186	.0185	.0182	.0178	.0170	.0152	.0036	.0020	.0014	
0.8	.0169	.0167	.0167	.0165	.0162	.0158	.0151	.0135	.0034	.0019	
0.9	.0153	.0152	.0152	.0150	.0149	.0146	.0143	.0136	.0122	.0031	
1.0	.0141	.0140	.0139	.0138	.0137	.0136	.0133	.0130	.0124	.0111	

variable considered. However this change does not materially affect the method of analysis, which will be discussed in regard to the equations relating to a flexible raft.

Accurate representation of the boundary conditions for Eq. (1) is difficult to achieve by the method of finite differences due to the presence of the stress singularity at the edge of the raft. Hence it was decided to treat the equations by means of the method of finite integrals (Brown and Trahair, 1968). This involves the use of $f=d^2\phi/dr^2$ as the basic dependent variable and determination of $d\phi/dr$ and ϕ by numerical integration of f .

Thus

$$\frac{d\phi}{dr} = \int_0^r f dr + A \quad (4)$$

and

$$\phi = \int_0^r \int_0^r f \, dr \, dr + Ar + B \quad (5)$$

however when

$$r=0, \quad \phi=0 \quad \text{hence} \quad B=0$$

and when

$$r=a, \quad \frac{d\phi}{dr} + \frac{\nu_r}{a}\phi = 0$$

hence

$$A = \frac{-\int_0^a f \, dr - \frac{\nu_r}{a} \int_0^a \int_0^r f \, dr \, dr}{1 + \nu_r} \quad (6)$$

Substitution of Eqs. (4) and (5) into Eq. (1) leads to

$$f + \frac{1}{r} \int_0^r f \, dr - \frac{1}{r^2} \int_0^r \int_0^r f \, dr \, dr = -\frac{Q}{D} \quad (7)$$

The values of Q required for use in this equation may readily be evaluated from the known values of reaction pressure.

The numerical integration, which is carried out by a modified form of Simpson's Rule (Brown and Trahair, 1968), and involves steps of unequal size, may be expressed in matrix form as

$$\int_0^r f \, dr = [N][f] \quad (8)$$

$$\int_0^r \int_0^r f \, dr \, dr = [N][N][f] \quad (9)$$

and substitution of these expressions into Eq. (7) produces a set of simultaneous equations which may be solved for the values of f . Subsequent integration of f then provides values of $d\phi/dr$ and ϕ for evaluation of the bending moments.

Clearly the use of only ten reaction zones will not produce results of high accuracy, although the percentage error in bending moment may be expected to decrease as the magnitude of the moment decreases. The magnitude of this error for a homogeneous soil and for each plate stiffness has been determined by taking the ratio of accurate results (Brown, 1969b) to results using ten reaction zones for a deep layer using a technique similar to that described in an earlier paper (Brown 1969a). The values of this ratio are 1.15, 1.14, 1.12, 1.03 for $K=\infty, 10, 1, 0.1$ respectively. These factors have been used to correct all values of bending moment for the corresponding raft stiffness. This will lead to accurate values of the larger moments, but will give rise to some over-estimation of the smaller values of moment, which is not considered serious.

RESULTS

Reaction distributions under a rigid raft for the whole range of degrees of soil inhomogeneity, when $\nu_s=0.5$, are shown in Fig. 1. When $G(o)/G(a)=0$ the reaction is uniform as predicted by the Winkler model. For all other cases up to $G(o)/G(a)=1$, i.e., the homogeneous case, the reaction distribution is non-uniform and includes a stress singularity at the edge of the raft.

Radial and tangential moment distributions in a uniformly loaded rigid raft for the whole range of degrees of soil inhomogeneity, when $\nu_s=0.5$ are shown in Fig. 2. When $G(o)/$

Fig. 1. Reaction distributions under a rigid raft for various degrees of soil inhomogeneity

Fig. 2. Moment distributions in a uniformly loaded rigid raft for various degrees of soil inhomogeneity

Fig. 3. Variation of maximum bending moment with degree of soil inhomogeneity for $\nu_s = 1/2$

Fig. 4. Variation of maximum bending moment with degree of soil inhomogeneity for $\nu_s = 0$ to $1/3$

$G(a)=0$ the moment is zero everywhere as this corresponds to the Winkler model. As the soil approaches homogeneity ($G(o)/G(a)=1$) the magnitudes of the moments increase, although the form of the distribution does not alter greatly.

The variation of maximum bending moment in uniformly loaded circular rafts for the whole range of degrees of inhomogeneity and a wide range of raft stiffnesses is shown in Fig. 3 and Fig. 4 for Poisson's ratio of the soil respectively 0.5 and 0 to $1/3$. The relative raft stiffness is defined as

$$K = \frac{E_r(1-\nu_s^2)}{E_s(o)} \left(\frac{t}{a} \right)^3 = \frac{E_r(1-\nu_s)}{2G(o)} \left(\frac{t}{a} \right)^3$$

where t is the raft thickness. The effect of any change in the value of ν_s is generally not large, and when $G(o)/G(a)$ approaches zero, the moments tend to zero when $\nu_s=0.5$ (the

Winkler result). For ν_s less than 0.5 and finite K , when $G(o)=0$, $E_r t^3$ must be zero and consequently the bending moments are also zero. However for ν_s less than 0.5 and $K=\infty$ the bending moments are finite when $G(o)=0$. The rapid decrease in moment as K decreases from 10 to 0.1 and the comparatively uniform decrease in moment as $G(o)/G(a)$ decreases, will readily be seen. The dotted lines indicate the approximate combination of values of K and $G(o)/G(a)$ at which the maximum moment starts to move from the centre of the raft. Above the dotted line the maximum moment occurs at the centre of the raft.

The variation of the differential displacement between the centre and edge of a uniformly loaded very flexible raft is shown in Fig. 5, and the variation of displacement of a rigid raft is shown in Fig. 6. In both figures, except for values of $G(o)/G(a)$ less than about 0.2, the curves lie quite close together indicating that dividing the displacements by $(1-\nu_s)$ largely accounts for the variation of Poisson's ratio. The displacements have also been multiplied by $G(a)$ since use of the modulus at this particular depth largely accounts for the effect of inhomogeneity. When $G(o)/G(a)=0$ and $\nu_s=0.5$ the displacement of the edge of the raft is multi-valued, however as can be seen from Brown and Gibson (1972) the value resulting from the analysis is half the central displacement. This serves to demonstrate the fact that results for differential displacement are almost meaningless for values of $G(o)/G(a)$ less than about 0.1 as is also indicated by the dotted curves in Fig. 5 which represent the differential displacement between the centre of the raft and a point at 0.95 of the raft radius. When $G(o)/G(a)$ is less than 0.1 the slope of the edge of the raft becomes quite large.

Fig. 5. Variation of differential displacement with degree of soil inhomogeneity for a very flexible raft

Fig. 6. Variation of rigid raft displacement with degree of soil inhomogeneity

DISCUSSION

Application of the foregoing results to practical situations should be carried out with caution. Although in the unloaded state the soil may have a modulus which varies almost linearly with depth, the changes in stress level due to application of load to the raft may cause significant changes to the form of variation of the soil modulus. In addition when $G(o)$ is small the surface strength is likely to be small and local yielding near the edge of the raft may significantly modify the raft bending moments and rigid raft displacements if the safety factor is near unity.

However the general trends of the results are of practical significance in showing firstly that bending moments are significantly less than for a homogeneous soil if the degree of inhomogeneity is significant, and secondly that if the modulus $G(a)$ is used neither differential displacement of a flexible raft nor displacement of a rigid raft is very greatly affected by a wide practical range of degree of inhomogeneity.

While it is interesting that there is an exact correspondence between an inhomogeneous soil and the Winkler model in the limiting case when $G(o)=0$ and $\nu_s=0.5$, it is strongly emphasised that this should not be used as a basis for concluding that the Winkler model is better than the homogeneous isotropic elastic model for practical problems. In distributed loading problems for which homogeneous and Winkler model results are very different, the theoretical investigation reported in this paper has shown that for an inhomogeneous soil the solution may be significantly different from that produced by either of the models.

Even in the limiting case when $G(o)=0$, if the soil is purely cohesive and cohesion/ G is a constant, the Winkler model does not correctly predict the raft behaviour. Local yield occurs at the edges when any load is applied and the moments are only zero when the load is zero. As the load is increased the moments, which are negative in sign, increase in magnitude until at general failure the reaction distribution is triangular with the maximum at the centre (Davis and Booker, 1973). For this inhomogeneous situation the effect of local yield is to increase the magnitudes of the bending moments, in contradistinction to the homogeneous case when bending moments are reduced by local yielding (Brown, 1968).

For foundation problems, the Winkler assumption remains what it has always been, one of mathematical convenience rather than physical reality.

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NOTATION

A =constant of integration
 a =raft radius
 B =constant of integration
 b =radius of loaded area
 D =raft stiffness

E_r = Young's modulus of raft

E_s = Young's modulus of soil

$f = d\phi^2/dr^2$

G = shear modulus of soil

K = stiffness of raft relative to soil = $\frac{E_r(1-\nu_s)}{2G(o)}\left(\frac{t}{a}\right)^3$

M_r = radial bending moment per unit length

M_t = tangential bending moment per unit length

m = rate of increase of soil shear modulus with depth

N = integration matrix

p = reaction pressure

q = loading pressure

r = radial coordinate

t = raft thickness

z = depth coordinate

β = parameter of soil inhomogeneity = $G(o)/m$

ν_r = Poisson's ratio of raft

ν_s = Poisson's ratio of soil

ϕ = raft slope

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APPENDIX

The equations which express the fact that soil and raft displacements relative to the centre of the raft are equal may be set up in the following way.

Soil displacements at the disc centre and mid-radii of the annular rings may be written

$$[w] = [In][p]$$

where $[In]$ is a matrix of displacement coefficients. Then soil displacements relative to the centre are given by

$$[w] - [w]_o = ([In] - [In]_o)[p]$$

which may be written

$$[v] = [J][p] \quad (A1)$$

where the o subscript denotes a matrix whose rows are all identical with the first row of

the corresponding unsubscripted matrix.

The behaviour of the raft is described by Eq. (7) namely

$$f + \frac{1}{r} \int_0^r f dr - \frac{1}{r^2} \int_0^r \int_0^r f dr dr = -\frac{Q}{D}$$

then using

$$\int_0^r f dr = [N][f], \quad \text{and} \quad \int_0^r \int_0^r f dr dr = [N]^2[f]$$

this becomes

$$\left([I] + \frac{1}{r} [N] - \frac{1}{r^2} [N]^2 \right) [f] = -\frac{1}{D} [Q]$$

which may be written

$$[F][f] = -\frac{1}{D} [Q]$$

whence

$$[f] = -\frac{1}{D} [F]^{-1} [Q]$$

Then from Eq. (5) and (6), namely

$$\phi = \int_0^r \int_0^r f dr dr + Ar, \quad \text{since } B=0$$

and

$$A = \frac{-\int_0^a f dr - \frac{\nu_r}{a} \int_0^a \int_0^r f dr dr}{1 + \nu_r}$$

it can be seen that

$$[\phi] = [N]^2[f] - r \frac{[N]_n[f] + \frac{\nu_r}{a} [N]_n^2[f]}{1 + \nu_r}$$

which may be written

$$[\phi] = [E][f]$$

where the subscript n denotes a matrix whose rows are all identical to the last row of the corresponding unsubscripted matrix.

Hence

$$[\phi] = -\frac{1}{D} [E][F]^{-1} [Q] \quad (\text{A2})$$

Values of Q may be expressed in terms of the load and reactions in the form

$$-[Q] = q[T] - [An][p] \quad (\text{A3})$$

Then, equating displacements relative to the centre of the raft, using Eq. (A1), (A2) and (A3), it can be seen that

$$\begin{aligned} [J][p] &= [N][\phi] \\ &= \frac{1}{D} [N][E][F]^{-1} (q[T] - [An][p]) \end{aligned}$$

Whence, by collecting terms in the unknown reaction p , the final equation becomes

$$\left([J] + \frac{1}{D} [N][E][F]^{-1} [An] \right) [p] = \frac{q}{D} [N][E][F]^{-1} [T]$$

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