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SETTLEMENT OF BURIED LOADED AREAS IN NORMALLY CONSOLIDATED CLAY DEPOSITS

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ABSTRACT

Non-dimensional curves for computation of consolidation settlement of square, rectangular and strip loaded areas in normally consolidated clay deposits of semi-infinite extent have been developed. Two types of stress distribution, viz., Boussinesq and Mindlin, have been used to evaluate their significance. At higher embedments settlement computations with Boussinesq's equation for stress distribution, as expected, are found to be much larger than the settlement computations if Mindlin's equation is used. While the settlement phenomenon beneath a loaded area might continue up to large depths, it has been shown that a major portion of the settlement occurs within a small depth below the loaded area. This makes the design curves reasonably applicable to loaded areas on confined layers also. The usage of the design curves for use in practice has been demonstrated with a numerical example.

Key words: clay, deep foundations, deformation, settlement, settlement problem, shallow foundations

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INTRODUCTION

In spite of the many recent advances made in the theory of consolidation, Terzaghi's one dimensional consolidation theory is still widely used for prediction of consolidation settlement of footings and pile groups in clay, due to its simplicity and reasonably satisfactory results obtained for practical purposes. The two aspects to be known before hand in order to calculate the settlement are the stress distribution in the medium and consolidation characteristics of the deposits. The stress distribution is obtained from formulae based on the theory of elasticity and standard oedometer tests are conducted on undisturbed samples to get the consolidation characteristics of the soil. Boussinesq's equation is ordinarily used for stress distribution calculations. Whereas the Boussinesq's equation gives the stress distribution due to a load at the surface of a semi-infinite elastic half space, Mindlin's equation gives the stress distribution due to a load at the interior of the half space and thus more realistically simulates the conditions of foundation structures. But the tedium involved tends to make less popular the use of Mindlin's equation for stress distribution though it would give more

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realistic results. Either of the two parameters, coefficient of volume compressibility (m_v) and compression index (C_c), borne out of oedometer test results, is frequently used to define the consolidation characteristics of the soil. Since C_c is a constant the computation procedure using C_c is simpler than that which makes use of m_v which is dependent upon the stress level.

The conventional procedures used for the calculation of settlement of pile groups in clay consider the pile group as an equivalent footing transferring the load either at the level of the tip of the piles or at two-third the length of the piles. Apart from this modification in the depth of load transfer there is hardly any difference between the procedures of calculation of settlement of footings and pile groups in the conventional approach. The non-dimensional curves developed hereunder for rectangular, square and strip loaded areas are based on Boussinesq's and Mindlin's equations, separately, for stress distribution. The normally consolidated deposit has been assumed to be semi-infinite in extent.

SETTLEMENT OF LOADED AREAS

Settlement of Rectangular Loaded Areas

Fig. 1. Loaded area in the interior of the semi-infinite deposit

The compression, $\Delta\rho$ of an element dz with initial void ratio e_0 , (Fig. 1) vertically below the centre of a uniformly loaded rectangular area of $2a \times 2b$ embedded at a depth h below the surface is given by

$$\Delta\rho = \frac{C_c}{1+e_0} \log \frac{p_z + \sigma_z}{p_z} dz \quad (1)$$

where p_z and σ_z are respectively over burden pressure and increase in pressure in the element at z . Fig. 1 shows the origins of ordinate z , left half of the figure being for Boussinesq type of stress distribution and right half for Mindlin stress distribution. The expression for vertical stress at depth z according to Boussinesq's equation (σ_{zB}) is given by (Harr, 1966).

$$\sigma_{zB} = \frac{2q}{\pi} \left[\frac{abz(a^2+b^2+2z^2)}{(a^2+z^2)(b^2+z^2)\sqrt{(a^2+b^2+z^2)}} + \sin^{-1} \frac{ab}{\sqrt{a^2+z^2}\sqrt{b^2+z^2}} \right] \quad (2)$$

where q = intensity of loading.

Mindlin's equation for vertical stress (σ_{zM}) is given by (Skopek, 1960),

$$\begin{aligned} \sigma_{zM} = \frac{q}{\pi(1-\mu)} & \left[(1-\mu) \left\{ \arctg \frac{ab}{(z-h)R_1} + \arctg \frac{ab}{(z+h)R_2} \right\} \right. \\ & + \frac{(z-h)aR_1}{2br_1^2} - \frac{a(z-h)^3}{2br_3^2R_1} + \frac{\{(3-4\mu)z(z+h)-h(5z-h)\}aR_2}{2(z+h)br_2^2} \\ & - \frac{\{(3-4\mu)z(z+h)^2-h(z+h)(5z-h)\}a}{2br_4^2R_2} + \frac{2hz(z+h)aR_2^3}{b^3r_2^4} \\ & \left. + \frac{3hz a R_2 r_6^2}{(z+h)b^3r_2^2} - \frac{hz(z+h)^3a}{b r_4^4 R_2} \left\{ \frac{2b^2-(z+h)^2}{b^2} - \frac{a^2}{R_2^2} \right\} \right] \quad (3) \end{aligned}$$

where

$$R_1^2 = a^2 + b^2 + (z-h)^2$$

$$r_1^2 = a^2 + (z-h)^2$$

$$r_3^2 = b^2 + (z-h)^2$$

$$R_2^2 = a^2 + b^2 + (z+h)^2$$

$$r_2^2 = a^2 + (z+h)^2$$

$$r_4^2 = b^2 + (z+h)^2$$

$$r_5^2 = b^2 - (z+h)^2$$

and μ = Poisson's ratio.

The settlement at the centre of the loaded area, ρ , is obtained by integration of Eq. (1) substituting for σ_z value.

$$\rho = \frac{1}{C} \int_0^\infty \log \left(1 + \frac{\sigma_{zB}}{\gamma(z+h)} \right) dz \quad (4)$$

for Boussinesq stress distribution and

$$\rho = \frac{1}{C} \int_h^\infty \log \left(1 + \frac{\sigma_{zM}}{\gamma \cdot z} \right) dz \quad (5)$$

for Mindlin stress distribution. In Eqs. (4) and (5), $C = (1+e_0)/C_c$ is assumed a constant and γ is the bulk density of the soil. The following substitutions are made in Eqs. (4) and (5).

$$a = AD \quad (6a)$$

$$b = BD \quad (6b)$$

$$z = ZD \quad (6c)$$

$$h = HD \quad (6d)$$

$$q = Q \gamma D \quad (6e)$$

where A , B , Z , H and Q are non-dimensional parameters and D is a constant having unit of length. Following these substitutions, Eqs. (4) and (5) transform to,

$$\rho = \frac{D}{C} S_F \quad (7)$$

where S_F is a dimensionless settlement factor. S_F in Eq. (7) with Boussinesq and Mindlin stress distribution is respectively given by,

$$S_F = \int_0^\infty \log \left[\frac{2Q}{(Z+H)\pi} \left\{ \frac{ABZ(A^2+B^2+2Z^2)}{(A^2+Z^2)(B^2+Z^2)\sqrt{(A^2+B^2+Z^2)}} \right. \right. \\ \left. \left. + \sin^{-1} \frac{AB}{\sqrt{A^2+Z^2}\sqrt{B^2+Z^2}} \right\} + 1 \right] dZ \quad (8)$$

and

$$S_F = \int_H^\infty \log \left[\frac{Q}{\pi(1-\mu)} \left\{ \frac{(1-\mu)}{Z} \left\{ \operatorname{arctg} \frac{AB}{(Z-H)R_{11}} + \operatorname{arctg} \frac{AB}{(Z+H)R_{22}} \right\} \right. \right. \\ + \frac{(Z-H)AR_{11}}{2Br_{11}^2Z} - \frac{A(Z-H)^3}{2Br_{33}^2R_{11}Z} + \frac{\{(3-4\mu)Z(Z+H) - (5Z-H)H\}AR_{22}}{2(Z+H)Br_{22}^2Z} \\ - \frac{\{(3-4\mu)Z(Z+H)^2 - H(Z+H)(5Z-H)\}A}{2Br_{44}^2R_{22}Z} + \frac{2H(Z+H)AR_{22}^3}{B^3r_{22}^4} \\ \left. \left. + \frac{3HA R_{22} r_{55}^2}{(Z+H)B^3 r_{22}^2} - \frac{(Z+H)^3 A H}{B r_{44}^4 R_{22}} \left\{ \frac{2B^2 - (Z+H)^2}{B^2} - \frac{A^2}{R_{22}^2} \right\} \right\} + 1 \right] dZ \quad (9)$$

where

$$R_{11}^2 = A^2 + B^2 + (Z-H)^2$$

$$\begin{aligned}
r_{11}^2 &= A^2 + (Z-H)^2 \\
r_{33}^2 &= B^2 + (Z-H)^2 \\
R_{22}^2 &= A^2 + B^2 + (Z+H)^2 \\
r_{22}^2 &= A^2 + (Z+H)^2 \\
r_{44}^2 &= B^2 + (Z+H)^2 \\
\text{and} \\
r_{55}^2 &= B^2 - (Z+H)^2
\end{aligned}$$

Settlement of Strip Loaded Areas

The expression for settlement at the centre of strip loaded area is same as in Eq. (7), where S_F is the appropriate dimensionless settlement factor. S_F for strip loaded areas with Boussinesq and Mindlin stress distributions is respectively given by,

$$S_F = \int_0^\infty \log \left[1 + \frac{2Q}{(Z+H)\pi} \left(\frac{AZ}{A^2+Z^2} + \sin^{-1} \frac{A}{\sqrt{A^2+Z^2}} \right) \right] dZ \quad (10)$$

and

$$\begin{aligned}
S_F = \int_H^\infty \log \left[1 + \frac{Q}{Z\pi} \left(\arctg \frac{A}{Z-H} + \arctg \frac{A}{Z+H} + \frac{A(Z-H)}{2\{A^2+(Z-H)^2\}(1-\mu)} \right. \right. \\
\left. \left. + \frac{A\{(3-4\mu)Z+H\}}{2\{A^2+(Z+H)^2\}(1-\mu)} + \frac{2HZ(Z+H)A}{\{A^2+(Z+H)^2\}^2(1-\mu)} \right) \right] dZ \quad (11)
\end{aligned}$$

Effect of Rigidity of Foundation Structures

The theory dealt with in the preceding paragraphs concerns with stresses and settlements due to uniform loading of an area. However, foundations in practice tend to impose conditions closer to uniform displacement than to uniform loading. For accurate settlement predictions it is desirable to allow for rigidity of foundations. In elastic displacement theory an approximation to the uniform displacement is obtained from the maximum and minimum displacements of a uniformly loaded area (Davis and Poulos, 1968). This approximation is obtained in two stages. First, the rigid footing displacement is known to be close to mean displacement of the uniformly loaded area (Fox, 1948). Second, the approximate mean displacement is obtained by assuming the displaced surface under the loaded area to be parabolic, in which case the centre and edge or corner displacements define the mean displacement completely. These approximations give :

$$\text{For a rectangle, mean displacement} \approx 1/3(2\rho_{\text{centre}} + \rho_{\text{corner}}) \quad (12)$$

$$\text{For a strip, mean displacement} \approx 1/2(\rho_{\text{centre}} + \rho_{\text{edge}}) \quad (13)$$

Adopting the above approximations for consolidation settlement also the mean settlement, ρ_m , of the loaded area is given by,

$$\rho_m = \frac{D}{C} S_{Fm} \quad (14)$$

where S_{Fm} is the mean settlement factor.

$$\text{For a rectangle, } S_{Fm} \approx 1/3(2S_{F\text{-centre}} + S_{F\text{-corner}})$$

$$\text{For a strip, } S_{Fm} \approx 1/2(S_{F\text{-centre}} + S_{F\text{-edge}})$$

Eqs. (8), (9), (10) and (11) give expressions for $S_{F\text{-centre}}$. Similar expressions for $S_{F\text{-corner}}$ and $S_{F\text{-edge}}$ can be found for the two types of approaches for stress distribution using which the mean settlement factor can be finally computed.

RESULTS

Calculations for mean settlement factor for rectangular and strip loaded areas are made

for the two types of stress distribution. S_F values are evaluated by numerical integration of the expressions for S_F using Simpson's one-third rule formula. The settlement factors are calculated up to a Z value after which the addition in settlement factor due to the next segment does not exceed 0.001. S_F values are calculated keeping A to be unity. B is varied from 1 to 10, Q from 2 to 100 and H from 0.25 to 10. $B=1$ represents the case of a square loaded area. μ is taken as 0.5 for solutions with Mindlin stress. Variation in μ is found not to affect the values of S_F significantly. The value of Z after which the increase in settlement factor due to the next segment does not exceed 0.001 has been considered as the value for hundred per cent settlement to occur. Similarly calculations have been made for values of Z at which 95 and 90 per cent of the above settlement factor are reached. These values are respectively designated as Z_{100} , Z_{95} and Z_{90} for Boussinesq stress distribution but the value of H is deducted from the respective values in order to obtain the Z_{100} , Z_{95} and Z_{90} values for Mindlin stress distribution. Thus these values refer to Z values only below the level of load transfer.

Fig. 2 shows the variation of S_{Fm} , the mean settlement factor with Q for a square loaded

Fig. 2. S_{Fm} vs. Q for square loaded areas using Boussinesq stress distribution

Fig. 3. S_{Fm} vs. Q for square loaded areas using Mindlin stress distribution

Fig. 4. S_{Fm} vs. Q for strip loaded areas using (a) Boussinesq stress distribution and (b) Mindlin stress distribution

Fig. 5. S_{Fm} vs. H for the two types of stress distribution for square loaded areas

Fig. 7. Z vs. Q for strip loaded areas for the two types of stress distribution

(a)

(a) for Boussinesq stress distribution

(b)

(b) for Mindlin stress distribution

Fig. 6. Z/B vs. H for rectangular loaded areas

area for Boussinesq stress distribution, each curve in it being the variation for one value of H . Fig. 3 shows the respective results for a square loaded area when Mindlin stress distribution is applied. The results for strip loaded areas with Boussinesq stress distribution and Mindlin stress distribution are reported in Figs. 4 (a) and 4 (b) respectively. In Fig. 5 S_{Fm} versus H plot has been obtained for $B=1$, each curve in it being for one value of Q . The curves in full line in it are for Boussinesq stress distribution and those in dashed line are for Mindlin's solution. This helps to distinctly comprehend the influence of the type of stress distribution on S_{Fm} -values. Since the current practice is to use Boussinesq's type of stress distribution, the use of more appropriate Mindlin's type distribution could be accounted by a reduction factor F , defined as $S_{Fm}\text{-Mindlin}/S_{Fm}\text{-Boussinesq}$. The variation of F with H for $Q=10$ is reported as an inset in Fig. 5. From the Z_{100} value calculated for each case, the corresponding Z_{100}/B is calculated for rectangular loaded areas. The variation of Z_{100}/B with H is not found to be large, but is significant with Q .

This phenomenon of less variation with H and more with Q is found to be more marked as the value of B increases. Hence average values for Z_{100}/B (averaged over the range of values of H studied) have been adopted in Figs. 6 (a) and 6 (b) which show the variation of Z/B with B for square and rectangular loaded areas, each figure being for one type of stress distribution and each curve in full line in them shows the variation of Z_{100}/B for a particular value of Q . Similarly from the Z_{95} values calculated, the corresponding Z_{95}/B values have been calculated and are found not to significantly vary with both Q and H . Hence the average values of Z_{95}/B have been shown in dashed lines in Figs. 6 (a) and 6(b). The Z_{100} values for strip loaded areas have been not only found to be not affected by H but also are the same for both types of stress distribution. Fig. 7 shows the variation of Z_{100} with Q for strip loaded areas for both types of stress distribution. But the Z_{95} values are found to vary with Q and H and hence not adaptable for simple illustration.

DISCUSSION

From Fig. 5 and similar plots for rectangular and strip loaded areas it is found that for values of H below 1 there is no significant difference between the S_F values computed with Boussinesq's type of stress distribution and Mindlin's type of stress distribution, other conditions remaining the same. But beyond $H=1$ there is a considerable increase in the difference between S_F values computed with the two types of stress distribution. Hence the settlements computed based on Boussinesq's equation are likely to err appreciably on the conservative side. But the Mindlin's equation may also be said strictly not to be applicable to soils. In the elastic theory the half space takes up tension also, but in soils the tensile capacity is very limited. This tends to produce a stress distribution intermediate between those given by Boussinesq's equation and Mindlin's equation. Consequently the S_F values will be bounded by the values computed with Boussinesq stress distribution and Mindlin stress distribution with those for deep footings and pile groups approaching values with Mindlin stress distribution. It may be said that, for pile groups the S_F values with Mindlin's equation may be justifiably used since the conventional analyses neglect the skin friction and consider the entire load to be transferred through an equivalent footing at some level of the pile. Figs. 6 (a) and 6 (b) show that, while the settlement phenomenon may continue up to very large depths below loaded areas the major portion of the settlement occurs within a shallow depth. This depth is nearly four times the sides for square loaded areas, nearly six times the breadth (least dimension) for areas with side ratio 1 : 2 when 95% of their ultimate consolidation settlement is considered. These figures can be considered as theoretical guidelines to determine the depth of exploration. However, in practice only a very small fraction of these depths are being considered. Since the settlement is found to predominantly occur within a short depth below the loaded area the design curves given from Figs. 2 to 4 (developed for semi-infinite continuum) can be used for foundations on confined layers also when the hard stratum is at some large depth below the loaded area. The use of the curves in Figs. 2 to 4 to compute the consolidation settlement is illustrated with the following example.

Data: Size of footing $(2a \times 2b) = 4\text{m} \times 4\text{m}$.

$h = 4\text{m}$; $q = 10\text{t/m}^2$

$e_0 = 1.235$; $r = 1.7\text{t/m}^3$; $C_c = 0.32$

The depth of the stratum is assumed to extend to infinity.

Computation of settlement :

$a = A$, $D = 2\text{m}$.

Since $a = 1$, $D = 2\text{m}$.

Since $b=2$ m, $B=1$ Similarly $H=2$

$$Q = \frac{q}{\gamma D} = \frac{10}{1.7 \times 2} = 2.94$$

$$C = \frac{1+e_0}{C_c} = 6.98$$

To calculate the settlement using Boussinesq's equation for stress distribution:

From Fig. 2, $S_{Fm}=0.4$

$$\text{Mean settlement, } \rho_m = \frac{D}{C} \times S_{Fm} = 11.4 \text{ cm.}$$

From Fig. 6 (a), 95% of this settlement occurs within $\frac{Z_{95}}{B} = 7.75$

$$Z_{95} = Z_{95} D = 7.75 \quad BD = 15.5 \text{ m.}$$

To calculate the settlement using Mindlin's equation for stress distribution:

From Fig. 3, $S_{Fm}=0.275$

Mean settlement, $\rho_m = 7.9$ cm.

From Fig. 6 (b) the depth below the level of load transfer within which 95% of this settlement occurs is found to be 16.5 m.

NOTATION

a = half the breadth of the loaded area

A = non-dimensional breadth parameter ($=1$)

b = half the length of the loaded area

B = non-dimensional length parameter

C_c = compression index

$$C = \frac{1+e_0}{C_c}$$

D = non-dimensional parameter having unit of length

e_0 = average initial void ratio

h = depth of embedment of loaded area

H = non-dimensional depth of embedment parameter

m_v = coefficient of volume compressibility

P_z = overburden pressure at z

q = intensity of loading

Q = non-dimensional loading parameter

S_F = non-dimensional settlement factor

$S_{F\text{-edge}}$ = non-dimensional settlement factor at the edge of the loaded area

$S_{F\text{-centre}}$ = non-dimensional settlement factor at the centre of the loaded area

$S_{F\text{-corner}}$ = non-dimensional settlement factor at the corner of the loaded area

z = depth co-ordinate

Z = non-dimensional depth parameter

z_{100}, z_{95}, z_{90} = non-dimensional depth parameters for 100%, 95% and 90% settlement respectively

ρ = settlement

$\rho_{\text{centre}}, \rho_{\text{corner}}, \rho_{\text{edge}}$ = settlement of the loaded area at the centre, corner and edge respectively

ρ_m = mean settlement

σ_z = increase in vertical stress at depth z

σ_{zB} = increase in vertical stress at depth z according to Boussinesq's equation
 σ_{zM} = increase in vertical stress at depth z according to Mindlin's equation
 μ = Poisson's ratio
 γ = bulk density of soil

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