SHEAR MODULUS AND SHEAR STRENGTH OF COHESIVE SOILS

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ABSTRACT

The purpose of this paper is to evaluate the relation between initial shear modulus and shear strength of cohesive soils. The initial shear moduli were obtained from the results of the well-shooting tests by means of shear waves, while the shear strength could be obtained from the results of laboratory tests conducted on undisturbed soil samples collected at the same site as the well-shooting tests. Taking into consideration the fact that there have been a number of instances where the well-shooting test and the standard penetration test were conducted on the same sites, the authors conducted their research to seek some relationships between the initial shear modulus and the \textit{N}-value of the standard penetration test, between the shear strength and the \textit{N}-value, and between the initial shear modulus and the shear strength. The research works have finally led the authors to finding out several relations among initial shear modulus, shear strength and \textit{N}-value.

\textbf{Key words:} cohesive soil, elasticity, geophysical exploration, in-situ test, laboratory test, penetration test, shear strength

\textbf{IGC:} D7/D6

INTRODUCTION

In Japanese cities, for lack of suitable building sites, there has recently been a marked tendency to plan construction of large and tall buildings on soft grounds which were never considered in the past as a building site. Every structure is influenced by the local ground behavior to a considerable extent during earthquakes, especially when built on a soft ground. Since any structure built on such a ground is inevitably subject to considerable displacement, particularly at its foundation, it is desirable that an earthquake-proof design follows a dynamic analysis by means of a simulation model of structure-foundation-ground system. Needless to mention, understanding by experiment of the dynamic properties of soils is required for such a dynamic analysis. These properties are characterized by the following items:

1. Initial shear modulus
2. Shear strength
3. Non-linear characteristics
4. Damping characteristics

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In order to research the above problems, it is desirable to make the best of the merits of the in-situ test and the laboratory test. By in-situ tests we are able to grasp directly the mechanical properties of soils at natural soil deposit, but the test conditions are not always clear. On the other hand, conditions of the laboratory test are clear, and the laboratory test enables us to obtain the mechanical properties at large deformation, but its weak point is disturbance of soil structure during the sampling and testing processes.

Therefore the in-situ and the laboratory tests have their merits and demerits. In order to take advantage of the merits, the authors took notice of the following items:

1. There are many cases where soil stiffness increases in general with strength according to the experience of laboratory or in-situ test.

2. Wilson and Dietrich presented many data on Young's modulus $E$ and undrained shear strength $S_u$ by the laboratory test. Furthermore, D'Apporona et al. showed a number of data on Young's modulus $E$ obtained from initial settlement curve and undrained shear strength $S_u$. It is recognized that the values of ratio $E/S_u$ calculated from their data are within a certain limited region.

3. The shear strength of soil can be calculated from the values of effective overburden pressure, angle of internal friction and cohesion, and the angle of internal friction depends upon the value of plasticity index, that is, it is affected by a number of factors. Furthermore, both dimensions of initial shear modulus and of shear strength are same. It is not without reason, therefore, to consider that initial shear modulus is closely related to shear strength.

The authors have tried, from the above mentioned reasons, to review the relation between initial shear modulus and shear strength of cohesive soils on the basis of the results of their own in-situ test (well-shooting test, standard penetration test) and laboratory test (triaxial compression test).

INITIAL SHEAR MODULUS OF SOILS

Well-Shooting Method

Miller et al. showed that the geophysical exploration test comprises several types of practical methods (cross-hole, down-hole, up-hole, surface refraction and surface-vibration methods).

The down-hole method is to generate shear waves by hitting horizontally a wooden plate placed on the ground surface, so that velocity transducers clamped in a borehole receive such shear waves. To say more precisely, several velocity transducers are concurrently clamped in the borehole, which allow velocity measurement of a shear wave passing through a section between one transducer and the another.

When the shear wave velocity $V_s$ is measured, shear modulus $G$ can be calculated by the following equation:

$$G = \rho V_s^2 \text{(kg/cm}^2\text{)}$$

where $\rho$: mass density.

This is the procedure called the well-shooting test by means of shear waves, on which Katsuizekai, Shima et al. and Imai et al. have already published their respective research papers.

Shear Strain Amplitude in Soil Deposits

Initial shear modulus can be defined as shear modulus at infinitely small shear strain amplitude. It is therefore necessary to obtain shear strain amplitude in soil deposits when the well-shooting test by means of shear waves is to be conducted.

The result of the well-shooting test which was performed in a soft clay deposit, as an example, is shown in Fig.1. The authors tried to calculate shear strain amplitude in a soft soil deposit from wave forms shown in Fig.1 with the following procedure:

1. Calibration curves of the velocity transducers were firstly determined by shaking table
in the laboratory.

(2) On the assumption that principal shear wave should be represented by that of harmonic motion, the maximum shear displacement amplitude \( \Delta x \) was estimated by the use of the calibration curves to be the following value:

\[
\Delta x = 0.0000144 \text{ (m)} \tag{2}
\]

(3) On the other hand, thickness of soil deposit \( \Delta H \) subject to shear displacement of \( \Delta x \) can be calculated as follows:

\[
\Delta H = V_s \Delta t = 90 \times 0.008 = 0.72 \text{ (m)} \tag{3}
\]

where \( V_s \): shear wave velocity.

(4) Shear strain amplitude \( \tau \) can thus be calculated as follows:

\[
\tau = \frac{1.44 \times 10^{-6}}{0.72} = 2.0 \times 10^{-6} \tag{4}
\]

Shear strain amplitude obtained in the above procedure are plotted in Fig. 2, which shows that shear strain amplitude thus obtained by the well-shooting test can be assumed to be less than \( 10^{-8} \) percent.

In addition, when shear strain amplitude is \( 10^{-8}\% \), shear stress as calculated below is so small that the shear modulus obtained by the well-shooting test can be regarded as initial shear modulus:

\[
\tau = \gamma G = 10^{-8} \times 110 = 0.0011 \text{ (kg/cm}^2) \tag{5}
\]

Shear Strength of Soil

The soil element shown in Fig. 3. is deformed due to random vibrations propagating from bedrock to surface during an earthquake. In this case, the deformation excited most significantly is mainly attributable to the shear wave.

The initial stress condition of the soil element is the \( K_0 \)-condition, which means that the initial effective stresses in vertical and horizontal direction are \( \sigma'_v \) and \( K_0 \sigma'_v \) respectively.

The shear modulus obtained from the well-shooting test is calculated from the velocity of the shear wave, which propagate through the soil elements vertically in the \( K_0 \)-condition.
Fig. 3. Conceptual field loading condition

Fig. 4. Mohr's circles and maximum shear stress

Therefore, it is reasonable that the shear strength of a soil deposit should be obtained by taking into account its initial stress condition. These problems have been sufficiently treated by Duncan\(^7\), Seed\(^8\) and Hardin\(^9\). According to these researches, when the soil element shown in Fig. 3 comes to failure, the maximum lateral shear stress \((\tau_{hv})_{\text{max}}\) is given on Mohr's circles as represented in Fig. 4. Taking the geometric relation in Fig. 4 into consideration, the maximum lateral shear stress \((\tau_{hv})_{\text{max}}\) will be

\[
S_v = (\tau_{hv})_{\text{max}} = \left\{ \left( \tau_{\text{max}} \right)^2 - \left( \frac{1-K_3}{2} \sigma' \right) \right\}^{1/2}
\]

where \(K_3\): coefficient of earth pressure at rest
\(\sigma'_v\): effective overburden pressure.
In addition, if the cohesion \( c \) and the angle of internal friction \( \phi \) of soil are given, the maximum shear stress \( \tau_{\text{max}} \) can be expressed by the following equation:

\[
\tau_{\text{max}} = \frac{1 + K_o}{2} \sigma_v \sin \phi + c \cos \phi
\]  

(7)

Substituting Eq. (7) into Eq. (6), the shear strength \( S_u \) of the soil element shown in Fig. 4 can be given as follows:

\[
S_u = \left( \frac{1 + K_o}{2} \sigma_v \sin \phi + c \cos \phi \right)^{1/2} - \left( \frac{1 - K_o}{2} \sigma_v \right)^{1/2}
\]  

(8)

However, the actual shear strength in soil deposit cannot be expressed so easily as above. For example, the process of development of the pore water pressure in the soil element vary with respective earthquakes, so that the effective overburden pressure and coefficient of earth pressure at rest are not constant, varying from the state of rest to the state of failure. The shear strength in a soil deposit cannot be exactly obtained for the above reason. Accordingly, it is realistic that the shear strength of a soil is calculated based on the assumption that the initial stress condition continues to the state of failure. The soil parameters in Eq. (8) are determined by the following conditions:

1. The cohesion \( c \) and the angle of internal friction \( \phi \) are determined by the expression of total stress of the unconsolidated-undrained triaxial compression test.
2. Brooker and Ireland\(^{19}\) have reported that the close relation were observed among the coefficient of earth pressure at rest \( K_o \), the plasticity index \( I_p \), and the overconsolidated ratio OCR as shown in Fig. 5. The value of \( K_o \) to be required in Eq. (8) was determined by the relation between \( I_p \) and OCR shown in Fig. 5.

**Soil Classification**

The data on initial shear modulus, shear strength and \( N \)-value of the standard penetration test were obtained from 25 sites, which consisted of 15 alluvial deposits, 9 diluvial deposits, and 1 tertiary deposit.

In order to describe the physical properties of soils used for this investigation, the triangular classification chart for the results of mechanical analysis is shown in Fig. 6, the results of plasticity and liquid limit test are shown in Fig. 7 by the plasticity chart, while the relation between overconsolidation ratio and plasticity index are shown in Fig. 8, and the relation between degree of saturation and void ratio in Fig. 9.

![Fig. 5. Relationship between \( K_o \), \( I_p \) and OCR (From Brooker and Ireland, 1965)](image)

![Fig. 6. Triangular classification chart](image)
Fig. 7. Relation between $I_p$ and $W_L$.

Fig. 8. Relation between OCR and $I_p$.

Fig. 9. Relationship between $S_r$ and $e$. 

$\pi_p = 0.73(W_L - 20)$
The physical properties of the soil used in this paper are summarized as follows from Figs. 6, 7, 8 and 9:

1. Soil belongs to the category of cohesive soil.
2. The plasticity index has a wide range of distribution.
3. The values of overconsolidation ratio range from 1.0 to 3.0.
4. The values of void ratio range from 0.5 to 3.0, and most of the data are from 1.0 to 2.0.
5. The values of the degree of saturation range from 90% to 100%, and most of the data nearly 100%.

**INITIAL SHEAR MODULUS AND N-VALUE**

The well-shooting test is gradually being adopted widely, because it is proven to be provided with such advantageous features that:

1. eliminate any necessity of securing an extensive area for measurement;
2. ensure higher accuracy of measurement, compared with surface refraction method;
3. allow wave velocity measurement in a soft soil layer, even if there exists a hard layer above it.

A review of the data on the well-shooting tests conducted in the past has led the authors to take notice that there have been a number of instances where the well-shooting test and the standard penetration test were conducted on the same site. It is therefore necessary to make an investigation into the relationship between the shear modulus and the N-value of soil deposits obtained respectively by the well-shooting test and the standard penetration test, so as to get a better knowledge of elastic properties of soil. When the value of initial shear modulus and N-value are to be sampled from the existing data, it is necessary, however, to set up certain criteria to avoid any partial data sampling.

**Criteria of Data Sampling**

In the case of the authors' investigation, criteria of data sampling were set up so that such data as might meet the following requirements could only be adopted:

1. Soil has to fall under the category of cohesive soil.
2. Shear wave velocity has to be measured by way of the well-shooting test.
3. Uniform section of shear wave velocity values and N-values has to be more than 2.0 m, as shown in Fig. 10 “An Example of Data Sampling.”

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**Fig. 10. An example of data sampling**

**Fig. 11. Relationship between \( G_0 \) and N-value**
(4) The well-shooting test and the standard penetration test have to be conducted on the same site.

Relation between Initial Shear Modulus and $N$-Value

The data on initial shear modulus $G_0$ and $N$-value thus sampled in conformity with the above criteria of data sampling are plotted on a logarithmic graph as shown in Fig. 11. Fig. 11 shows that $G_0$ and $N$-value is in approximate proportion to each other on the logarithmic graph.

Accordingly, on the assumption that the first order equation should be $G_0 = aN^b$, the following empirical formula was obtained by calculating $a$ and $b$ by the least squares method:

\[ G_0 = 158N^{0.648} \text{(kg/cm}^2\text{)} \]  \hspace{1cm} (9)

where coefficient of correlation $\rho_{xy} = 0.88$.

Comparison with Other Investigations

Ohsaki and Iwasaki\textsuperscript{11} have evaluated the relation between $G_0$ and $N$-value by arranging the data obtained from well-shooting tests and standard penetration tests, which were collected by the Evaluation Committee on High-Rise Building Structures, the Building Center of Japan. With classification of soil into cohesive soil and sandy soil with a view to facilitating evaluation of the relation between $G_0$ and $N$-value, they have shown for cohesive soils that the following empirical formula existed between $G_0$ and $N$-value:

\[ G_0 = 140N^{0.735} \text{(kg/cm}^2\text{)} \]  \hspace{1cm} (10)

where coefficient of correlation $\rho_{xy} = 0.90$. Graphical expression of Eqs. (9) and (10) show that there is quite a similarity between them, as illustrated in Fig. 11.

SHEAR STRENGTH AND $N$-VALUE

Now that the relation between shear modulus $G_0$ and $N$-value of the standard penetration test has been evaluated in the preceding section, the relation between shear strength $S_u$ and $N$-value has to be evaluated, too, in a similar way.

Shear strength $S_u$ can be obtained from Eq. (8) by use of the results of triaxial compression test on undisturbed soil samples. When the relation between shear strength and $N$-value is to be evaluated, particular attention has to be paid to soil sampling practice and accuracy of soil test. With this taken into account, the authors have sampled data solely from such basic soil exploration reports as considered to be under stringent supervision for securing as high accuracy as possible of soil test and, especially, the results of such soil tests as were conducted with special attention paid to sampling method\textsuperscript{13} or by use of block samples, since soil samples are easy to be disturbed, if $N$-value exceeds 10.

Criteria of Data Sampling

Criteria of data sampling was set up so that the following requirements could be satisfied:

1. Soil has to fall under the category of cohesive soil.
2. Soil sampling and standard penetration test have to be carried out in the same borehole.
3. Any data with $N$-value of less than 1.0 shall be rejected, because there exists no coefficient of correlation between shear strength and $N$-value, if the latter is less than 1.0.

Relation between Shear Strength and $N$-Value

Just as in the preceding section, the data on shear strength and $N$-value thus sampled are plotted on a logarithmic graph shown in Fig. 12. On the assumption that the first order equation should be $S_u = aN^b$, the following empirical formula was obtained by calculating $a$ and $b$ the method of least squares:
where coefficient of correlation: $\rho_{xy} = 0.93$.

**Comparison with Some Other Investigations**

The shear strength of clay can be approximately obtained from the unconfined compression strength $q_u$ by the following equation.

$$S_u = \frac{1}{2} q_u \text{(kg/cm}^2\text{)}$$  \hspace{1cm} (12)

Mikasa\[15\] has shown the range of $N$-value and unconfined compression strength of cohesive soil on the basis of his investigation into typical cohesive soil layers in Osaka district. Similarly, Yanase\[16\] has shown many data on $N$-value and unconfined compression strength of cohesive soil by way of his test conducted on such soil samples as considered to have been least disturbed during sampling. These data are plotted in Fig. 13 together with graphical expression of Eq. (11), which is expressed as a straight line passing near the center of those data shown by Mikasa and Yanase.

**INITIAL SHEAR MODULUS AND SHEAR STRENGTH**

So far, the authors' effort has been directed toward evaluating relationship between $N$-value of the standard penetration test and initial shear modulus or shear strength of cohesive soil.

In this section, however, their effort is to be directed solely to evaluating direct relation between initial shear modulus $G_0$ and shear strength $S_u$. It is to be noted here that criteria of data sampling can be more moderate in this section, because of reduction in number of data on initial shear modulus and $N$-value, compared with the previous sections.

The data for use in this section are from one of the investigations made in the past by other researchers\[9\] and such data sources as Kajima Institute of Construction Technology by which the authors are employed, involving 20 sites and 194 data in total.

**Criteria of Data Sampling**

Criteria of data sampling was set up so that the following requirements could be met:

1. Soil has to fall under the category of cohesive soil.
2. Shear wave velocity has to be measured by way of the well-shooting test.
(3) Values of $G_0$ and $S_u$ have to be those obtained by experiment at the same site and in the same depth. If there exist several values for $S_u$, corresponding number of data should be made available by combining each value of $S_u$ with the corresponding value of $G_0$.

**Relation between Initial Shear Modulus and Shear Strength**

Data on initial shear modulus $G_0$ and shear strength $S_u$ are plotted on a logarithmic graph shown in Fig. 14. On the assumption that as a first approximation equation should be $G_0 = aS_u^b$ and taking the data plotted in Fig. 14 into account, the following empirical formula was obtained by calculating $a$ and $b$ by the method of least squares:

$$G_0 = 516S_u^{1.012} \text{ (kg/cm}^2)$$  \((13)\)

where coefficient of correlation $\rho_{xy} = 0.95$.

Exponent of $S_u$ in the Eq. (13) is 1.012 or approximately equal to 1.0. $G_0$ can therefore be considered to be in proportion to $S_u$.

![Fig. 14. Relationship between $G_0$ and $S_u$](image1)

**Investigation of Normalized Shear Modulus**

Now that $G_0$ is confirmed to be in proportion to $S_u$, it is convenient to define the normalized shear modulus $G_n$ as follows:

$$G_n = \frac{G_0}{S_u}$$  \((14)\)

Fig. 15 shows a histogram of the normalized shear modulus drawn by calculating values of $G_n$ from the data plotted in Fig. 14. The value of $G_n$ are in the region ranging from 250 to 1430, with the mean value of 548 and the standard deviation of 211.

For comparison, the following values of $G_n$ can be quoted independently from the data obtained by Hardin et al., Wilson et al., and D'Appolonia et al.

1. Hardin et al.\(^9\) have obtained the initial shear modulus and the shear strength of the same cohesive soil by means of a resonant-column apparatus in the laboratory. The normalized shear modulus $G_n$ calculated from these values is ranging from 380 to 1500 with its mean value of about 760.

2. Wilson and Dietrich\(^9\) have obtained the values of initial Young's modulus, initial shear modulus and compressible strength by measuring a longitudinal or torsional natural
frequency of clay samples. The value of $G_n$ can be calculated from these measured data by assuming the Poisson's ratio as 0.5, and it is in the region ranging from 178 to 550 with its mean value of about 390.

(3) D'Appolonia et al. have shown eleven combinations of data on the initial Young's modulus and the shear strength by in-situ and laboratory tests.

The value of $G_n$ calculated from these values by assuming the Poisson's ratio as 0.5 is in the region ranging from 53 to 833 with its mean value about 420.

The results of these investigations are shown in Fig. 15 for comparison with the authors' result. The histogram in Fig. 15 shows that the peaks of frequency distribution curves are in the region ranging from 400 to 500 except Hardin's data.

**DISCUSSION AND CONCLUSIONS**

The standard penetration test is easy and less expensive to conduct mainly because of its standardization, though it has a drawback in rather low accuracy. Investigation into soil deposits by way of the standard penetration test allows a three-dimensional grasp of their structure, if the test is conducted as many times as possible, so that as many test data as possible can be obtained.

With due attention paid to such advantageous feature of the standard penetration test, the authors have finally succeeded in introducing Eqs. (9) and (11) by arranging in a statistical way the data on initial shear modulus $G_n$, shear strength $S_n$ and $N$-value of the standard penetration test.

On the other hand, the authors have also evaluated the relation between $G_n$ and $S_n$, so that it turns out to be expressed in Eq. (13). Elimination of $N$ from the Eqs. (9) and (11) allows the relation between $G_n$ and $S_n$ to be expressed in the following equation:

$$G_n = 487 S_n^{0.928} \text{(kg/cm}^2)$$

Eq. (15) is represented in Fig. 14, so that it can be compared with Eq. (13). A close resemblance existing between the two equations is self-explanatory of mutual relationship among initial shear modulus, shear strength and $N$-value.

The investigations covered in this paper are summarized as follows.

(1) Shear strain amplitude generated in a very soft soil deposit by the well-shooting test was in general found less than $10^{-4}\%$. It follows from this that shear modulus obtained by in-situ tests can be regarded as the initial shear modulus.

(2) As a result of investigating the mutual relationship among initial shear modulus, shear strength and $N$-value, it became clear that the initial shear modulus of cohesive soil is proportional to its shear strength obtained under the $K_s$-condition.

(3) The ratio between the initial shear modulus and the shear strength can be estimated to be about 500 from Eq. (13) and Fig. 15.

(4) It is simple and convenient if the initial shear modulus is estimated from $N$-value. However, we should not resort to $N$-value only, because errors of the standard penetration test become large, especially when $N$-value is less than 2.

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NOTATION

\( S_u \) = undrained shear strength in kg/cm²
\( \sigma_v' \) = vertical effective stress in kg/cm²
\( K_o \) = coefficient of lateral pressure at rest

\( OCR \) = overconsolidation ratio

\( C \) = cohesion obtained from unconsolidated-undrained triaxial compression test in kg/cm²
\( \phi \) = angle of internal friction obtained from unconsolidated-undrained test in degrees
\( N \) = number of blows per foot in the standard penetration test
\( G_s \) = initial shear modulus in kg/cm²
\( V_s \) = shear wave velocity in m/sec
\( \rho \) = coefficient of correlation
\( G_n \) = normalized shear modulus (\( G_s / S_u \))

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