DILATANCY CHARACTERISTICS OF SOIL

HAJIME MATSUOKA*

ABSTRACT

The formulas of the relationship between stress ratios and strains due to dilatancy derived from the microscopic analysis based on observation of the particulate nature of soil (Matsuoka, 1973; Matsuoka, 1974) have been verified with respect to the measured data of drained triaxial compression and extension tests on sand and clay. From the test results, it is judged that dilatancy of soil may be governed uniquely by the shear-normal stress ratio (τ/σ_N) on the mobilized planes or the principal stress ratios (σ_1/σ_3 , σ_1/σ_2 , σ_2/σ_3) as indicated by the above formulas. This concept differs from the view advocated by Shibata, Karube et al. (1963, 1964, 1965) that the volumetric strain caused by dilatancy is governed by the principal stress difference vs. mean effective principal stress ratio ($(\sigma_1 - \sigma_3)/\sigma_m \equiv q/p$). Further, it has been demonstrated that the stress-strain relationship in drained triaxial compression tests under a constant lateral pressure (σ_3) and the stress path in undrained triaxial tests can be calculated by assuming the superposition of the above-mentioned strains due to dilatancy and the strains due to consolidation. Thus, it is inferred that dilatancy of soil as well as shear resistance (strength) may obey Coulomb's law of friction.

Key words: clay, dilatancy, drained shear, sand, soil structure, stress path, stress-strain curve, triaxial compression test

IGC: D6/D3

INTRODUCTION

Dilatancy is one of the most essential phenomena of soil relating to the particulate nature. Dilatancy of clay was investigated by Shibata (1963), Shibata and Karube (1964, 1965a, 1965b), and Karube and Kurihara (1966). The principal stress difference versus mean effective principal stress ratio ($(\sigma_1 - \sigma_3)/\sigma_m$) was proposed by them as a parameter governing dilatancy of clay. This parameter was also investigated with respect to sand by Murayama and Yagi (1964, 1965), Yagi (1967, 1970) and others.

The formulas of the stress-strain relationships proposed by the author (1973, 1974) suggest that the volumetric strain due to dilatancy is governed by the principal stress ratios (σ_1/σ_3 , σ_1/σ_2 , σ_2/σ_3) or the shear-normal stress ratio (τ/σ_N) on the mobilized planes. This variable τ/σ_N (or σ_1/σ_3) is different from the above-mentioned parameter $(\sigma_1 - \sigma_3)/\sigma_m (\equiv q/p)$. Thus, the measured data of the volumetric strain caused by dilatancy in drained triaxial compression and extension tests on sand and clay will be compared with these two variables. Further, the stress paths in undrained tests will be calculated on condition that the sum of the volumetric strain due to dilatancy and that due to consolidation is zero, and will be verified with respect to the measured values of undrained triaxial compression and extension tests on sand and clay.

* Assistant Professor, Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto.
Written discussions on this paper should be submitted before July 1, 1975.

PREPARATION OF TEST SPECIMENS

Sagami River sand and Fujinomori clay were investigated here. Data of Sagami River sand are those published by Tatsuoka and Shiba (1972) and Tatsuoka and Ishihara (1973). The sand has the average grain size of 0.42mm, the uniformity coefficient of 2.2, the specific gravity of 2.68, and the maximum and the minimum void ratios of 1.08 and 0.53, respectively. The test specimens were formed in a cylindrical shape, 5cm in diameter and 10cm in height. To minimize possible friction on the upper and lower end faces of the test specimen, rubber membranes coated with silicone grease are inserted (Tatsuoka and Shiba, 1972).

Fujinomori clay is available commercially in the form of dry powder at Fujinomori in Kyoto. This clay was kneaded with water and then subjected to preliminary consolidation (under a pressure of about 0.5kg/cm²) prior to use. The clay has L.L. of 44%, P.L. of 26% and the specific gravity of 2.68. In terms of the triangular classification chart, it falls in the category of silty clay loam. The test specimens were prepared in cylinders, 3.5cm in diameter and 8cm in height. Also to minimize the friction on the upper and lower end faces, rubber membranes coated with silicone grease were inserted between the loading cap and the upper end of the test specimen and between the lower end of the test specimen and an Araldite disc mounted on a porous stone, respectively. To provide a test piece with drainage, not only the lateral face of the test specimen but also that of the porous stone were wrapped by filter paper. In the drained triaxial tests on clay, the shear speed is fixed at a sufficiently low level of 7200.01 min/10 mm.

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Consolidation is considered to be the phenomenon of volumetric change due to the change of mean effective principal stress, and dilatancy to be that caused by the change of effective principal stress ratio. In the field of soil mechanics, it has been customary for the convenience of treatment to regard the strain of soil as summation of the strain due to consolidation and the strain due to shear (or dilatancy). The formulas of the principal strains ϵ_1 , ϵ_2 and ϵ_3 derived in the previous papers (Matsuoka, 1972; Matsuoka, 1973; Murayama and Matsuoka, 1973; Matsuoka, 1974) are considered to represent the principal strains $(\epsilon_1)_d$, $(\epsilon_2)_d$ and $(\epsilon_3)_d$ due to dilatancy in view of the basic concept underlying the derivation of the said formulas. They are restated below in a general form under three different principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$).

$$\left. \begin{aligned} (\epsilon_1)_d &= f\left(\frac{\sigma_1}{\sigma_3}\right) + f\left(\frac{\sigma_1}{\sigma_2}\right) \\ (\epsilon_2)_d &= f\left(\frac{\sigma_2}{\sigma_3}\right) + g\left(\frac{\sigma_1}{\sigma_2}\right) \\ (\epsilon_3)_d &= g\left(\frac{\sigma_1}{\sigma_3}\right) + g\left(\frac{\sigma_2}{\sigma_3}\right) \end{aligned} \right\} \quad (1)$$

where the following relationships are assumed to hold for $X \equiv \sqrt{\sigma_i/\sigma_j} - \sqrt{\sigma_j/\sigma_i}$ ($i, j=1, 2, 3, i \leq j$).

$$\begin{aligned} f\left(\frac{\sigma_i}{\sigma_j}\right) &= \frac{\gamma_0 \cdot \exp\left(-\frac{\mu}{\mu' - \mu}\right)}{2} \cdot \exp\left\{\frac{X}{2(\mu' - \mu)}\right\} \\ &\quad \cdot \left\{\frac{X^2}{8} + \left(\frac{1}{2} - \frac{1}{\lambda} - \frac{\mu' - \mu}{2}\right) \cdot X + (\mu' - \mu)^2 - (\mu' - \mu) + \frac{2\mu'}{\lambda} + 1\right\} \quad (i < j) \\ g\left(\frac{\sigma_i}{\sigma_j}\right) &= \frac{\gamma_0 \cdot \exp\left(-\frac{\mu}{\mu' - \mu}\right)}{2} \cdot \exp\left\{\frac{X}{2(\mu' - \mu)}\right\} \\ &\quad \cdot \left\{-\frac{X^2}{8} + \left(\frac{1}{2} - \frac{1}{\lambda} + \frac{\mu' - \mu}{2}\right) \cdot X - (\mu' - \mu)^2 - (\mu' - \mu) + \frac{2\mu'}{\lambda} - 1\right\} \quad (i < j) \end{aligned}$$

$$f\left(\frac{\sigma_i}{\sigma_j}\right) = g\left(\frac{\sigma_i}{\sigma_j}\right) = 0 \quad (i=j)$$

In the case of a triaxial compression test ($\sigma_1 \geq \sigma_2 = \sigma_3$), therefore, the principal strains and the volumetric strain caused by dilatancy can be expressed as follows:

$$(\epsilon_1)_d = 2f\left(\frac{\sigma_1}{\sigma_3}\right), \quad (\epsilon_2)_d = (\epsilon_3)_d = g\left(\frac{\sigma_1}{\sigma_3}\right) \quad (2)$$

$$\left(\frac{\Delta V}{V}\right)_d = (\epsilon_1)_d + 2(\epsilon_3)_d = 2f\left(\frac{\sigma_1}{\sigma_3}\right) + 2g\left(\frac{\sigma_1}{\sigma_3}\right) \quad (3)$$

In the case of a triaxial extension test ($\sigma_1 = \sigma_2 \geq \sigma_3$), the principal strains and the volumetric strain due to dilatancy are to be expressed as follows:

$$(\epsilon_1)_d = (\epsilon_2)_d = f\left(\frac{\sigma_1}{\sigma_3}\right), \quad (\epsilon_3)_d = 2g\left(\frac{\sigma_1}{\sigma_3}\right) \quad (4)$$

$$\left(\frac{\Delta V}{V}\right)_d = 2(\epsilon_1)_d + (\epsilon_3)_d = 2f\left(\frac{\sigma_1}{\sigma_3}\right) + 2g\left(\frac{\sigma_1}{\sigma_3}\right) \quad (5)$$

The preceding expressions indicate that the volumetric strain due to dilatancy in the case of triaxial compression and extension tests can be uniquely expressed as a function of the principal stress ratio (σ_1/σ_3) and, more generally, the shear-normal stress ratio (τ/σ_N) on the mobilized planes. This variable τ/σ_N (or σ_1/σ_3) is different from the ratio of principal stress difference to mean effective principal stress, $(\sigma_1 - \sigma_3)/\sigma_m (\equiv q/p)$ which has been proposed by Shibata (1963), Shibata and Karube (1964, 1965a, 1965b), Karube and Kurihara (1966) as a parameter governing the dilatancy of clay. The parameter $(\sigma_1 - \sigma_3)/\sigma_m$ has also been investigated with respect to sand by Murayama and Yagi (1964, 1965), Yagi (1967, 1970) and Tatsuoka and Shiba (1972). Besides, Hata and Ohta (1968), and Ohta (1971) have derived stress-strain relationships of soil by assuming that the dilatancy of soil is expressed in a linear relationship with respect to this parameter $(\sigma_1 - \sigma_3)/\sigma_m$.

Figs. 1 and 2 represent the data (Tatsuoka and Shiba, 1972; Tatsuoka and Ishihara,

Fig. 1. Relationship between $(\Delta V/V)_d$ and $(\sigma_1 - \sigma_3)/\sigma_m$ in triaxial compression and extension tests on Sagami River sand ($\sigma_m = 1.0 \text{ kg/cm}^2$)

Fig. 2. Relationship between $(\Delta V/V)_d$ and $(\sigma_1 - \sigma_3)/\sigma_m$ in triaxial compression and extension tests on Sagami River sand ($\sigma_m = 3.0 \text{ kg/cm}^2$)

1973) of triaxial compression tests ($\circ$ marks; $\sigma_m=1.0, 3.0 \text{ kg/cm}^2$, loose) and triaxial extension tests ($\bullet$ marks; $\sigma_m=1.0, 3.0 \text{ kg/cm}^2$, loose) on Sagami River sand as plotted in terms of the relationship between $(\Delta V/V)_d$ and $(\sigma_1 - \sigma_3)/\sigma_m$. Fig. 1 shows the results of the tests under $\sigma_m=1.0 \text{ kg/cm}^2$. In the diagram, the solid lines represent the curves of values calculated by Eqs. (3) and (5). In the related calculations, the coefficients $\lambda=1.2$, $\mu=0.28$, $\mu'=0.46$ proper to Sagami River sand and $\gamma_0=0.13\%$ for triaxial compression and $\gamma_0=0.3\%$ for triaxial extension were used. Fig. 2 shows the results of the tests under $\sigma_m=3.0 \text{ kg/cm}^2$. In the diagram, the curves of calculated values were obtained by presuming $\gamma_0=0.46\%$ for triaxial compression and $\gamma_0=0.59\%$ for triaxial extension, respectively.

When the same data as those of Figs. 1 and 2 are rearranged in terms of the relationship between $(\Delta V/V)_d$ and τ/σ_N (corresponding to σ_1/σ_3) according to Eqs. (3) and (5), the results are as shown in Fig. 3. This figure represents the results of triaxial compression tests ($\circ$ and $\triangle$ marks; loose and medium; $\sigma_m=\text{const.}$) and triaxial extension tests ($\bullet$ marks; loose; $\sigma_m=\text{const.}$) on the same Sagami River sand. The numbers in the diagram denote the values of $\sigma_m(\text{kg/cm}^2)$. In this diagram, it is observed that the curves of dilatancy $(\Delta V/V)_d$ are inverted from the compression direction to the expansion direction at practically the same value of τ/σ_N . Further, the curves in which the maximum compression value of $(\Delta V/V)_d$ is the same are considered to represent a test specimen with an equal value of γ_0 . It is interesting to note that, in such a case, both triaxial compression and triaxial extension describe practically the same curves, supporting the appropriateness of Eqs. (3) and (5). In Fig. 4, representative data of compression ($\sigma_m=3.0 \text{ kg/cm}^2$) and extension ($\sigma_m=2.0 \text{ kg/cm}^2$), and those of compression ($\sigma_m=2.0 \text{ kg/cm}^2$) and extension ($\sigma_m=1.0 \text{ kg/cm}^2$), respectively, selected from the data of Fig. 3 are compared with the curves of values (indicated by solid lines) calculated in accordance

Fig. 3. Relationship between $(\Delta V/V)_d$ and τ/σ_N in triaxial compression and extension tests on Sagami River sand

Fig. 4. Theoretical and experimental relationship between $(\Delta V/V)_d$ and τ/σ_N

with Eqs. (3) and (5). In these calculations, the coefficients $\lambda=1.2$, $\mu=0.28$, $\mu'=0.46$ proper to Sagami River sand and $\tau_0=0.3\%$ and $\tau_0=0.46\%$ were used. Comparison of these two diagrams with Figs. 1 and 2 clearly indicates that the dilatancy of sand is uniquely controlled not by $(\sigma_1-\sigma_3)/\sigma_m (\equiv q/p)$ but by the ratio of τ/σ_N (or σ_1/σ_3). This relationship is predicted also from Eqs. (3) and (5).

Here, the author tries to calculate the value of τ/σ_N (or σ_1/σ_3) at the maximum compression point (minimum point) of dilatancy. At the minimum point of dilatancy, the equation $d(\Delta V/V)_d = d(\epsilon_1)_d + 2d(\epsilon_3)_d = 0$ is established for triaxial compression and the equation $d(\Delta V/V)_d = 2d(\epsilon_1)_d + d(\epsilon_3)_d = 0$ is established for triaxial extension. In the previous papers (Matsuoka, 1973; Matsuoka, 1974), the following formulas have been derived for triaxial compression and triaxial extension, respectively.

$$\frac{d\epsilon_3}{d\epsilon_1} = \frac{1}{2} \cdot \frac{\sigma_1/\sigma_3 - 2\mu \cdot \sqrt{\sigma_1/\sigma_3} + (\lambda-1)}{(1-\lambda) \cdot \sigma_1/\sigma_3 - 2\mu \cdot \sqrt{\sigma_1/\sigma_3} - 1} \quad (6)$$

$$\frac{d\epsilon_3}{d\epsilon_1} = 2 \cdot \frac{\sigma_1/\sigma_3 - 2\mu \cdot \sqrt{\sigma_1/\sigma_3} + (\lambda-1)}{(1-\lambda) \cdot \sigma_1/\sigma_3 - 2\mu \cdot \sqrt{\sigma_1/\sigma_3} - 1} \quad (7)$$

Therefore, the following expression for the minimum point of dilatancy can be derived from the above formulas.

$$\frac{\sigma_1}{\sigma_3} + \frac{4\mu}{\lambda-2} \cdot \sqrt{\frac{\sigma_1}{\sigma_3}} - 1 = 0 \quad (8)$$

Since Sagami River has the coefficients $\lambda=1.2$ and $\mu=0.28$, $\sigma_1/\sigma_3=3.69$ and consequently $\tau/\sigma_N=0.70$ are obtained from Eq. (8) for the minimum point of dilatancy. It is observed that this value naturally corresponds to Figs. 3 and 4.

Fig. 5 shows the data of triaxial compression and triaxial extension tests on normally consolidated Fujinomori clay ($\sigma_m=1.0, 2.0, 3.0 \text{ kg/cm}^2$) as plotted in terms of the relationship of $(\Delta V/V)_d$ versus $(\sigma_1-\sigma_3)/\sigma_m$. When the same data as those of Fig. 5 are rearranged in terms of the relationship between $(\Delta V/V)_d$ and τ/σ_N , the results are as shown in Fig. 6. In the

Fig. 5. Relationship between $(\Delta V/V)_d$ and $(\sigma_1-\sigma_3)/\sigma_m$ in triaxial compression and extension tests on Fujinomori clay

Fig. 6. Relationship between $(\Delta V/V)_d$ and τ/σ_N in triaxial compression and extension tests on Fujinomori clay

diagram, the solid line represents the curve of values calculated in accordance with Eqs. (3) and (4). It appears from these diagrams that the dilatancy of clay is also governed by τ/σ_N (or σ_1/σ_3) and not by the ratio of $(\sigma_1 - \sigma_3)/\sigma_m$ ($\equiv q/p$).

UNDRAINED SHEAR CHARACTERISTICS OF SAND AND CLAY

As already pointed out in the preceding paragraph, the following equations can be derived by assuming that the strain generally produced in the soil element is the combination of the strain ascribable to consolidation with the strain resulting from dilatancy.

$$\frac{\Delta V}{V} = \left(\frac{\Delta V}{V}\right)_c + \left(\frac{\Delta V}{V}\right)_d \quad (9)$$

$$\epsilon_1 = (\epsilon_1)_c + (\epsilon_1)_d \quad (10)$$

$$\epsilon_3 = (\epsilon_3)_c + (\epsilon_3)_d \quad (11)$$

wherein the suffix "c" stands for the strain due to consolidation and the suffix "d" for the strain caused by dilatancy (or shear).

The volumetric strain $(\Delta V/V)_c$ and the principal strains $(\epsilon_1)_c$, $(\epsilon_2)_c$ and $(\epsilon_3)_c$ caused by consolidation can be expressed by the following formulas, as is universally known.

$$\left(\frac{\Delta V}{V}\right)_c = \frac{C_c}{1+e_i} \log_{10} \frac{\sigma_m}{\sigma_{mi}} \quad (12)$$

$$(\epsilon_1)_c = (\epsilon_2)_c = (\epsilon_3)_c = \frac{1}{3} \frac{C_c}{1+e_i} \log_{10} \frac{\sigma_m}{\sigma_{mi}} \quad (13)$$

wherein C_c stands for compression (or expansion) index, e_i and σ_{mi} for the initial void ratio and the initial mean effective principal stress existing prior to consolidation. The volumetric strain $(\Delta V/V)_d$ and the principal strains $(\epsilon_1)_d$ and $(\epsilon_3)_d$ caused by dilatancy can be expressed by Eqs. (2) and (3) in the case of a triaxial compression test and by Eqs. (4) and (5) in the case of a triaxial extension test. This means that the relationship of the principal stress ratio (σ_1/σ_3) , the principal strains (ϵ_1, ϵ_3) and the volumetric strain $(\Delta V/V)$ involving the both phenomena of consolidation and shear can be found by calculation.

Since the compressibility is generally small (C_c being small) in the case of sand, it is inferred that the difference between the test under a constant mean principal stress σ_m and the test under a constant lateral pressure σ_3 is not too conspicuous as pointed out in the previous papers (Matsuoka, 1973; Matsuoka, 1974). The difference in the stress-strain curves ascribable to such difference in stress conditions will be discussed in the case of clay. Here, the author limits his discussion to the behavior of sand under undrained conditions, viz., conditions which do not permit drainage during shear. Since undrained conditions are such that the soil specimen as a whole is not permitted to undergo any volumetric change, it is only natural to interpret that if the volume tends to contract because of negative dilatancy, the effective confining pressure decreases so as to induce a proportional volumetric expansion, and if the volume tends to expand because of positive dilatancy, then the effective confining pressure increases to induce a proportional volumetric contraction. According to Eq. (9), therefore, this relationship may be expressed as follows:

$$\left(\frac{\Delta V}{V}\right)_c + \left(\frac{\Delta V}{V}\right)_d = 0 \quad (14)$$

When Eqs. (3), (5) and (12) are substituted in Eq. (14) on the basis of the concept mentioned above, the formulated relationship of effective stresses during shear in triaxial compression tests and triaxial extension tests, namely, the formula indicative of the effective stress path, can be calculated. In the case of a triaxial compression test, for example, since $\sigma_m = (\sigma_1 + 2\sigma_3)/3$, the following equation can be derived from Eq. (14).

$$2\left\{f\left(\frac{\sigma_1}{\sigma_3}\right)+g\left(\frac{\sigma_1}{\sigma_3}\right)\right\}+\frac{C_c}{1+e_i}\left\{\log_{10}\frac{\sigma_1/\sigma_3+2}{3\sigma_{mi}}+\log_{10}\sigma_3\right\}=0 \quad (15)$$

From this equation, the value of σ_3 and consequently the value of σ_1 are determined for a given value of σ_1/σ_3 . In the preceding formula, σ_1 , σ_3 and σ_{mi} stand for effective stresses. Fig. 7 compares the results of calculation performed in the manner just mentioned with the measured values. The measured values are the data (by Tatsuoka and Shiba, 1972; Tatsuoka and Ishihara, 1973) of triaxial compression and triaxial extension tests on Sagami River sand. In the related calculations, the coefficients $\lambda=1.2$, $\mu=0.26$, $\mu'=0.44$, $\gamma_0=0.35\%$ (for triaxial compression) or 0.6% (for triaxial extension), $C_c=0.06$ and $e_i=0.74$ were used. It is observed from the diagram that, for both triaxial compression and triaxial extension, curves of calculated values and those of measured values are in satisfactory agreement. It is interesting to note that these curves are inverted from left to right under the influence of dilatancy. The minimum point of σ_m on this stress path is called "the point of phase change" by Ishihara, Tatsuoka and Yasuda (1972), and is considered to correspond to the minimum point of volumetric strain $(\Delta V/V)_d$ due to dilatancy as described previously. Therefore, the principal stress ratio at this point can be determined in accordance with Eq. (8).

Fig. 7. Relationship between $(\sigma_1 - \sigma_3)$ and σ_m in undrained triaxial compression and extension tests on Sagami River sand

In the graph of the $(\sigma_1 - \sigma_3)$ vs. σ_m relationship represented in Fig. 7, the curves of measured values for triaxial compression and those for triaxial extension are considerably separated from each other. When these values are rearranged in terms of the τ vs. σ_N relationship on the mobilized plane, the results are as shown in Fig. 8. It appears from this diagram that the curves representing the values for triaxial compression and triaxial extension are both

Fig. 8. Relationship between τ and σ_N in undrained triaxial compression and extension tests on Sagami River sand

inverted from left to right at an equal value of τ/σ_N and they have the same maximum value of τ/σ_N (peak strength). Here, the conversion of the curves from left to right at the same value of τ/σ_N is evident from Eq. (8) and the acquisition of the same maximum value of τ/σ_N is considered to indicate the faithful accordance with Mohr-Coulomb's failure criterion. It is also observed that the curves of calculated values (indicated by solid lines) obtained by using the same coefficients as those for Fig. 7 show the same trend. These observations may support the hypothesis that the mechanical properties of soil are governed by τ/σ_N on the mobilized plane and not by the ratio of $(\sigma_1 - \sigma_3)/\sigma_m (\equiv q/p)$.

Since clay has a relatively large compressibility (or expansibility), it is considered to depend more on the effect of strain due to consolidation than sand. Thus, the author compared the data of the test involving a fixed lateral pressure σ_3 considered to induce both phenomena of consolidation and dilatancy with the data of the test involving a fixed mean effective principal stress σ_m considered to induce only the phenomenon of dilatancy and, at the same time, compared these data with the values calculated in the method described above. Fig. 9 represents the data of the drained triaxial compression test under $\sigma_m = 1.0 \text{ kg/cm}^2$ and $\sigma_3 = 1.0 \text{ kg/cm}^2$ as plotted in terms of the $\sigma_1/\sigma_3 - \epsilon_1 - \Delta V/V$ relationship in conjunction with curves of calculated values. In the related calculations, the coefficients $\lambda = 1.1$, $\mu = 0.30$, $\mu' = 0.42$, $r_0 = 1.0\%$, $C_e = 0.24$ and $e_i = 0.897$ appropriate to the test specimen were used. From this diagram, it is noted that the relationship between σ_1/σ_3 and ϵ_1 is not appreciably different under two sets of conditions. Fig. 10 shows the measured values relative to the ϵ_3 vs. ϵ_1 relationship and the curves of values obtained by calculations. These two diagrams seem to suggest a possibility that the calculated values according to the said concept will approximately express the trend of the measured values of the tests involving a fixed value of σ_m and a fixed value of σ_3 .

In the case of the undrained test, necessary calculations can be carried out according to Eq. (14). Fig. 11 represents the measured values (by Murayama, Sekiguchi and Ueda, 1972) of the undrained triaxial compression test on Fujinomori clay in comparison with the curves (solid lines) of calculated values. It is noted from the graph that the trends of both curves are substantially in agreement. In the calculations mentioned above, the coefficients $\lambda = 1.1$, $\mu = 0.30$, $\mu' = 0.42$, $r_0 = 0.7\%$, $C_e = 0.24$ and $e_i = 0.776$ were used. Fig. 12 shows a typical

Fig. 9. Relationship among σ_1/σ_3 , ϵ_1 and $\Delta V/V$ for $\sigma_m = \text{const.}$ and $\sigma_3 = \text{const.}$ tests on Fujinomori clay

Fig. 10. Relationship between ϵ_1 and ϵ_3 for $\sigma_m = \text{const.}$ and $\sigma_3 = \text{const.}$ tests on Fujinomori clay

Fig. 11. Relationship between $(\sigma_1 - \sigma_3)$ and σ_m in the undrained triaxial compression test on Fujinomori clay

calculation of the stress path on the stress plane proposed by Rendulic. The coefficients which were used in the calculation were $\lambda=1.1$, $\mu=0.30$, $\mu'=0.42$, $\gamma_0=1.0\%$, $C_c=0.24$ and $e_i=0.897$, with σ_m taken as a variable. The similarity between the curves reminds us of Henkel's data (Henkel, 1960). It is also interesting to note that the stress path is inverted from left to right on the compression side as well as on the extension side.

What has been discussed to this point verifies that the stress-strain relationship under general conditions inducing the phenomena of consolidation and shear can substantially be explained adequately in accordance with the formulas of the stress-strain relationships due to shear which has been derived in the present work through the microscopic analysis of the mechanism of soil dilatancy and the formulas of the stress-strain relationships due to consolidation which has heretofore been adopted, on condition that the strain is the combination of the strain due to shear with

Fig. 12. An example of calculated stress path on Rendulic's stress plane

the strain due to consolidation. The stress path is inverted from left to right on the compression side as well as on the extension side.

the strain due to consolidation. It has further been confirmed that, as a special case, the stress path in undrained shear can be calculated on condition that the sum of volumetric strains due to the two phenomena is 0.

CONCLUSIONS

The results which have been ascertained in this paper may be summarized as follows :

(1) The formulas of the stress-strain relationships proposed by the author suggest that the volumetric strain due to dilatancy is governed by the principal stress ratios (σ_1/σ_3 , σ_1/σ_2 , σ_2/σ_3) or the shear-normal stress ratio (τ/σ_N) on the mobilized planes. This concept differs from the view advocated by Shibata, Karube et al. (1963, 1964, 1965) that the said volumetric strain is governed by the principal stress difference versus mean effective principal stress ratio ($(\sigma_1 - \sigma_3)/\sigma_m$) or the octahedral shear-normal stress ratio (τ_{oct}/σ_m). Based on the data of triaxial compression and extension tests on sand and clay, it is judged that dilatancy of soil may be governed uniquely by τ/σ_N (or σ_1/σ_3) and not by the ratio of $(\sigma_1 - \sigma_3)/\sigma_m (\equiv q/p)$.

(2) It has been demonstrated that the stress-strain relationship under general stress conditions inducing both phenomena of consolidation and shear, for example, the stress-strain relationship by the test involving a fixed lateral pressure (σ_3) can be calculated by assuming the superposition of the strain due to consolidation and the strain due to dilatancy. It has been ascertained that the results of the calculation can substantially give a satisfactory explanation to the measured values.

(3) The stress path in the undrained test has been calculated on the assumption that the sum of the volumetric strain due to consolidation and the volumetric strain caused by dilatancy is 0. When the results of this calculation are compared with the results of the undrained triaxial compression and extension tests on sand and clay, the trends of the two are found to correspond in a satisfactory manner.

ACKNOWLEDGEMENTS

The author sincerely acknowledges his indebtedness to Professor Sakuro Murayama of Kyoto University, for his continual guidance and encouragement given throughout the present study. A grateful acknowledgement is made to Professor Toru Shibata of Kyoto University, Assistant Professor Norio Yagi of Kanazawa University, Assistant Professor Daizo Karube of Kobe University and Mr. Norio Kurihara of Japan Highway Public Cooperation for their helpful advice, criticisms and encouragement.

The author also wishes to express his thanks to Messrs. Chokkan Suematsu, Ichiro Kamo, Hideo Sekiguchi, Shin-ichi Isoda, Ryosuke Kitamura and others, who were colleagues at the Professor Murayama's laboratory, for their daily discussions and support. The author would like to express his appreciation to Dr. Fumio Tatsuoka of the Ministry of Construction for his kind discussion and the offer of his published experimental data.

NOTATION

C_c = compression (or expansion) index

e_i = initial void ratio

$\frac{\Delta V}{V}$ = volumetric strain

$\left(\frac{\Delta V}{V}\right)_c$ = volumetric strain due to consolidation

$\left(\frac{\Delta V}{V}\right)_d$ = volumetric strain due to dilatancy

γ = shear strain on the mobilized plane

$\gamma_0 = \gamma$ at the maximum compression point of ϵ_N

ϵ_N = normal strain on the mobilized plane

$\epsilon_1, \epsilon_2, \epsilon_3$ = major, intermediate and minor principal strains

$(\epsilon_1)_c, (\epsilon_2)_c, (\epsilon_3)_c$ = major, intermediate and minor principal strains due to consolidation

$(\epsilon_1)_d, (\epsilon_2)_d, (\epsilon_3)_d$ = major, intermediate and minor principal strains due to dilatancy

λ = a constant decided by the value of μ and taking an approximate value (1.1 to 1.5)

μ = coefficient of interparticle friction ($\equiv \tan \phi_\mu$)

μ' = one of the coefficients of soil

$\sigma_1, \sigma_2, \sigma_3$ = major, intermediate and minor effective principal stresses

σ_m = mean effective principal stress

σ_{mi} = initial mean effective principal stress

τ/σ_N = shear-normal stress ratio on the mobilized plane

$(\sigma_1 - \sigma_3)/\sigma_m$ ($\equiv q/p$) = principal stress difference vs. mean effective principal stress ratio

τ_{oct}/σ_m = octahedral shear vs. normal stress ratio

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(Received December 5, 1973)