MICROSCOPIC DEFORMATION MECHANISM OF GRANULAR MATERIAL IN SIMPLE SHEAR

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ABSTRACT

The deformation of granular material was simulated by an assembly of cylinders (made of photoelastic material) packed at random in a two-dimensional simple shear apparatus. From the detailed observations on the change of granular fabric due to shear and from the analyses of interparticle contact force among grains, we obtained some experimental facts which must be taken into account in proposing a granular model of sand.

With the increase of shear stress in the simple shear test and direct shear test, the normals N_i to the contact planes tend to concentrate toward the maximum principal stress axis. This tendency of the concentration of N_i is not determined by the magnitude of applied shear displacement but by the intensity of mobilized stress ratio τ/σ_N . The preferred direction of N_i gradually rotates with the rotation of principal stress axes during shear deformation. The sliding in microscopic scale at contact is, at any one instant, confined at some preferred contacts. According to the microscopic study of granular fabric of sand deformed in the direct shear apparatus, the similar characters are also observed.

Key words: direct shear test, model test, microscopy, progressive failure, sand, soil structure

IGC: D6/D3

INTRODUCTION

The direct shear- and the simple shear-apparatus with their rigid confinement parallel to the shear direction have similarities to many engineering problems which fail under plane strain. Thus it is an immediate object in soil mechanics to discuss the deformation and strength behaviour of sand directly sheared in these apparatus.

With the increase of shear stress in these equipments, the granular fabric of sand gradually changes in order to increase its ability to withstand the increasing shear stress, accompanying with the increase of shear strain. This process is called "strain hardening" of granular material. From the microscopic studies on the deformation mechanism of sand in the triaxial test ($\sigma_1 > \sigma_2 = \sigma_3 = \text{constant}$), Oda (1972 and 1974) has shown that the geometrical parallel arrangement of contact planes has an important bearing on the mechanism of the strain hardening or strain softening of sand. The contacts whose normals N_i are parallel to the direction of axial compression are most effective in supporting the axial load, and the ability to support the axial load at the contact gradually decreases with the increase of the angle between the direction of axial compression and the normal. It must be noted, however, that this conclusion came from the results obtained by the triaxial compression

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test in which no rotation of principal stress axes occurred. It has been shown that the rotation of the principal stress axes takes place in the simple shear test (Roscoe, et al., 1967; Roscoe, 1970). Therefore more attention must be paid to predict the mechanical behaviours of granular material in the simple shear and direct shear tests.

In this paper, the process of strain hardening of sand will be discussed on the basis of some experimental studies of simple shear test by two dimensional model and of direct shear test of sand.

SIMPLE SHEAR TEST OF TWO DIMENSIONAL MODEL

Testing Material and Testing Procedures

Testing material and testing procedures have already been described in some detail (Konishi, 1972 and 1973). Thus, the brief outlines will be described in this paper.

Right cylinders with the radii of 0.3, 0.4 and 0.5 cm and the length of 1.9 cm were cut out from a plate of epoxy resin (photoelastic material) and the residual stresses in them were removed completely by the thermal treatment. To simulate a granular material by a two-dimensional model, the cylinders were arranged in random packing in a loading frame. The mixing ratio of cylinders with these three different radii was 25 : 15 : 8 by the number. This mixture of cylinders was put into the plane polarized light and uniaxially loaded through a loading arm P up to 40 kg (normal force) in the rigid frame of two-dimensional simple shear apparatus with no lateral expansion. The shear force was then applied through a loading arm L (Photo. 1). At each application of step load, the shear displacement, the rate of thickness change and the shear force were measured and at the same time photographs showing the photoelastic isochromatics were taken to analyze the photoelastic stress concentration at the contacts between the cylinders, as shown in Photos. 2 and 3.

Fig. 1 shows the relations between the shear force S , the shear distortion γ and the rate of thickness change $\Delta H/H$ for the models packed to the initial void ratio of 0.22 (dense packing model) and to that of 0.26 (loose packing model). From this figure, the following significant natures are to be found:

(1) The assembly is compressed at first and becomes to dilate when the shear distortion becomes about 5%.

Photo. 1. Two dimensional simple shear apparatus for an assembly of cylinders

(2) The dense assembly has a stronger tendency to dilate than the loose one.

(3) The values of internal friction angle calculated from the stress ratio τ/σ_N at the point of minimum volume during the test are 20° and 22° for the loose and dense models, respectively. These values are almost equal to the interparticle friction angle of 22° * (Kirkpatrick, 1957).

Since these characters are commonly observed features of sand sheared in the simple shear test, we may infer that the two-dimensional model used here is sufficient to simulate the mechanical behaviour of sand deformed in the simple shear test.

* The interparticle friction angle of testing material was determined by sliding them on the plate of the same material. According to the experiment, the angle ranges from 18° to 25° whose mean value is approximately 22° .

Photo. 2-1 D-2

Photo. 2-2 D-5

Photo. 2-3 D-10

Photo. 2-4 D-16

Photo. 2. Photoelastic isochromatics for dense model

Photo. 3-1 L-2

Photo. 3-2 L-3

Photo. 3-3 L-10

Photo. 3-4 L-16

Photo. 3. Photoelastic isochromatics for loose model

Fig. 1. Relations between shear force, shear distortion and rate of thickness change for two-dimensional simple shear tests of dense and loose models

Fabric Analyses

According to the previous studies by Oda (1972), Murayama and Matsuoka (1970 and 1973) and Konishi (1972), the most important aspect of granular fabric is the preferred orientation of contact plane. This preferred orientation of contact plane is clearly shown by the two- or three-dimensional distribution of the normals to the contact planes.

The current reference axes X , Y and Z are defined as shown in Fig. 2. The axis Y corresponds to the negative direction of the applied shear stress τ . The axis Z is the vertical direction (upward being positive) which corresponds to the negative direction of the applied normal stress σ_N . The axis X is perpendicular to both of axes Y and Z .

Consider a contact C_i in Fig. 2. The normal N_i to the contact plane at its contact is defined by its inclination angle β_i to the axis Z . The angle β_i was measured at all contacts between grains by using the photographs shown by Photos. 2 and 3. The frequency distributions of N_i were determined as shown in Fig. 3 for the experimental series of the dense model (D-2, D-5, D-8, D-10, D-12 and D-16 in Fig. 1) and in Fig. 4 for the experimental series of the loose model (L-2, L-3, L-5, L-10, L-12 and L-16 in Fig. 1).

The following important facts in regard to the distribution of N_i are found for the experimental series of densely packed

Fig. 2. Reference axes X , Y and Z and inclination angles of N_i and F_i to the reference axis Z

Fig. 3. Frequency distributions of N_i for experimental series of dense model

Fig. 4. Frequency distributions of N_i for experimental series of loose model

model. The frequency of N_i for D-2 shown in Fig. 3 is high within the range of angle -50° to $+50^\circ$, whose centre almost coincides with the axis Z . This means that the normals concentrate near the axis Z when the particle assembly is uniaxially loaded up to 40 kg without lateral expansion. When the shear stress is applied, the frequency distribution changes and the centre of distribution gradually rotates from 0° to 45° as shown by D-2 $\rightarrow$ D-5 $\rightarrow$ D-8 $\rightarrow$ D-10 $\rightarrow$ D-12 $\rightarrow$ D-16 in Fig. 3. The similar characteristics, i. e., the gradual concentration of N_i and the gradual rotation of the preferred direction, are also observed in the process of simple shear of loose model as shown in Fig. 4.

Interparticle Contact Forces between Particles

Consider the contact C_i between grains G_i and G_i' (Fig. 2). The contact force transmitted through the contact from the grain G_i to G_i' is represented by the vector F_i which inclines to the axis Z at θ_i as shown in Fig. 2. The angles β_i and θ_i are positive when measurements are clockwise and the angle $\delta_i (= \theta_i - \beta_i)$ is tentatively called "mobilized angle". It must be noted that when the absolute value of the mobilized angle $|\delta_i|$ is equal to the interparticle friction angle $\phi_\mu (= 22^\circ)$ the interparticle sliding must occur at the contact.

From Fig. 5 which shows the frequency distribution of the mobilized angle δ_i , the following facts are observed :

Fig. 5. Frequency distribution of mobilized angle δ in two-dimensional granular assembly of loose model

(1) The value of δ ranges from -25° to $+25^\circ$ and their frequency curves are unimodal ones with the distinct peak at $\delta=0^\circ$.

(2) There are some critical contacts satisfying the relation of $|\delta|=\phi_\mu$ even in the granular assembly which was compressed uniaxially without any application of shear force (see L-2 in Fig. 5).

(3) Contrary to our expectation, the contacts satisfying the relation of $|\delta|=\phi_\mu$ is a few even when the granular assembly was sheared (see L-9 and L-12 in Fig. 5). This means that the slidings among particles during the progressive shear deformation are not, at any one instant, occurring in the majority of contacts in the assembly. Therefore it is very probable that "deformation occurs as relative motion between instantaneously rigid groups of particles, these groups continuously reforming by division and coalescence (Horne, 1965)".

Consider the plane (a-b) which is inclined to the preferred direction of N_i at right angle (Fig. 6). The centres of grains $G_1, G_2, \dots$ and G_i exist above the plane. These grains contact at the contacts $C_1, C_2, \dots$ and C_i with the grains $G'_1, G'_2, \dots$ and G'_i whose centres exist below the plane. The symbol F_i in Fig. 6 is the contact force transmitted through the contact C_i . According to the analysis of contact force, the direction of resultant vector $F(=\sum F_i)$ is approximately perpendicular to the plane (a-b). For example, F inclines to the plane at 88° in the case of L-12 and at 86° in the case of L-16. This means that the direction of resultant vector F is agreement with the preferred direction of N_i . It is worthy of note that the plane (a-b) must be the principal plane on which the component of shear force is nearly absent, because the resultant vector F is inclined to the plane about 90° . Therefore, we can conclude that the preferred direction of N_i agrees nearly with the axis of maximum principal stress.

From detailed microscopic studies on the deformation mechanism of sand in the triaxial test ($\sigma_1 > \sigma_2 = \sigma_3 = \text{constant}$), Oda (1972 and 1974) has said that the concentration of N_i to the maximum principal stress direction plays an essential role to harden or soften the granular material. In the simple shear test, the process of fabric change is also characterized by the concentration of N_i into the narrow domain whose centre is nearly the maximum principal stress axis. Therefore we conclude that the same principle in regard to the mechanism of strain hardening can be applied not only to the triaxial test but also to the

Fig. 6. Sum of interparticle contact forces transmitted through plane perpendicular to preferred direction of N_i

simple shear test. The apparent difference between these two types of test must be only due to the rotation of the principal stress axes in the simple shear test.

DIRECT SHEAR TEST OF SAND

Material and Testing Procedures

Characteristics of the sand which was tested are as follows: grain size=0.84 to 0.95 mm, mineral composition=90% quartz and 10% feldspar, particle shape=subrounded to subangular, limiting void ratio=0.96 to 0.67, interparticle friction angle= 26° (=the interparticle friction angle used in a previous paper (Oda 1972) to fit a theoretical equation to the experimental values for this sand).

The following two series of experimental works were done by a conventional direct shear test apparatus (Taylor, 1948) whose upper frame was fixed in place by a horizontal arm.

(1) P-series: The oven dried sand was poured into a shear box (6 cm in internal diameter and 4.3 cm in height) and was compacted by the plunging method (Oda, 1972a). The specimens (P-specimens) compacted by this method had an initial void ratios ranging from 0.67 to 0.71.

(2) L-series: The oven dried sand was spooned into the shear box to a height of 2 cm as loosely as possible (void ratios ranging from 0.79 to 0.84). The specimens made by this method are called L-specimens.

After normal stress of 0.6, 1.1, 2.1 or 3.1 kg/cm² was applied to the potential shear plane, the specimens were sheared at a rate of relative displacement of 0.3 mm/min. The shear displacement and the change of thickness ΔH of the specimen were measured by a horizontal and a vertical dial gauges, respectively. Each test was interrupted at a different stage of shear displacement. The composite shear stress-displacement curves thus obtained are shown in Figs. 7 and 8, and the positions at which tests were terminated are also indicated.

After the shear stress was released as shown by the dotted lines in Figs. 7 and 8, the specimens were fixed by polyester resin without any disturbance of the granular fabric (Oda, 1972). The fixed specimen was cut with a diamond saw along the longitudinal, transversal and horizontal planes (Fig. 9). Three thin sections (about 0.09 mm in thickness) which are parallel to these three planes were made by the ordinary petrographic method.

Fig. 7. Relations between τ/σ_N , shear displacement and change of thickness for sand compacted by plunging method

Fig. 8. Relations between τ/σ_N , shear displacement and change of thickness for sand packed as loosely as possible

Fabric Analyses and Results

The values of angles α and β (Fig. 10) were measured by the polarized microscope equipped with a universal stage, to define the direction of N_i at about 160 contacts selected at random from the homogeneously sheared zone of the longitudinal thin section. The directions of N_i were plotted on the Schmidt's equal area net to show their three-dimensional distribution (Oda, 1972). The resultant fabric diagrams are shown in Fig. 11 for the P-specimens and in Fig. 12 for the L-specimens. To obtain the true N_i -distribution, the contacts on which the angles α and β are measured must be selected at random on the three thin sections at least which are mutually perpendicular. It must be noted, however, that these fabric diagrams shown in Figs. 11 and 12 were made on the assumption that

Fig. 9. Preparation of thin sections and reference axes X , Y and Z .

Fig. 10. Solid angles α and β to define normal direction N_i .

Fig. 11. Fabric diagrams of P-specimens to show three-dimensional distribution of N_i (P-UND, P-2, P-4 and P-6 indicate the positions at which tests are interrupted as shown in Fig. 7.)

Fig. 12. Fabric diagrams of L-specimens to show three-dimensional distribution of N_i (L-UND, L-1, L-2 and L-3 indicate the positions at which tests are interrupted as shown in Fig. 8.)

the N_i -distribution near the axis X does not change remarkably during the increase of shear strain because the axis X corresponds to the direction of zero elongation.

The distribution of N_i for the P-specimen consolidated one-dimensionally (Fig. 11-a) does not show any reproducible local concentration and is very similar to that of unconsolidated P-specimen (Oda, 1972). Therefore, we can say that the character of the granular fabric does not change markedly but the initial void ratio decreases slightly by the one-dimensional consolidation. This is quite different from the remarkable orientation of flaky clay minerals toward the direction perpendicular to the consolidation axis (Quigley, et al., 1966; Sides, 1971).

By the comparison of the fabric diagram of one-dimensionally consolidated P-specimen (Fig. 11-a) with those of P-specimens (Fig. 11-b to 11-d) deformed to the specific values of shear displacement, it can be concluded that N_i gradually concentrates within the narrow domains enclosed by broken lines in these figures whose centers are represented by the principal direction P. The tendency of the concentration of N_i to the narrow domain disappears when the shear displacement increases to the residual strength state. Therefore it is reasonable to say that the degree of this concentration has no relation to the magnitude of applied shear displacement but is determined by the intensity of mobilized stress ratio.

Even though the N_i -distribution of the consolidated P-specimen shows entirely different characters from that of consolidated L-specimen as shown by the comparison of Fig. 11-a with Fig. 12-a, both of them have the similar tendency to concentrate N_i to the narrow domains during the shear displacement and results almost in the same fabric diagram as shown by Figs. 11-d and 12-d.

From the comparison of the direct shear test of sand with the simple shear test by the two-dimensional model, we can conclude that the concentration of the normal N_i is the essential factor to strengthen the granular fabric in both of tests.

CONCLUSIONS

The two-dimensional granular model by the photoelastic cylinders packed at random in the loading frame is useful to simulate the shear deformation of sand, because the experimental results obtained from the two-dimensional model show the same characteristics in regard to the deformation and strength behaviour in microscopic scale as those of deformed sand. The study on the two-dimensional model explains the following significant facts:

(1) The normals to the contact planes tend to concentrate toward the maximum principal stress axis during the increase of shear force in the simple and direct shear tests as in the case of the triaxial compression test of sand. This tendency of the concentration of N_i has no relation to the magnitude of applied shear displacement but is determined by the intensity of mobilized stress ratio. This concentration must have an important bearing on the mechanism of strain hardening of granular material.

(2) The preferred direction of N_i gradually rotates from $\beta=0^\circ$ to $\beta=45^\circ$ when the shear stress is gradually applied up to the peak value. This rotation is due to the rotation of principal stress axes described by Roscoe, et al. Thus we must take account of the rotation of principal stress axes when we intend to propose a useful granular model.

(3) The slidings in microscopic scale at contacts are not, at any one instant, occurring in the majority of contacts in the assembly, but is confined at some preferred contacts. This fact confirms Horne's statement that the deformation of granular material occurs as relative motion between instantaneously rigid groups of particles, which are continuously reformed by the division and coalescence.

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NOTATION

F_i =vector of contact force acting on a contact C_i

F =summation vector of the contact forces acting on a given plane

N_i =direction normal to tangential plane at a given contact C_i

X, Y, Z =reference axes

α, β =angles defining the direction of N_i

γ =shear distortion in simple shear tests

δ =mobilized angle ($=\theta_i - \beta_i$)

θ_i =inclination angle of the vector F_i to the vertical at a given contact C_i

τ, σ_N =shear and normal stresses acting on an apparent shear plane in simple shear tests

ϕ_μ =interparticle friction angle

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