THEORY OF ONE-DIMENSIONAL CONSOLIDATION OF CLAYS WITH CONSIDERATION OF THEIR RHEOLOGICAL PROPERTIES

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ABSTRACT

The paper concerns the problem of one-dimensional consolidation for clays exhibiting creep under constant effective stress. A stress-strain-time relation relevant to the consolidation process is deduced, by assuming K_0 -conditions, from the stress-strain-time theory developed by Murayama et al. (1974) and Sekiguchi (1974). A numerical solution to the mathematical equations governing the consolidation process is presented, and based on the predicted settlement-time factor relations a method of determining the coefficient of consolidation is proposed. Discussion is made of the influence of both the duration of secondary compression during the preceding load and the maximum drainage distance on the consolidation behavior of such clays during the following increment of loading. In addition, it is shown that there is reasonable agreement between the measured settlement and pore pressure dissipation-time curves and the predicted ones for laboratory consolidation tests on specimens of a saturated clay for each of pressure increment ratios of 0.1, 0.5 and 1.0.

Key words: clay, consolidation, pore pressure, rheology, secondary compression, time effect

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INTRODUCTION

Studies have so far been made on the rheological properties of normally consolidated clays under axi-symmetric loading conditions to develop a stress-strain-time model for them (Murayama, Sekiguchi and Ueda, 1974 and Sekiguchi, 1974). As the first step toward wider use of the stress-strain-time model, an attempt will be made in this paper to apply it to the problem of one-dimensional consolidation for clays exhibiting creep under constant effective stress, considering that for this problem the Terzaghi theory of consolidation (Terzaghi, 1923) and other refined theories of consolidation (for example, Mikasa (1963) and Davis and Raymond (1965)) which assume unique relationships between the vertical effective stress and the vertical strain are invalid. The consolidation process of such clays as just cited may be characterized by the following aspects:

- (a) The settlement does not cease even when substantial dissipation of pore pressures has occurred, but is generally followed by so called secondary compression which continues in most cases in proportion to the logarithm of elapsed time (for example, Buisman, 1936 and Gray, 1936).
- (b) The proportion of secondary to primary settlement increases with decreasing pres-

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sure increment ratio (for example, Newland and Allely, 1960), and also with decreasing maximum drainage distance (for example, Barden, 1965).

- (c) The mid-plane pore pressure dissipates more rapidly with decreasing pressure increment ratio in the early stage of the consolidation process (for example, Leonards and Girault, 1961).
- (d) The prolonged duration of preceding secondary compression gives rise to the increased proportion of secondary to primary settlement and also to the initially rapid dissipation of mid-plane pore pressure under a following increment of loading (for example, Barden, 1969).

It is the scope of this paper to present a numerical solution to the mathematical equations governing the consolidation process, to discuss the role of each parameter included in the equations, particularly $-(\partial e/\partial T)_o/\lambda$, to propose a method of determining the coefficient of consolidation C_v , and to present laboratory test results as a check on the reliability of the numerical solution and of the proposed method for the determination of C_v .

It may be appropriate here to mention that there recently appeared some theories of consolidation in which emphasis was placed on the use of the known characteristics (including the creep effect) of one-dimensional compression of clays (Garlanger (1972), Hawley and Borin (1973), Lowe (1974), and Mesri and Rokhsar (1974) among others). The effective stress-strain-time (or strain-rate) relations in these theories are adopted for explaining only the behavior of clays under conditions of no lateral strain, so that by these theories the characteristic of secondary compression cannot be related to other important properties such as stress relaxation and creep rupture that will be exhibited by the same clays, in most cases, under undrained conditions. In this respect, the present theory of consolidation formulated below could provide insight besides some new results concerning the role of the parameter $-(\partial e/\partial T)_o/\lambda$ and the method of determining C_v .

THEORETICAL CONSIDERATION

The clay is assumed to be saturated with pore water and to be homogeneous, and the compressibility of the clay particles and that of the pore water are assumed to be negligible compared with that of the clay skeleton. Body forces resulting from self weight are supposed to be neglected. In addition, infinitesimal strains are assumed to occur in the process of one-dimensional consolidation. Compressive stresses and strains are taken as positive.

Equations of Equilibrium and Continuity for One-Dimensional Consolidation

Let us consider a stratum of clay that has undergone creep (secondary compression) under a vertical total stress σ_{v0} which is equal to or only slightly greater than the vertical effective stress $\sigma_{v'0}$. When an increment of vertical total stress $\Delta\sigma_v$ is applied to the stratum and then maintained constant under conditions of one-dimensional compression and drainage, the related equilibrium equation is expressed as

$$\left. \begin{aligned} \sigma_v' + u &= \sigma_{v0} + \Delta\sigma_v \doteq \sigma_{v'0} + \Delta\sigma_v \\ \partial\sigma_v'/\partial t &= -\partial u/\partial t \end{aligned} \right\} \quad (1)$$

where σ_v' is the vertical effective stress, u the excess pore pressure, and t the time.

The continuity equation for the process of one-dimensional consolidation (Terzaghi, 1923) is expressed as follows:

$$\frac{\partial e}{\partial t} = \frac{1+e_0}{\gamma_w} \frac{\partial}{\partial z} \left(k \frac{\partial u}{\partial z} \right) \quad (2)$$

where e is the void ratio, e_0 the value of e immediately before the change of loading, γ_w the unit weight of pore water, z the vertical coordinates, and k the coefficient of permea-

bility. Let us assume that the variation of k during the consolidation process obeys the following relation:

$$k = k_0 \exp\{-(e_0 - e)/C_k^*\} \quad (3)$$

with reference to Figs. 6.9 and 6.10 of Taylor (1948) and also to Fig. 3 of Mesri and Rokhsar (1974), where k_0 is the value of k at $e=e_0$ and C_k^* a material constant. Using this relation, the continuity equation (2) is rewritten as

$$\frac{\partial e}{\partial t} = \frac{(1+e_0)k_0}{r_w} \cdot \exp\left(-\frac{e_0-e}{C_k^*}\right) \cdot \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{C_k^*} \frac{\partial e}{\partial z} \frac{\partial u}{\partial z}\right) \quad (4)$$

Stress-Strain-Time Relation under K_0 -Conditions

Experimental evidence from Akai and Adachi (1965a and b), Moore and Spencer (1972) and others has shown that the ratio of horizontal to vertical effective stress, denoted by σ_H'/σ_V' , does not change appreciably in the process of one-dimensional consolidation under σ_V' exceeding a pre-consolidation pressure and is reasonably assumed to be equal to the coefficient of earth pressure at rest, K_0 . It seems reasonable, therefore, to suppose that the following relations hold during the consolidation process.

$$\left. \begin{aligned} \sigma_m'/\sigma_{m'o} &= \sigma_V'/\sigma_{V'o} \\ \eta &= \eta_o \end{aligned} \right\} \quad (5)$$

where σ_m' is the mean effective stress defined by $\sigma_m' = (\sigma_V' + 2\sigma_H')/3$, η is the effective stress ratio defined by $\eta = (\sigma_V' - \sigma_H')/\sigma_m'$, and each of $\sigma_{m'o}$ and η_o refers to that of σ_m' and η , respectively, immediately before the change of loading. The corresponding strain parameter under K_0 -conditions may be the volumetric strain ε_v defined by $\varepsilon_v = (e_0 - e)/(1 + e_0)$.

Sekiguchi (1974) has extended the stress-strain-time model proposed by Murayama, Sekiguchi and Ueda (1974) to include the time-dependency of dilatancy of normally consolidated clays and obtained a relation predicting the time-dependent variation of volumetric strain under axi-symmetric conditions of sustained loading. The relation is rewritten here in the form:

$$\begin{aligned} \varepsilon_v = & \frac{\lambda}{1+e_0} \ln\left(\frac{\sigma_m'}{\sigma_{m'o}}\right) + D(\eta - \eta_o) + \frac{3}{2} \bar{H}_o \left\{ D + \frac{\lambda}{(1+e_0)N} \right\} \\ & \times \ln \left\{ \left(\frac{\partial \varepsilon_v}{\partial t} \right)_{\sigma_{m'o}, \eta_o} / \left(\frac{\partial \varepsilon_v}{\partial t} \right)_{\sigma_m', \eta} \right\} \end{aligned} \quad (6)$$

where λ is the compression index, D the coefficient of dilatancy (Shibata, 1963), $\bar{H}_o$ the relaxation spectrum of a box-type which is normalized by the consolidation pressure, N the extra stress ratio, $(\partial \varepsilon_v / \partial t)_{\sigma_{m'o}, \eta_o}$ the volumetric strain rate immediately before the change of loading, and $(\partial \varepsilon_v / \partial t)_{\sigma_m', \eta}$ the volumetric strain rate after the change of loading. The derivation of Eq. (6) is given in APPENDIX. It may be appropriate to mention that both $\bar{H}_o$ and N have been shown by Murayama et al. (1974) and Sekiguchi (1974) not to be affected by the variation of η and σ_m' . It may also be noted that if the third term on the right-hand side of Eq. (6) vanishes the resulting relation becomes quite similar to the one deduced by Roscoe, Schofield and Thurairajah (1963).

Substituting Eq. (5) into Eq. (6) and taking notice of the aforementioned definition of ε_v , we have

$$e = e_0 - \lambda \ln(\sigma_V'/\sigma_{V'o}) + C_a^* \ln\{(\partial e / \partial t)_{\sigma_V'} / (\partial e / \partial t)_o\}, \quad (7)$$

in which

$$C_a^* = (3/2) \bar{H}_o \{D(1+e_0) + \lambda/N\} \quad (8)$$

where $(\partial e / \partial t)_{\sigma_V'}$ denotes the rate of change in void ratio under K_0 -conditions with σ_V' being held constant, and $(\partial e / \partial t)_o$ refers to the rate of change in void ratio immediately

before the change of loading, as illustrated in Fig. 1. Solving the differential equation (7),

$$e = e_o - \lambda \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right) - C_a^* \ln\left[-\frac{(\partial e / \partial t)_o t}{C_a^*} + \exp\left\{-\frac{\lambda}{C_a^*} \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right) + \frac{e_o - e_i}{C_a^*}\right\}\right] \quad (9)$$

where e_i is the constant of integration representing the value of void ratio immediately after the change of loading. Eq. (9) indicates that void ratio exhibits a tendency to decrease in proportion to $\ln(t)$ when the elapsed time t increases to such a large value that the second term in the square brackets is negligible compared with the first one in them. Thus C_a^* is shown to correspond to the so-called rate of secondary compression, $-de/d\ln(t)$.

Now, let us suppose that the immediate change in void ratio is given by

$$e_o - e_i = \kappa^* \ln(\sigma_v' / \sigma_{v'o}) \quad (10)$$

where κ^* is a material constant defined as the slope of the straight line AB in Fig. 3. Substitution of this equation into Eq. (9) yields

$$e = e_o - \lambda \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right) - C_a^* \ln\left[-\frac{(\partial e / \partial t)_o t}{C_a^*} + \exp\left\{-\frac{\lambda - \kappa^*}{C_a^*} \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right)\right\}\right] \quad (11)$$

Hence, the rate of the total change in void ratio is given by

$$\frac{\partial e}{\partial t} = \left(\frac{\partial e}{\partial t}\right)_{\sigma_v'} + \left(\frac{\partial e}{\partial \sigma_v'}\right)_t \frac{\partial \sigma_v'}{\partial t} \quad (12)$$

The first term on the right-hand side of the above equation stands for the creep rate and is obtained from Eq. (7) as follows:

$$\left(\frac{\partial e}{\partial t}\right)_{\sigma_v'} = \left(\frac{\partial e}{\partial t}\right)_o \exp\left\{-\frac{e_o - e - \lambda \ln(\sigma_v' / \sigma_{v'o})}{C_a^*}\right\} \quad (13)$$

The second term on the right-hand side of Eq. (12) is related to the time rate of σ_v' , and the coefficient $(\partial e / \partial \sigma_v')_t$ is obtained by differentiating Eq. (11) with respect to σ_v' with t being fixed and then by eliminating t . Hence,

$$\left(\frac{\partial e}{\partial \sigma_v'}\right)_t = -\frac{\lambda}{\sigma_v'} \left[1 - \left(1 - \frac{\kappa^*}{\lambda}\right) \exp\left\{-\frac{C_a^*}{e_o - e} + \frac{\kappa^*}{C_a^*} \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right)\right\}\right] \quad (14)$$

Substitution of Eqs. (13) and (14) into Eq. (12) leads to

$$\begin{aligned} \frac{\partial e}{\partial t} = & -\frac{\lambda}{\sigma_v'} \frac{\partial \sigma_v'}{\partial t} \left[1 - \left(1 - \frac{\kappa^*}{\lambda}\right) \exp\left\{-\frac{e_o - e}{C_a^*} + \frac{\kappa^*}{C_a^*} \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right)\right\}\right] \\ & + \left(\frac{\partial e}{\partial t}\right)_o \exp\left\{-\frac{e_o - e}{C_a^*} + \frac{\lambda}{C_a^*} \ln\left(\frac{\sigma_v'}{\sigma_{v'o}}\right)\right\} \end{aligned} \quad (15)$$

This is the stress-strain-time relation under K_o -conditions used later.

Governing Equations of One-Dimensional Consolidation

The basic equations (1), (4) and (15) are reduced to dimensionless forms for the convenience of obtaining a solution to them. Let us define the following dimensionless parameters:

Fig. 1. Schematic illustration of the variation of rate of reduction in void ratio before and after the change of loading

$$\bar{z} = z/H \quad (16)$$

$$T = C_v t / H^2 \quad (17)$$

$$\bar{u} = u / \Delta \sigma_v \quad (18)$$

$$U_e = (e_o - e) / \{\lambda \ln(1 + \Delta \sigma_v / \sigma_v' o)\} \quad (19)$$

in which $\bar{z}$ is the dimensionless vertical coordinates, H the maximum drainage distance, T the time factor, C_v the coefficient of consolidation, $\bar{u}$ the dimensionless excess pore pressure, and U_e the local degree of consolidation with respect to the vertical strain. It should be noted that the denominator of Eq. (19) represents the change in void ratio between two states with the same value of $-\partial e / \partial t$ (see the points A and C in Fig. 3). It should also be noted that C_v in Eq. (17) is defined in the form:

$$C_v = \frac{(1 + e_o) k_o \Delta \sigma_v}{r_w \lambda \ln(1 + \Delta \sigma_v / \sigma_v' o)} \quad (20)$$

by taking the secant modulus between the two states A and C in Fig. 3 as the measure of compressibility.

Using the dimensionless parameters defined above and putting the pressure increment ratio $\Delta \sigma_v / \sigma_v' o$ as ϕ , Eq. (4) is reduced to

$$\frac{\partial U_e}{\partial T} = - \left\{ \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \frac{\lambda}{C_v^*} \frac{\partial \bar{u}}{\partial \bar{z}} \frac{\partial U_e}{\partial \bar{z}} \ln(1 + \phi) \right\} \cdot (1 + \phi)^{-U_e \lambda / C_v^*} \quad (21)$$

In the same way, it follows from Eqs. (1) and (15) that

$$\begin{aligned} \frac{\partial \bar{u}}{\partial T} = & (1/\phi + 1 - \bar{u}) \left[- \frac{\partial U_e}{\partial T} \ln(1 + \phi) - \frac{(\partial e / \partial T)_o}{\lambda} \cdot (1 + \phi)^{-U_e \lambda / C_v^*} \right. \\ & \times \left\{ 1 + (1 - \bar{u}) \phi \right\}^{\lambda / C_v^*} \left. \right] \left[1 - (1 - \kappa^* / \lambda) \cdot (1 + \phi)^{-U_e \lambda / C_v^*} \right. \\ & \times \left. \left\{ 1 + (1 - \bar{u}) \phi \right\}^{\kappa^* / C_v^*} \right] \end{aligned} \quad (22)$$

NUMERICAL ANALYSIS

Eqs. (21) and (22) appear too complex to yield an exact solution, so that an approximate solution will be obtained using a method of finite differences.

Boundary and Initial Conditions

The stratum of clay considered has an initial thickness of H , as illustrated in Fig. 2. The upper surface of the stratum ($\bar{z} = 0$) is assumed to be impermeable and the lower surface of it ($\bar{z} = 1$) is supposed to be permeable.

It has been supposed in the paper that the initial state of the stratum (i. e., the state immediately before the change of loading) is determined by a set of values, e_o , $\sigma_v' o$ and $-(\partial e / \partial t)_o$, independently of the $\bar{z}$ -coordinates. It is, however, exact to take into account the excess pore pressure u_o that has existed before the change of loading (Hawley and Borin, 1973). Use of Eqs. (4), (16), (17) and (18) allows u_o to be expressed in the following dimensionless form:

$$\frac{u_o}{\Delta \sigma_v} = - \frac{(\partial e / \partial T)_o \cdot (1 - \bar{z}^2)}{2 \lambda \ln(1 + \phi)} \quad (23)$$

Consequently, the dimensionless excess pore pressure immediately after the change of loa-

Fig. 2. A stratum of clay together with the computational grid

ding $\bar{u}_i$ is given by

$$\left. \begin{aligned} \bar{u}_i &= 1 + u_0 / \Delta \sigma_v \text{ at } 0 \leq \bar{z} < 1 \\ \bar{u}_i &= 0 \text{ at } \bar{z} = 1 \end{aligned} \right\} \quad (24)$$

It may be appropriate here to point out that in some of the predicted curves of $\bar{u}_m (= \bar{u}$ at $\bar{z}=0$) versus t the values of $\bar{u}_m$ slightly exceed 1.0 in the early portion of the curves because of the introduction of such residual excess pore pressures.

Finite Difference Approximation

The finite difference approximations to the derivatives in Eqs. (21) and (22) are

$$(\partial U_e / \partial T)_{i,j} = (U_{e,i,j+1} - U_{e,i,j}) / \Delta T \quad (25)$$

$$(\partial U_e / \partial \bar{z})_{i,j} = (U_{e,i+1,j} - U_{e,i-1,j}) / (2 \Delta \bar{z}) \quad (26)$$

$$(\partial \bar{u} / \partial T)_{i,j} = (\bar{u}_{i,j+1} - \bar{u}_{i,j}) / \Delta T \quad (27)$$

$$(\partial \bar{u} / \partial \bar{z})_{i,j} = (\bar{u}_{i+1,j} - \bar{u}_{i-1,j}) / (2 \Delta \bar{z}) \quad (28)$$

$$(\partial^2 \bar{u} / \partial \bar{z}^2)_{i,j} = (\bar{u}_{i+1,j} - 2\bar{u}_{i,j} + \bar{u}_{i-1,j}) / (\Delta \bar{z})^2 \quad (29)$$

in which the first subscript i refers to the i -th spatial grid line and the second subscript j to the j -th time grid line. The computational grid is shown in Fig. 2. It is apparent that the simple explicit process of numerical analysis is applicable to Eqs. (21) and (22) except in the extreme case in which κ^* is equal to zero.

The simple explicit process proceeds as follows.

- The initial values of U_e along the grid line $j=1$ are zero except at $\bar{z}=1$ and equal κ^* / λ at $\bar{z}=1$. The initial values of $\bar{u}$ are given in Eq. (24). Using these values for $U_{e,i,j}$ and $\bar{u}_{i,j}$, $\partial U_e / \partial T$ can be computed from Eqs. (21), (26), (28) and (29).
- The term $\partial \bar{u} / \partial T$ can then be calculated using Eq. (22).
- Using the values of $(\partial U_e / \partial T)_{i,j}$ and $(\partial \bar{u} / \partial T)_{i,j}$ obtained in steps (a) and (b), $U_{e,i,j+1}$ and $\bar{u}_{i,j+1}$ can be computed from Eqs. (25) and (27).
- The average degree of consolidation in the j -th step, denoted by $(U_e)_j$, can be calculated from

$$(U_e)_j = \int_0^1 U_{e,i,j} d\bar{z} \quad (30)$$

The time factor T is computed from

$$T = \sum_j (\Delta T)_j \quad (31)$$

where $(\Delta T)_j$ denotes the increment of T in the j -th step and is determined as follows:

$$(\Delta T)_j = R (\Delta \bar{z})^2 [1 - (1 - \kappa^* / \lambda) \exp \{-U_{e,i,j} \lambda \ln(1 + \psi) / C_\alpha^*\}] / \{(1 + 1/\psi) \ln(1 + \psi)\} \quad (32)$$

where R is a scalar factor. (It was found appropriate to choose a value 0.1 for R from viewpoints of the convergence of the numerical solution and of the shortening of the computation time.)

- Steps (a)-(d) are repeated until the time factor reaches a prescribed value.

For the extreme case in which κ^* is equal to zero a method of the simple explicit process with a relaxation process is applied, and the details of the method have been described elsewhere (Sekiguchi, 1974).

NOTES ON DETERMINATION OF PARAMETERS IN THE THEORY

It is seen that Eqs. (21) and (22) include the following parameters: $\psi = \Delta \sigma_v / \sigma_{v'0}$; C_α^* / λ ; $-(\partial e / \partial T)_0 / \lambda$; C_α^* / λ ; κ^* / λ . In this way, the compression index λ itself is not included in

Determination of λ : —. Fig. 3 schematically illustrates a process of one-dimensional consolidation when subjected to an increment of loading from $\sigma_{v'o}$ to $\sigma_{v'f} = \sigma_{v'o} + \Delta\sigma_v$. It is seen that each element of a stratum of clay when entering the stage of secondary compression after a long time follows the state path along BD as shown in Fig. 3, and that at an intermediate point C on the path BD the value of the average rate of reduction in void ratio becomes equal to $-(\partial e / \partial t)_o$, which stands for the rate of reduction in void ratio immediately before the change of loading. The value of the average void ratio at the point C denoted by e_c in Fig. 3, may be measured easily in laboratory consolidation tests. Thus it follows from Eq. (13) that the measured reduction in the average void ratio, $e_o - e_c$, divided by $\ln (\sigma_{v'f} / \sigma_{v'o})$ yields the value of λ .

Determination of κ^/λ :*— The term κ^* refers to the compression index which is associated with the one-dimensional compression behavior immediately after the change of loading. While little information on the measured values of κ^* exists, it may be reasonable to assume that such an immediate response in the compression behavior is essentially to elastic nature. As a consequence, the values of κ^*/λ are assumed here to be equal of those of the ratio of swelling index κ to the compression index λ .

Determination of $\Delta\sigma_v/\sigma_v'$:— The values of the pressure increment ratio can be controlled arbitrarily in laboratory consolidation tests, while routine tests employ a specific value 1.0 for the pressure increment ratio.

$$-(\partial e/\partial T)_o/\lambda = -(\partial e/\partial t)_o H^2/(C_V \lambda) \quad (33)$$

Procedures of measuring $-(\partial e/\partial t)_0$ and H are trivial in laboratory consolidation tests, and the method of determining λ has already been described, so that there arises a problem of determining C_v in conjunction with the determination of $-(\partial e/\partial T)_0/\lambda$. This problem will be discussed in the next section.

Fig. 3. Schematic representation of a process of one-dimensional consolidation

A METHOD OF DETERMINING COEFFICIENT OF CONSOLIDATION

It has been very common practice in the determination of the coefficient of consolidation to use either the square root of time fitting method known as the Taylor construction or the logarithm of time fitting method known as the Casagrande construction. However, these methods appear to lack a logical consistency, in the strict sense, in the case of clays exhibiting creep under constant effective stress, because they place their theoretical basis on the Terzaghi theory of consolidation that does not account for the creep effect. It was, therefore, decided to develop a method of determining C_v for clays exhibiting creep under constant effective stress, without resort to the Terzaghi theory.

Let us focus the investigation on the theoretical relationship between $U_{\bar{\epsilon}}$ and T under the pressure increment ratio of 1.0 by taking notice of a common procedure in routine one-dimensional consolidation tests. Fig. 4(a) presents one of such typical results plotted as $\log(U_{\bar{\epsilon}})$ - $\log(T)$ relation for various values of $-(\partial e/\partial T)_0/\lambda$. It appears clear from the figure that the reduction in $-(\partial e/\partial T)_0/\lambda$ allows a plot of $\log(U_{\bar{\epsilon}})$ versus $\log(T)$ to shift upwards on the figure with no significant variation of its shape, and that each plot can be approximated by two straight lines of distinct slopes; the slope of the straight line in the earlier stage is equal to $1/2$ and the slope in the later stage, which is generally affected by the variation of C_{α}^*/λ , is considerably smaller than $1/2$. Let us denote the time factor at a

Fig. 4 (a) A theoretical plot of $\log(U_{\bar{\epsilon}})$ versus $\log(T)$ for $\Delta\sigma_v/\sigma'_{v0}=1.0$

Fig. 4(b) A laboratory plot of $\log(\bar{\epsilon})$ versus $\log(t)$ for $\Delta\sigma_v/\sigma'_{v0}=1.0$ obtained for a specimen of the remolded Osaka-Nanko clay

point of intersection of the two straight lines as T_* . It is, then, found from Fig. 4(a) that T_* ranges from 1.0 to 1.2 within the hundredfold variation of $-(\partial e/\partial T)_o/\lambda$.

Fig. 4(b) indicates, for a comparison, a typical laboratory consolidation curve plotted as $\log(\bar{e}) - \log(t)$ relation for $\Delta\sigma_v/\sigma_v' = 1.0$ for a specimen of the remolded Osaka-Nanko clay mentioned later. On such a laboratory plot a straight line with a slope of 1/2 may be drawn for the early portion of it, and another straight line for the later portion of it may then be drawn in close agreement with the observed points, as demonstrated in Fig. 4(b). Let us denote the real time at a point of intersection of such two straight lines as t_* . It is immediately evident that there are remarkable resemblances between the theoretical curves and the laboratory one in both Figs. 4(a) and (b). Consequently, the coefficient of consolidation for the curve of Fig. 4(b) can be determined, by remembering Eq. (17), from

$$C_v = H^2 T_*/t_* \quad (34)$$

where it is allowed to use $T_*=1.2$ with no serious error, because experimental evidence shown later in the paper has indicated that a reasonable value of $-(\partial e/\partial T)_o/\lambda$ for the laboratory consolidation tests on thin specimens was roughly in the order of 10^{-3} and therefore the theoretical results shown in Fig. 4(a) have led to $T_*=1.2$ with a reasonable accuracy.

It is important to note that related numerical computations have provided similar results to those shown in Fig. 4(a) and confirmed the approximation of $T_*=1.2$ under the following variation of the parameters: $C_a^*/\lambda = 0.025$ to 0.040 ; $\kappa^*/\lambda = 0$ to 0.3 ; $-(\partial e/\partial T)_o/\lambda = 4.33 \times 10^{-4}$ to 4.33×10^{-2} . It is of interest to mention that the range for each of C_a^*/λ and κ^*/λ appears to cover the range for each of C_a^*/λ and κ/λ that has been observed for a relatively wide class of normally consolidated clays exhibiting secondary compression (see, for example, Mesri and Rokhsar, 1974). Thus, we lead to a method of determining the coefficient of consolidation, without resort to the prediction from the Terzaghi theory, for a relatively wide class of normally consolidated clays exhibiting creep under constant effective stress; the basic formula of the method is Eq. (34) with $T_*=1.2$.

ROLE OF THE PARAMETER $-(\partial e/\partial T)_o/\lambda$

Eq. (33) indicates that the main factors affecting $-(\partial e/\partial T)_o/\lambda$ are $-(\partial e/\partial t)_o$ and H . In this section we discuss the role of these factors in conjunction with the influence resulting from both a prolonged duration of preceding secondary compression and a decreased maximum drainage distance.

Fig. 5 presents theoretical settlement-time factor curves for a clay with model constants as indicated in the figure at various values of $-(\partial e/\partial T)_o/\lambda$ at each of the pressure increment ratios. It is apparent from Fig. 5 that the reduction in $-(\partial e/\partial T)_o/\lambda$ leads to the suppression of the settlement at a fixed value of T for each of the pressure increment ratios, although the slope of the secondary portion of each of all the curves remains unaffected by the variation of $-(\partial e/\partial T)_o/\lambda$ and also by the change in $\Delta\sigma_v/\sigma_v'$. This trend is interpreted below from two viewpoints. First, consider the case in

Fig. 5. Theoretical effects of $-(\partial e/\partial T)_o/\lambda$ on settlement-time factor relations for a clay with model constants as indicated

which $-(\partial e/\partial t)_0$ is varied while H is held constant. In this case, the trend shown in Fig. 5 leads to the prediction that a prolonged duration of preceding secondary compression suppresses to some extent the settlement under a following increment of loading, while the rate of secondary compression is left unaffected. This predicted characteristic may be understood as a kind of time-hardening and related to so-called quasi-preconsolidation effect discussed in detail by Bjerrum (1967). Secondly, consider the case in which H is varied while $-(\partial e/\partial t)_0$ is maintained constant. In this case, the following prediction is possible; the ratio of secondary to primary settlement under a pressure increment ratio increases with decreasing maximum drainage distance, in other words the creep-type settlement becomes more significant in thin strata of clay than in thick ones. This prediction is qualitatively in agreement with the observations made by Barden (1965 and 1968).

Fig. 6 presents typical results concerning the role of $-(\partial e/\partial T)_0/\lambda$ in rates of pore pressure dissipation for a clay with a set of typical constants as indicated in the figure. It is clearly seen that the reduction in $-(\partial e/\partial T)_0/\lambda$ leads to the rapid dissipation of mid-plane pore pressure $\bar{u}_m$ in the early stages of the consolidation process, although later owing to the lasting secondary compression the predicted $\bar{u}_m$ tends to be larger than predicted by the Terzaghi theory. Fig. 7 presents, for a comparison, the measured effects of preceding secondary compression on rates of dissipation of $\bar{u}_m$ obtained by Barden (1969) for remolded Kaolin. Barden (1969) states regarding the results quoted in Fig. 7 as follows: "...the initial pore pressure drop is much more pronounced following even a 7 day secondary stage than when none is allowed to develop. As can be seen, the effect is accentuated by a small value of H ." Taking notice of this statement and recalling the definition of $-(\partial e/\partial T)_0/\lambda$ given in Eq. (33), the theoretical results shown in Fig. 6 become evident in qualitative agreement with the measured ones shown in Fig. 7.

As a summary of this section, it can be said that discussion of the role of $-(\partial e/\partial T)_0/\lambda$ leads to a better understanding of the influence of both the duration of preceding secondary compression and the maximum drainage distance on the settlement and pore pressure dissipation behavior of clays. It may be appropriate to supplement that this understanding is of practical significance to correlate properly the results from laboratory one-dimensional consolidation tests on thin specimens with the long-term field behavior of thick strata of clay undergoing one-dimensional consolidation.

Fig. 6. Theoretical effects of $-(\partial e/\partial T)_0/\lambda$ on $\bar{u}_m$ -log(T) relations for a clay with model constants as indicated

Fig. 7. Measured effects of a prolonged duration of preceding secondary compression on rate of pore pressure dissipation (after Barden, 1969)

EXPERIMENTAL ASSESSMENT

A description is made in this section of the clay, apparatus and testing procedure used for the one-dimensional consolidation tests carried out by the authors, and then comparisons are made of the results from the consolidation tests with the predicted ones to assess the validity of the numerical solution and of the proposed method for the determination of C_v .

The Clay Used

The clay used is a saturated, reconstituted clay called the remolded Osaka-Nanko clay, whose basic rheological properties have been studied by Murayama, Sekiguchi and Yoshida (1973) and Sekiguchi (1974). The index properties of the clay are as follows: the liquid limit=110.5%; the plastic limit=41.0%; the clay fraction=16%; the specific gravity=2.69.

Apparatus and Testing Procedure

All the tests were carried out at a controlled room temperature of $19.5 \pm 0.5^\circ\text{C}$, using a triaxial cell of the NGI type inside which a brass hollow cylinder was installed to wrap the pedestal so as to prevent lateral displacements during the consolidation process. The inner surface of the cylinder was coated with a thin layer of silicone grease to minimize the problem of side friction. The specimens used were either 3.57cm or 4.00cm in diameter, and the initial heights were adjusted to be in the neighborhood of 2cm. During the tests drainage was allowed only from the upper surface of each specimen and measurements of pore pressures were made at the bottom of it using an electric pressure transducer under a back pressure of 1.5kg/cm^2 throughout the consolidation process.

Material Constants Determined

Table 1 lists the material constants of the remolded Osaka-Nanko clay determined by following the aforementioned procedure. The value of C_a^*/λ given in the table was chosen to be equal to the one evaluated by Sekiguchi (1974) for the clay by making use of Eq. (8).

Table 1. Material constants of the remolded Osaka-Nanko clay

λ	C_a^*/λ	κ^*/λ	C_k^*/λ
0.364	0.032	0.216	1.0

Coefficient of Consolidation Determined

The proposed method of determining C_v was applied to the results from the consolidation tests with $\Delta\sigma_v/\sigma_{v'0}$ equal to 1.0 on four specimens of the remolded Osaka-Nanko clay, and the values of the coefficient of permeability were also evaluated. The results are summarized in Table 2 together with the values of C_v determined by means of the square root of time fitting method. Table 2 indicates that the proposed method yields, in this case,

Table 2. Coefficient of consolidation and that of permeability measured by means of the proposed method for the specimens of the remolded Osaka-Nanko clay for $\sigma_{v'0}=1\text{kg/cm}^2$ and $\Delta\sigma_v/\sigma_{v'0}=1.0$

Test No.	$-(\partial e/\partial T)_0/\lambda$	$C_v(\text{cm}^2/\text{min})$	$k_0(\text{cm}/\text{min})$	$C_v^*(\text{cm}^2/\text{min})$
6-2	8.07×10^{-3}	2.89×10^{-2}	2.66×10^{-6}	1.56×10^{-2}
7-2	2.81×10^{-3}	3.18×10^{-2}	2.92×10^{-6}	1.91×10^{-2}
8-2	8.35×10^{-3}	3.38×10^{-2}	3.12×10^{-6}	1.76×10^{-2}
11-2	1.60×10^{-3}	3.40×10^{-2}	3.12×10^{-6}	2.92×10^{-2}

† Evaluated by means of $\sqrt{t}$ —fitting method.

somewhat larger values of C_v than does the square root of time fitting method. In addition, the values of the coefficient of permeability given in Table 2 allow Eq. (3) to be expressed for the remolded Osaka-Nanko clay in the form:

$$\frac{k}{2.95 \times 10^{-6}} = \exp\left(\frac{e-1.744}{0.364}\right) \quad (k \text{ in cm/min}) \quad (35)$$

In this way, once k is determined as a function of e , the values of C_v in any combinations of $\sigma_{v'0}$ and $\Delta\sigma_v/\sigma_{v'0}$ for the clay can be evaluated from Eq. (20).

Subsequently, we could use the values of C_v evaluated by means of the proposed method.

Comparisons of Theoretical and Experimental Results

The settlement-time and the mid-plane pore pressure-time curves were computed in relation to the experimental results under such loading conditions as listed in Table 3. As can be seen, each specimen of the set (I) with a value of $\Delta\sigma_v/\sigma_{v'0}$ has a smaller value of $-(\partial e/\partial T)_0/\lambda$ than does the corresponding one of the set (II) with the same value of $\Delta\sigma_v/\sigma_{v'0}$.

Fig. 8 presents the observed settlement-time curves together with the predicted ones for the three specimens of the set (I), and similarly Fig. 9 does for the specimens of the

Table 3. Loading conditions for two sets of three specimens of the remolded Osaka-Nanko clay

	Test No.	e_0	H(cm)	$\sigma_{v'0}$ (kg/cm ²)	$\psi^\dagger$	$-(\partial e/\partial T)_0/\lambda$	C_v (cm ² /min)
(I)	4-3	1.725	1.786	1.1	0.1	2.56×10^{-3}	2.42×10^{-2}
	12-2	1.693	1.931	1.0	0.5	1.40×10^{-3}	2.34×10^{-2}
	11-2	1.752	2.041	1.0	1.0	1.60×10^{-3}	3.40×10^{-2}
(II)	5-3	1.706	2.206	1.5	0.1	1.43×10^{-2}	3.11×10^{-2}
	9-3	1.565	2.063	1.5	0.5	5.05×10^{-3}	2.36×10^{-2}
	8-2	1.734	2.161	1.0	1.0	8.35×10^{-3}	3.38×10^{-2}

$^\dagger \psi = \Delta\sigma_v/\sigma_{v'0}$.

Fig. 8. Observed and predicted settlement-time curves for the specimens of the set (I)

Fig. 9. Observed and predicted settlement-time curves for the specimens of the set (II)

Fig. 10. Observed and predicted pore pressure dissipation-time curves for the specimens of the set (I)

Fig. 11. Observed and predicted pore pressure dissipation-time curves for the specimens of the set (II)

set (II). It is evident from Figs. 8 and 9 that there is good agreement between the predicted and observed settlement-time curves for all the specimens.

On the other hand, Fig. 10 for the three specimens of the set (I) and Fig. 11 for the three specimens of the set (II) indicate that the predicted values of $\bar{u}_m$ when $\Delta\sigma_v/\sigma_v'0$ is equal to 0.1 or 0.5 are somewhat larger than the measured ones in the early stages of the consolidation process though later they are reasonably close to the measured ones. It is also evident from each of Figs. 10 and 11 that when $\Delta\sigma_v/\sigma_v'0$ is equal to 1.0 the predicted pore pressure-time curve is in reasonable agreement with the observed one in the whole stage of the consolidation process. It may also be noted that the predicted and measured results shown in each of Figs. 10 and 11 confirm that $\bar{u}_m$ dissipates more rapidly in the early stage of the consolidation process with decreasing pressure increment ratio (Leonards and Girault, 1961).

CONCLUSIONS

The principal conclusions drawn from the present study may be summarized as follows:

- (1) A stress-strain-time relation relevant to the process of one-dimensional consolidation for clays exhibiting creep under constant effective stress can be deduced, by assuming K_0 -conditions, from the stress-strain-time model developed by Murayama et al. (1974) and Sekiguchi (1974).
- (2) A study of both the predicted $\log(\bar{u}_m) - \log(T)$ relations for $\Delta\sigma_v/\sigma_v'0 = 1.0$ and the corresponding $\log(\bar{\epsilon}) - \log(t)$ plot leads to a method of determining C_v for a relatively wide class of normally consolidated clays; the basic formula of the method is Eq. (34) with $T_* = 1.2$.
- (3) Discussion of the role of $-(\partial e/\partial T)_0/\lambda$ indicates that the present theory of consolidation provides a quantitative basis to a better understanding of the influence resulting from both a prolonged duration of preceding secondary compression and a decreased maximum drainage distance.
- (4) The proposed theory of consolidation in combination with the proposed method of determining C_v gives reliable predictions of both the settlement-time and the mid-plane pore pressure-time curves for the laboratory consolidation tests on the specimens of a saturated clay for each of pressure increment ratios of 0.1, 0.5 and 1.0.

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NOTATION

- C_k^* =material constant defined by Eq. (3)
- C_v =coefficient of consolidation
- C_α^* =material constant defined by Eq. (8)
- $-(\partial e/\partial t)_i$ =rate of reduction in void ratio immediately after the change of loading
- $-(\partial e/\partial t)_o$ =rate of reduction in void ratio immediately before the change of loading
- $-(\partial e/\partial T)_o/\lambda$ =parameter defined by Eq. (33)
- D =coefficient of dilatancy
- e =void ratio
- e_c =void ratio at a point C in Fig. 3
- e_i =void ratio immediately after the change of loading
- e_o =void ratio immediately before the change of loading
- H =maximum drainage distance
- $\bar{H}_o$ =relaxation spectrum of a box-type normalized by the consolidation pressure
- k =coefficient of permeability
- k_o =coefficient of permeability at $e=e_o$
- K_o =coefficient of earth pressure at rest
- N =extra stress ratio
- R =scalar factor in Eq. (32)
- t =real time
- t_* =real time at a point of intersection of two straight lines shown in Fig. 4(b)
- T =time factor
- T_* =time factor at a point of intersection of two straight lines shown in Fig. 4(a)
- u =excess pore pressure
- $\bar{u}$ =dimensionless excess pore pressure defined by Eq. (18)
- u_o =residual excess pore pressure immediately before the change of loading
- $\bar{u}_i$ =dimensionless excess pore pressure immediately after the change of loading
- $\bar{u}_m$ =dimensionless mid-plane excess pore pressure
- U_s =local degree of consolidation defined by Eq. (19)
- $U_{\bar{s}}$ =average degree of consolidation defined by Eq. (30)
- z =vertical coordinates
- $\bar{z}$ =dimensionless vertical coordinates defined by Eq. (16)
- γ_w =unit weight of pore water
- Δ =increment of any quantity
- $\bar{\epsilon}$ =settlement per unit height
- ϵ_v =volumetric strain
- η =effective stress ratio defined by $(\sigma_v' - \sigma_H')/\sigma_m'$
- η_o =effective stress ratio immediately before the change of loading
- κ =swelling index defined as a slope of $e - \ln(\sigma_m')$ plot
- κ^* =material constant defined by Eq. (10)
- λ =compression index defined as a slope of $e - \ln(\sigma_m')$ plot
- σ_m' =mean effective stress defined by $(\sigma_v' + 2\sigma_H')/3$

- $\sigma_{m'o}$ = mean effective stress immediately before the change of loading
 σ_v = vertical total stress
 σ_v', σ_h' = vertical and horizontal effective stress, respectively
 σ_{v0} = vertical total stress immediately before the change of loading
 $\sigma_{v'f} = \sigma_{v'o} + \Delta\sigma_v$
 ϕ = pressure increment ratio ($= \Delta\sigma_v / \sigma_{v'o}$) appearing in Eqs. (21), (22), (23) and (32)

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APPENDIX

Derivation of Eq. (6):— Let us consider the behavior of normally consolidated clay under axi-symmetric conditions of triaxial compression. Shibata (1963) has obtained the following relation expressing the change in volumetric strain when subjected to simultaneous variations of σ_m' and η ;

$$\epsilon_v = \epsilon_{vp} + \epsilon_{v\eta} \quad (\text{A-1})$$

in which

$$\epsilon_{vp} = \frac{\lambda}{1+e_o} \ln\left(\frac{\sigma_m'}{\sigma_{m'o}}\right) \quad (\text{A-2})$$

$$\epsilon_{v\eta} = D\eta \quad (\text{A-3})$$

where ϵ_{vp} refers to the volumetric strain due to isotropic compression and $\epsilon_{v\eta}$ stands for the volumetric strain due to dilatancy. Putting $\epsilon_v = 0$ in Eq. (A-1), we obtain the relation for the undrained effective stress path as follows (Hata and Ohta, 1968):

$$\eta = \frac{\sigma_d}{\sigma_m'} = \frac{D(1+e_o)}{\lambda} \ln\left(\frac{\sigma_{m'o}}{\sigma_m'}\right) \quad (\text{A-4})$$

where σ_d denotes $\sigma_v' - \sigma_h'$. It must be noted that Eqs. (A-1) through (A-4) are concerned with the volumetric behavior in states of equilibrium where the rate of strain is imperceptibly slow.

Let us modify Eq. (A-3) in the form:

$$\dot{\epsilon}_{v\eta} = D(\dot{\eta} - \dot{\eta}_R) \quad (\text{A-5})$$

in order to take the time-dependency of dilatancy into consideration, where the superposed dot refers to the time derivative, and η_R is defined as the time-dependent critical value of η below which no dilatancy occurs. Under conditions of sustained loading it follows from Eq. (A-5) that

$$\dot{\epsilon}_{v\eta} = -D\dot{\eta}_R \quad (\text{A-6})$$

In order to obtain the expression for η_R , we first extend Eq. (A-4) in the form:

$$\frac{\sigma_d - \sigma_d^R}{\sigma_m' - \sigma_m'^R} = \frac{\lambda}{D(1+e_o)} \ln\left(\frac{\sigma_{m'o}}{\sigma_m' - \sigma_m'^R}\right) \quad (\text{A-7})$$

where σ_d^R and $\sigma_m'^R$ refer to the extra stresses that depend on the change in time scale (Murayama et al., 1974 and Sekiguchi, 1974). Experimental evidence from stress relaxation tests under undrained conditions suggests that

$$\sigma_m'^R = \sigma_d^R / N \quad (\text{A-8})$$

where N is called the extra stress ratio and is found to take the values almost equal to 3 (see, for example, Fig. 4 of Murayama et al., 1974). Substituting Eq. (A-8) into Eq. (A-7) and then putting $\sigma_m' = \sigma_{m'o}$, we obtain the expression for η_R as follows:

$$\eta_R \left(= \frac{\sigma_d}{\sigma_{m'o}} \right) = \frac{\sigma_d^R}{\sigma_{m'o}} - \frac{\lambda}{D(1+e_o)} \left(1 - \frac{\sigma_d^R}{N\sigma_{m'o}} \right) \ln \left(1 - \frac{\sigma_d^R}{N\sigma_{m'o}} \right)$$

This relation can be approximated under the condition of $\frac{\sigma_d^R}{N\sigma_{m'o}} \ll 1$ in the form:

$$\eta_R \doteq \left\{ 1 + \frac{\lambda}{D(1+e_o)N} \right\} \frac{\sigma_d^R}{\sigma_{m'o}} \quad (\text{A-9})$$

Following the procedure described by Murayama et al. (1974), we can obtain $\sigma_d^R/\sigma_{m'o}$ under shearing at a constant rate of shear strain, when the shear strain $\gamma (= \varepsilon_v - \varepsilon_H)$ exceeds the critical shear strain γ_c , in the form:

$$\frac{\sigma_d^R}{\sigma_{m'o}} = \frac{3}{2} \bar{H}_o \left[\left\{ 1 - \exp \left(-\frac{\gamma_c}{\dot{\gamma} \lambda_u} \right) \right\} / \left(\frac{\gamma_c}{\dot{\gamma} \lambda_u} \right) - E_i \left(-\frac{\gamma_c}{\dot{\gamma} \lambda_u} \right) \right] \quad (\text{A-10})$$

where $\bar{H}_o (= H_o/\sigma_{m'o})$ is the relaxation spectrum of a box-type which is normalized by the consolidation pressure, $\dot{\gamma}$ is the rate of shear strain, λ_u the maximum relaxation time, and $E_i(\cdot)$ the exponential integral function. (N.B. In the second term on the right-hand side of Eq. (41) of Murayama et al. (1974), ε_{1c} and $\dot{\varepsilon}_1$ are used, respectively, instead of γ_c and $\dot{\gamma}$ for undrained steady shearing.) When $0 < \gamma_c/(\dot{\gamma} \lambda_u) \ll 1$, Eq. (A-10) can be approximated as

$$\frac{\sigma_d^R}{\sigma_{m'o}} = \frac{3}{2} \bar{H}_o \left\{ 1 - \gamma_E - \ln \left(\frac{\gamma_c}{\dot{\gamma} \lambda_u} \right) \right\}$$

where γ_E denotes the Euler's constant. Differentiating the above equation with respect to time and using Eq. (A-9), we get

$$\dot{\eta}_R = \frac{3}{2} \bar{H}_o \left\{ 1 + \frac{\lambda}{D(1+e_o)N} \right\} \frac{\ddot{\gamma}}{\dot{\gamma}} \quad (\text{A-11})$$

Let us suppose here that under conditions of sustained loading

$$\frac{\ddot{\varepsilon}_{v\eta}}{\dot{\varepsilon}_{v\eta}} = \frac{\ddot{\gamma}}{\dot{\gamma}} \quad (\text{A-12})$$

Then, it follows from Eqs. (A-6), (A-11) and (A-12) that

$$\dot{\varepsilon}_{v\eta} = -\frac{C_a^*}{1+e_o} \cdot \frac{\ddot{\varepsilon}_{v\eta}}{\dot{\varepsilon}_{v\eta}} \quad (\text{A-13})$$

where

$$C_a^* = (3/2) \bar{H}_o \{ D(1+e_o) + \lambda/N \} \quad (\text{A-14})$$

It should be noted here that Eq. (A-14) is the same as Eq. (8).

Under conditions of sustained loading we can put $\dot{\varepsilon}_{vp} = 0$ except at $t = +0$, so that Eq. (A-13) is rewritten as

$$\dot{\varepsilon}_v = -\frac{C_a^*}{1+e_o} \cdot \frac{\ddot{\varepsilon}_v}{\dot{\varepsilon}_v} \quad (\text{A-15})$$

Integrating (A-15), we have

$$\varepsilon_v = -\frac{C_a^*}{1+e_o} \ln(\dot{\varepsilon}_v) + C$$

where C is the constant of integration and is determined by assuming the following condition; when $\dot{\varepsilon}_v$ decreases with increasing time to the value of $\dot{\varepsilon}_v$ immediately before the change of loading (i.e. when $\dot{\varepsilon}_v = \dot{\varepsilon}_{v0}$), the value of ε_v becomes equal to $\frac{\lambda}{1+e_o} \ln \left(\frac{\sigma_{m'}}{\sigma_{m'o}} \right) + D(\eta - \eta_o)$. Thus, we finally obtain that

$$\varepsilon_v = \frac{C_a^*}{1+e_o} \ln \left(\frac{\dot{\varepsilon}_{v0}}{\dot{\varepsilon}_v} \right) + \frac{\lambda}{1+e_o} \ln \left(\frac{\sigma_{m'}}{\sigma_{m'o}} \right) + D(\eta - \eta_o) \quad (\text{A-16})$$

where

$$\dot{\epsilon}_{v0} = \left(\frac{\partial \epsilon_v}{\partial t} \right)_{\sigma_m', \eta_0}$$

$$\dot{\epsilon}_v = \left(\frac{\partial \epsilon_v}{\partial t} \right)_{\sigma_m', \eta}$$

It is, thus, evident that Eq. (A-16), with Eq. (A-14) for C_α^* , is the same as Eq. (6).

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