CONSOLIDATION OF EMBANKMENT FOUNDATION

MIKIO SHOJI* and TAKASHI MATSUMOTO**

ABSTRACT

In this paper a variational principle is introduced in which the Euler equations are the theoretical three-dimensional equations of consolidation in fully saturated and heterogeneous poro-elastic media with the fundamental consideration by Sandhu and Wilson. A finite element method is then used in both the space and time domains to develop a practical solution technique. Furthermore, in order to discuss the various problems and difficulties in applying this numerical method to the practical design of foundation settlements, the results of the analyses are compared with the observations on the Aiko test embankment for the construction of the Tomei Expressway. The calculated settlements and pore water pressure fields agree well with the measurements when the permeability coefficient can be predicted accurately, except the parts where plastic settlements occur. As previously pointed out, however, when the permeability coefficient obtained from the oedometer test is used, the field time-rates appear to be faster than indicated by the analyses.

Key words: consolidation, design, effective stress, elasticity, pore pressure, finite element method

IGC: E2

INTRODUCTION

In applying the one-dimensional theory of consolidation by Terzaghi (1924) to calculating settlements of embankment foundation, it has been pointed out that the settlements predicted from his theory do not agree with the actual ones since two- or three-dimensional deformations actually occur, e.g., Darragh (1964). Therefore, many studies on the three-dimensional consolidation have been conducted so far.

At first, three-dimensional consolidation problems were tackled by making simplifications and deriving an approximate one-dimensional problem which could be solved by using Terzaghi's theory. Subsequently, two theories have been proposed. The first, which is generally called the three-dimensional consolidation theory of Terzaghi, has been proposed by Rendulic (1935), Terzaghi (1948) and Barron (1948). The second has been developed by Biot in a series of papers (1941, 1955, 1956); in these papers we shall be concerned with Biot's equations for poro-elastic medium and their general solutions. In the latest studies, Akai (1972) has shown the effective stress path at every point in the ground, and compared the settlements due to shear deformation at the instant of loading with those due to the following compression by using the concept of compliance of soil skeleton. Yoshikuni (1972, 1973) has introduced the concept of "consolidation potential" in his funda-

* Senior Research Engineer, Kajima Institute of Construction Technology, Tobitakyu, Chofu, Tokyo.

** Research Engineer, Kajima Institute of Construction Technology, Tobitakyu, Chofu, Tokyo.

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mental equation of consolidation equivalent to Biot's equations, and has tried to elucidate systematically the previous one- or three-dimensional consolidation theories.

As mentioned above, the theories of three-dimensional consolidation are classified broadly into Terzaghi-Rendulic type and Biot type. The two categories have been compared frequently, e.g., by Gibson, Knight and Taylor (1963) or Yoshikuni (1972). It admits of no doubt from various points of view that the Biot type is superior to the Terzaghi-Rendulic type. However, Biot's theory has not been used practically, but Terzaghi-Rendulic theory has been frequently used. One of the reasons may be that no available solution of Biot's equation has been prepared. The analytical solutions of Biot's equations were proposed by McNamee and Gibson (1957, 1960) and others. However, they are for simple models, and it is impossible to obtain closed form solutions for heterogeneous systems with complex geometry and arbitrary conditions.

Today many of severe limitations and restrictions which have been imposed in previous treatments can be removed owing to improved analytical capability of modern digital computers and the powerful finite element method. The advantage of this method, as compared with other numerical approaches, are numerous. The method is completely general with respect to geometry and material properties. Since each element in the system may have different properties, complex bodies composed of many different layers and anisotropic materials are easily represented. Arbitrary boundary conditions of displacements, stresses or pore water pressures may be specified at any point in the finite element system. Mathematically, it can be shown that the method converges to the exact solutions as the number of elements is increased. Therefore, any desired degree of accuracy may be obtained.

Christian and Boehmer (1970, 1972) formulated a mixed type of finite element and finite difference method. Because of the nature of finite difference technique, fictitious elements outside the region must be introduced.

Sandhu and Wilson (1969) or Yokoo, Yamagata and Nagaoka (1971) formulated the same finite element discretization as the authors. However, their formulas appear to be difficult to represent faithfully the fundamental equations because of the nature of interpolation functions.

The above-mentioned studies were restricted to theoretical treatment and did not discuss the applicability of the theories to the practical problems. One of the reasons may be that the method to evaluate the mechanical constants necessary to Biot's theory such as elastic modulus or Poisson's ratio has not been established since the existing test of consolidation should be schemed out for the sake of Terzaghi's one-dimensional theory.

The object of this research is to obtain the accurate solutions of consolidation to make a practical design of embankment foundation with complex geometry and arbitrary boundary conditions. In order to accomplish the above mentioned object, a finite element method is used for Biot's three-dimensional consolidation theory. The procedure to determine the constants such as elastic modulus, Poisson's ratio and permeability coefficient from the results of both consolidation and unconfined compression tests is then discussed. Furthermore the results of the analyses are compared with the observations on a full size test embankment in the field (the Aiko test embankment), and the effects such as the lateral permeability or the intensity of both precompression load and applied load of embankment are discussed.

FUNDAMENTAL EQUATIONS OF THREE-DIMENSIONAL CONSOLIDATION

The procedure of analysis described in this and following sections is the same one developed by Sandhu and Wilson (1969). Then the authors will explain it briefly.

The equilibrium equations for fully saturated soil can be written as

$$(\bar{\sigma}_{ij} + u_w \delta_{ij}),_j + F_i = 0 \quad (1)$$

in which $\bar{\sigma}_{ij}$ = the components of the symmetric effective stress tensor; u_w = the excess pore water pressure; F_i = the components of the body force vector and δ_{ij} = the Kronecker delta. Subscript after a comma denotes spatial differentiation in the standard indicial notation used and repeated indices indicate summation.

Assuming small deformations, the components of the strain tensor ϵ_{ij} are

$$\epsilon_{ij} = 1/2(u_{i,j} + u_{j,i}) \quad (2)$$

in which u_i = the rectangular cartesian components of the displacement vector.

For linear elasticity, the effective stress and strain of soil skeleton are related as

$$\bar{\sigma}_{ij} = E_{ijkl} \epsilon_{kl} \quad (3)$$

in which E_{ijkl} = the components of the elasticity tensor which has the usual symmetric properties. For an anisotropic porous solid, Darcy's law can be written as

$$v_i = -k_{ij} u_{w,j} \quad (4)$$

in which v_i = the components of the stream velocity vector and k_{ij} = the components of the permeability tensor. When using finite element method, it is convenient to define that the tensile stress and tensile strain are positive. Then, the relation between volumetric strain and inflow due to the pore water pressure field can be written as

$$\dot{\epsilon}_{ii} = -v_{i,i} \quad (5)$$

in which a superposed dot denotes differentiation with respect to time.

The boundary conditions are

$$\left. \begin{aligned} u_i(x, t) &= \bar{u}_i(x, t) && \text{on } S_1 \times [0, \infty) \\ n_j \{ \bar{\sigma}_{ij}(x, t) + u_w(x, t) \delta_{ij} \} &= \bar{T}_i(x, t) && \text{on } S_2 \times [0, \infty) \\ -k_{ij} u_{w,j}(x, t),_j &= \bar{v}(x, t) && \text{on } \zeta_1 \times [0, \infty) \\ u_w(x, t) &= \bar{u}_w(x, t) && \text{on } \zeta_2 \times [0, \infty) \end{aligned} \right\} \quad (6)$$

in which x = the spatial co-ordinate vector; t = time and n_i = the direction cosine of outward normal to the boundary. S_1, S_2, ζ_1 and ζ_2 , respectively, denote the boundary surfaces on which are prescribed displacement $\bar{u}_i$, stress $\bar{T}_i$, stream velocity of water $\bar{v}$ and pore water pressure $\bar{u}_w$. For two sets A and B , $A \times B$ implies the cartesian product, e.g., $S_1 \times [0, \infty)$ is the set of all ordered pairs (x, t) such that $x \in S_1$ and $t \in [0, \infty)$. Eqs. (4) and (5) with initial condition $\epsilon_{ii} = 0$ are equivalent to the following equation (Gurtin, 1964).

$$\epsilon_{ii} = (k_{ij} * u_{w,j}),_i \quad (7)$$

The symbol (*) designates convolution integral which is defined as (Titchmarsh, 1926)

$$f * g = \int_0^t f(\tau) g(t - \tau) d\tau \quad (8)$$

VARIATIONAL FORMULATION AND FINITE ELEMENT DISCRETIZATION

Variational formulation

The variational principle equivalent to the fundamental equations (1), (6) and (7) is as follows: For the functional $\mathcal{Q}(t)$ defined by

$$\begin{aligned} \mathcal{Q}(t) &= \int_{S_1} 1 * \bar{v} * u_w dS - \int_{S_2} \bar{T} * u_i dS \\ &+ \int_V \left(\frac{1}{2} \bar{\sigma}_{ij} * \epsilon_{ij} + u_w * \epsilon_{ii} + \frac{1}{2} k_{ij} * u_{w,i} * u_{w,j} - F_i * u_i \right) dV \end{aligned} \quad (9)$$

The first variation $\delta Q(t)$ becomes, by the divergence theorem,

$$\begin{aligned}\delta Q(t) = & \int_{\zeta_1} 1 * (\bar{v} + n_i k_{ij} u_{w,j}) * \delta u_w dS \\ & + \int_{S_2} \{n_j (\bar{\sigma}_{ij} + u_w \delta_{ij}) - \bar{T}_i\} * \delta u_i dS \\ & - \int_V \{(\bar{\sigma}_{ij} + u_w \delta_{ij}),_j + F_i\} * \delta u_i dV \\ & + \int_V \{\epsilon_{ii} - (k_{ij} * u_{w,j}),_i\} * \delta u_w dV = 0\end{aligned}\quad (10)$$

By the fundamental lemma of calculus of variation which include convolution of Eq. (10) for each δu_i and δu_w which satisfy

$$\delta u_i = 0 \quad \text{on } S_1 \quad (11)$$

$$\delta u_w = 0 \quad \text{on } \zeta_2 \quad (12)$$

is equivalent to Eqs. (1), (6) and (7).

Finite element discretization

The spatial domain is divided into a finite number of nontrivial subregions or elements. The displacement and pore water pressure fields within each element are defined in terms of nodal point values.

$$\underline{u}^n = \underline{\Phi}_u^{nT} \underline{u} \quad (13)$$

$$\underline{u}_w^n = \underline{\Phi}_{u_w}^{nT} \underline{u}_w \quad (14)$$

in which $\underline{u}^n$ = the n -th component of the displacement vector in the element, $\underline{u}$ = the set of the n -th components of nodal point displacements for the entire system and $\underline{\Phi}_u^n$ = the set of displacement interpolation functions. A superposed T denotes the transposition. Similarly, $\underline{u}_w^n$, $\underline{u}_w$ and $\underline{\Phi}_{u_w}^n$ are, respectively, the scalar pore water pressure fields in the element, the set of nodal point pore water pressures and the set of pore water pressure interpolation functions. By proper differentiation of Eqs. (13) and (14), the reduced strain tensor in the element $\underline{\epsilon}^n$, the volumetric strain scalar $\underline{\epsilon}_v$ and the pressure gradient vector $\underline{\zeta}^n$ can be obtained as

$$\underline{\epsilon}^n = \underline{\Phi}_\epsilon^{nT} \underline{u} \quad (15)$$

$$\underline{\epsilon}_v^n = \underline{\Phi}_{\epsilon_v}^{nT} \underline{u} \quad (16)$$

$$\underline{\zeta}^n = \underline{\Phi}_\zeta^{nT} \underline{u}_w \quad (17)$$

Eqs. (3) and (15) yield the reduced stress tensor in the element $\underline{\sigma}^n$

$$\underline{\sigma}^n = \underline{E}^n \underline{\Phi}_\epsilon^{nT} \underline{u} \quad (18)$$

in which $\underline{E}^n$ = the reduced elasticity tensor for the element.

For a total element, using Eqs. (13) through (18), the functional in Eq. (9) can be written, in terms of nodal point values of pore water pressure and the components of displacement as

$$\begin{aligned}Q(t) = & \underline{u}_w^{T*} \underline{P}_1 - \underline{u}^{T*} \underline{P}_2 + \frac{1}{2} \underline{u}^{T*} \underline{K} \underline{u} \\ & + \underline{u}_w^{T*} \underline{M} \underline{u} + \frac{1}{2} \underline{u}_w^{T*} \underline{H} \underline{u}_w - \underline{u}^{T*} \underline{P}_3\end{aligned}\quad (19)$$

in which

$$\underline{P}_1 = \sum_n \int_{\zeta_1} \underline{\Phi}_{u_w}^n * \bar{v} dS \quad (20)$$

$$\underline{P}_2 = \sum_n \int_{S_2} \underline{\Phi}_u^{nT} \underline{T} dS \quad (21)$$

$$\underline{K} = \sum_n \int_{V^n} \underline{\Phi}_e^{nT} \underline{E} \underline{\Phi}_e^n dV \quad (22)$$

$$\underline{M} = \sum_n \int_{V^n} \underline{\Phi}_{u_w}^n \underline{\Phi}_{u_w}^{nT} dV \quad (23)$$

$$\underline{H} = \sum_n \int_{V^n} \underline{\Phi}_z^{nT} \underline{k} \underline{\Phi}_z^n dV \quad (24)$$

$$\underline{P}_3 = \sum_n \int_{V^n} \underline{\Phi}_u^{nT} \underline{F} dV \quad (25)$$

in which $\underline{T}$ and $\underline{F}$, respectively, are the set of nodal point applied surface force and body force. The first variation of the functional of Eq. (19) results in the integral equations governing the consolidation problem.

$$\underline{K}\underline{u} + \underline{M}^T \underline{u}_w = \underline{P}_1 + \underline{P}_2 \quad (26)$$

$$\underline{H}^* \underline{u}_w + \underline{M}\underline{u} + \underline{P}_3 = 0 \quad (27)$$

In physical terms, Eq. (26) is the equilibrium equations and Eq. (27) is the equations of continuity of flow.

From Eqs. (26) and (27), the equation with respect to $\underline{u}_w$ is given as

$$\underline{H}^* \underline{u}_w - \underline{Q}\underline{u}_w + \underline{R} = 0 \quad (28)$$

in which

$$\underline{Q} = \underline{M}\underline{K}^{-1}\underline{M}^T \quad (29)$$

$$\underline{R} = \underline{P}_3 + \underline{M}\underline{K}^{-1}(\underline{P}_1 + \underline{P}_2) \quad (30)$$

The step-by-step solution technique with respect to time developed by Zienkiewicz and Parekh (1970) is applied to Eq. (28) in order to obtain $\underline{u}_w$. Next $\underline{u}_w^{(n+1)}$ at time $t_n + \Delta t$ can be expanded in terms of $\underline{u}_w^{(n)}$ at time t_n .

$$(\Delta t/2 \underline{H} - \underline{Q}) \underline{u}_w^{(n+1)} = -(\Delta t/2 \underline{H} + \underline{Q}) \underline{u}_w^{(n)} + (\underline{R}^{(n+1)} - \underline{R}^{(n)}) \quad (31)$$

Interpolation function

A triangular element is employed in this analysis. Then, it is more general and convenient to use a linear interpolation for the displacement and pore water pressure fields (see Fig. 1).

$$\underline{\Phi}_u^n = \begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k \end{bmatrix} \quad (32)$$

Fig. 1. Typical triangular element

Fig. 2. Dimensions of embankment and thickness of various layers

$$\Phi_{uw}^{nT} = \{N_i N_j N_k\} \quad (33)$$

$$N_i = (a_i + b_i x + c_i z) / 2\Delta \quad (34)$$

$$2\Delta = \begin{vmatrix} 1 & x_i & z_i \\ 1 & x_j & z_j \\ 1 & x_k & z_k \end{vmatrix} \quad (35)$$

$$a_i = x_j z_k - x_k z_j$$

$$b_i = z_j - z_k$$

$$c_i = x_k - x_j \quad (36)$$

EXAMPLE-THE AIKO TEST EMBANKMENT

In order to discuss various problems and difficulties in applying the method mentioned in the previous section to a practical analysis of foundation settlements, the numerical results are compared with the observations of settlements and pore water pressures on the Aiko test embankment which was constructed on soft ground for Japan Highway Public Corporation.

The cross section of the test embankment as shown in Fig.2 is as follows: The height is 4.5m, the bottom width is 26.2m and the crown width is 10m. Fig. 2 also shows the locations of settlement and pore pressure gauges. The longitudinal crown width of embankment is 20m. In the finite element analysis of consolidation conducted during this investigation, plane strain deformation is assumed though it remains problematical to assume plane strain conditions for the test embankment of which length is not sufficiently greater than the cross-sectional dimensions.

Then, the various soil explorations, soil tests and observations used in this investigation are taken from "Report of the Aiko test embankment for the construction of the Tomei Super-highway, No.I" compiled by Japan Highway Public Corporation (1966).

Geotechnical outline

The test embankment was constructed in the Aiko district in the middle of northern Kanagawa Prefecture. Judging from a geological survey, the substratum is composed of

Fig. 3. Soil properties of test embankment foundation
(exploration by Japan Highway Public Co.)

the Tertiary formation called the Miura group and the Ashigara group. The soft ground of the alluvial deposit underlain by the substratum is composed of organic soil which was accumulated on the drowned valley formulated by eroding Kanto loam.

Fig. 3 shows the soil profile of the test embankment foundation. This figure also shows the effective vertical stress distribution on the 18th day of banking (at the end of the work) and on the 100th day calculated in Case III which will be discussed later. As seen

Fig. 4. Typical compression curves (tests by Japan Highway Public Corporation)

Fig. 5. Typical coefficient of consolidation versus effective stress (tests by Japan Highway Public Co.)

Fig. 6. Typical coefficient of volume compressibility versus effective stress (tests by Japan Highway Public Corporation)

Fig. 7. Typical permeability coefficient versus effective stress (tests by Japan Highway Public Corporation)

Fig. 8. Stress-strain curves of unconfined compression tests and tangent moduli E_{50} illustrated by straight lines (tests by Japan Highway Public Corporation)

Fig. 9. Results of consolidated-undrained triaxial tests (tests by Japan Highway Public Corporation)

from Fig. 3, the soft ground can be divided into three layers, that is, layer I (0.0-2.3m), layer II (2.3-5.5m) and layer III (5.5-10.0m).

A). Consolidation characteristics

Typical compression curves obtained from the oedometer test are shown in Fig. 4, which illustrates the remarkable difference among the three layers. Fig. 4 also shows the extent of the initial effective stress (the effective overburden pressure P_e) and the ultimate effective stress (P_e plus calculated effective vertical stress σ_v'). Also shown in Fig. 4 is the precompression load P_0 in each layer. As can be seen from this figure, $P_e + \sigma_v'$ that occurred in each layer is less than 1.0 kg/cm^2 , which demonstrates that the ultimate effective stress in layer III is in an over-consolidated state and in layer I and II a normally consolidated state.

Fig. 5 shows typical plots of the coefficient of consolidation versus effective stress. Fig. 6 and Fig. 7, respectively, show the coefficient of volume compressibility versus effective stress and the permeability coefficient versus effective stress. These figures also show the values of P_e , P_0 and $P_e + \sigma_v'$ as in Fig. 4.

B). Shear strength

The stress-strain curves in each layer obtained from the unconfined compression test are shown in Fig. 8, in which the thin straight lines illustrate the tangent moduli E_{50} . The strains at failure fall in the range of 4-8% in layer I, 10-15% in layer II and 2-4% in layer III. Fig. 9 shows the relation between shear strength and normal stress obtained from the consolidated-undrained triaxial tests.

Finite element idealization and boundary conditions

It is desirable that the lateral boundaries are far enough from the embankment to a point where any further increase in the distance would have a negligible effect on the values of stress, strain and displacement calculated at the boundary. However, since the amount of computer time and capacity should be kept to a minimum necessary for accurate representation of the system, the proper lateral extent of foundation layer should be determined before analyzing the consolidation. There are various criteria to determine the boundary position of lateral extent, e.g., Dunlop, Duncan and Seed (1968) or Christian and Bohmer (1972). In this report, the lateral extent is determined at the position of 40m from the axis of symmetry as shown in Fig. 2, which is equal to about three times the bottom width of the embankment. For such a region, Fig. 10 shows the finite element mesh containing 177 elements and 106 nodal points.

Fig. 10. Finite element idealization

The nodal points along the symmetric axis and the lateral extent are constrained from moving only horizontally and those along the bottom boundary are constrained from moving either horizontally or vertically. Drainage occurs from both the top and bottom surface of the foundation soil.

Determination of elastic modulus and permeability coefficient

In order to analyze consolidation problem using the method in this paper, the physical and mechanical constants required for calculation are the following three constants: modulus of elasticity, Poisson's ratio and permeability coefficient.

A). Modulus of elasticity and Poisson's ratio

The relation between stress and strain of soil is not linear-elastic during even the initial phase of loading so that the hysteresis loop characteristic occurs due to unloading. Then, it is impossible to get the modulus of elasticity from stress-strain curves in the strict sense of the term. However, in this analysis tangent modulus E_{50} on the unconfined compression test (straight lines in Fig. 8) should be used to analyze instead of modulus of elasticity. Since the initial shear stress can not be expressed in the unconfined compression test, the stress-strain relations from the test may be different from the real relation in real foundations. However, since it is very difficult to get practical stress-strain relations, the authors should judge from the results of their analyses whether or not the use of E_{50} is justified. Furthermore, in spite of assuming plane strain deformation in this investigation, the modulus of elasticity should be estimated from the unconfined compression test since it is not expedient from the design point of view to estimate stress-strain relation from plane

strain test because of the complexity of the scheme and apparatus used on the plane strain test.

On the other hand, the modulus of elasticity can also be predicted from the coefficient of volume compressibility on the oedometer test. It is a familiar concept that the volume change of clay consists of two components: compression and dilatancy. The volume change of clays due to dilatancy is a function defined by the applied shear stress and effective confined pressure, e.g., Shibata (1963) or Ohta (1971). In order to find the characteristics of dilatancy, the triaxial drained test should be repeated frequently. However, the authors should substitute the coefficient of volume compressibility m_v , from a practical point of view.

When plane strain deformation is assumed, the relation between the modulus of elasticity E and m_v can be written as

$$E = \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \frac{1}{m_v} \quad (37)$$

in which ν = the effective stress Poisson's ratio.

Since the unconfined compression test should be regarded as an undrained test, E_{50} is equivalent to the total stress elastic constant. Then, E_{50} must be converged into effective stress elasticity modulus because of introducing the concept of effective stress in this paper. The shear modulus, $G = E/2(1+\nu)$, must be the same for the material with or without pore fluid because there is no volume change, and the Poisson's ratio $\bar{\nu}$ in terms of total stress must be one-half. Equating the shear moduli in terms of total and effective stress yields the relation between effective stress constants and total stress constants for the undrained case

$$\frac{E}{E_{50}} = \frac{1+\nu}{1+\bar{\nu}} = \frac{1+\nu}{1.5} \quad (38)$$

The authors employ the mean value of moduli of elasticity predicted from E_{50} and m_v .

Fig. 11. Estimated moduli of elasticity

Since soil is not an elastic medium in the strict sense of the words as mentioned above, Poisson's ratio can not be measured directly. However, ν is related with the coefficient of earth pressure at rest K_0 as

$$K_0 = \nu / (1 - \nu) \quad (39)$$

Table 1. Moduli of elasticity in this investigation

	Elastic Modulus (kg/cm ²)	Poisson's Ratio
Layer I	6.0	0.333
Layer II	2.0	0.333
Layer III	8.5	0.333

K_0 remains to be nearly equal to one-half during consolidation process according to the study by Akai and Adachi (1965). The Poisson's ratio ν is equal to one-third. In this paper, Poisson's ratio of 1/3 is used.

Fig. 11 shows the moduli of elasticity of each layer obtained by putting E_{50} in Fig. 8, m_v in Fig. 6 and $\nu=1/3$ into Eqs. (37) and (38). From this figure, the moduli of elasticity in each layer used in this calculation are shown in Table 1.

B). Permeability coefficient

The typical relation between the permeability coefficient and the effective stress obtained from the oedometer test is shown in Fig. 7. As this figure demonstrates, the permeability becomes lower as the effective stress increases. Especially, the permeability coefficient decreases steeply when the effective stress becomes larger than the pre-compression load and comes into the normally consolidated state. However, the permeability coefficient should be regarded as constant during the over-consolidated state.

Now, since there might be many cases that the horizontal permeability is higher than the vertical permeability, it is questionable to use the permeability coefficient predicted from the oedometer test in which the drainage occurs only vertically.

In this analysis, consolidation problems are calculated for the following three cases: Case I-the permeability coefficient of the parts regarded as constant obtained from the oedometer test are used; Case II-horizontal permeability coefficients are ten times the vertical permeability coefficients; Case III-the permeability coefficients determined by trial and error to coincide with observations are used (see Table 2).

Table 2. Permeability coefficient in this investigation

	permeability coefficient (cm/sec)					
	case I		case II		case III	
	k_H	k_V	k_H	k_V	k_H	k_V
Layer I	2.1×10^{-7}	2.1×10^{-7}	2.1×10^{-6}	2.1×10^{-7}	2.1×10^{-6}	2.1×10^{-6}
Layer II	4.0×10^{-5}	4.0×10^{-5}	4.0×10^{-4}	4.0×10^{-5}	1.7×10^{-3}	4.0×10^{-4}
Layer III	2.8×10^{-5}	2.8×10^{-5}	2.8×10^{-4}	2.8×10^{-5}	2.8×10^{-5}	2.8×10^{-5}

Results of analyses and discussions

A). Pore water pressures and stresses

The variation of the excess pore water pressure at the depth of 2.25m, 4.3m and 7.2m under the center of embankment is shown in Fig.12. In Case III, the results of analyses agree well with measurements when the accuracy of pore pressure gauge is taken into consideration. In Cases I and II, the excess pore water pressures at the depth of 2.25m and 4.3m have a trend toward much later dispersion than the observations. This might suggest that the real ground has higher permeability coefficient than the specimen in the oedometer.

It is not correct to compare the pre-compression load P_0 with the effective overburden pressure plus calculated effective vertical stress ($Pe + \sigma_v'$), because the two stress states are not on the same principal stress plane. However, it may be possible to discuss them roughly for comparison. The stress of Pe plus σ_v' calculated in Case III exceeds P_0 and

Fig. 12. Observed and calculated variations of excess pore water pressure

comes into normally consolidated state on the 18th day of banking (at the end of the work) in layers I and II as shown in Fig. 3. The value of $P_e + \sigma_v'$ in layer III does not exceed P_0 during 100 days. As can be seen from Figs. 6 and 7, the coefficients of volume compressibility and of permeability decrease steeply when $P_e + \sigma_v'$ exceeds P_0 . The excess pore water pressures at the depth of 2.25m and 4.3m in Fig. 12 appear to dissipate faster than the observations after the end of banking (18 days). This indicates that the lower permeability coefficients than those in Table 2 should be used in calculation when $P_e + \sigma_v'$ exceeds P_0 . The discussions mentioned above should suggest that the intensity of pre-compression load exerts an influence on the determination of the moduli of elasticity and permeability coefficients.

The induced pore water pressure distribution on the 18th day of banking (at the end of the work) is obtained as shown in Fig. 13. The one-dimensional consolidation theory is frequently used to predict settlements under the embankment as a conventional technique of design of foundation settlements, though lateral deformation occurs in reality. As can be seen from this figure, the contour line of pore water pressure distribution and the surface line of foundation layer run parallel with each other within an extent of 1.0–2.0m from the axis of symmetry. Then, the conventional scheme, namely the one-dimensional theory, should be used within only the territory of 1/5–2/5 of the crown width as shown

Fig. 13. Excess pore water pressure distribution on 18th day calculated in Case III

Fig. 14. Variations of effective and total vertical stress distribution calculated in Case III

Fig. 15. Observed and calculated variations of settlement

Fig. 16. Observed and calculated variations of settlement in each layer

Fig. 17. Observed and calculated variations of vertical displacement away from toe of slope

Fig. 18. Observed and calculated variations of horizontal displacement away from toe of slope

by the broken line in Fig. 13.

Fig. 14 shows isochrones of the effective and total vertical stresses calculated for Case III. The excess pore water pressure that occurred due to the embankment load does not propagate to layer III on the tenth day of banking when the work reaches half height. After the 18th day when the banking work ends, the effective vertical stresses that occurred in layers I and II are equivalent to the pre-compression load. Though the vertical total stresses on the 100th day are fractionally larger than those on the 18th day, it appears to be warrantable that the total stress distributions may be regarded as constant with respect to time. As is evident from the effective stress and total stress distributions on the 100th day in this figure, the consolidation of clay layer comes almost to an end at this time.

B). Settlements and displacements

Fig. 15 shows the observed and calculated variations of settlements at both the center of embankment (point A) and the toe of slope (point B). The settlements were measured with the settlement plate. In Case I, as can be seen from this figure, the magnitude of calculated settlement of point A through the end of banking and subsequent 80 days is approximately 55% of the observed value so that the time-rate of settlement in Case I is slower than that of the observation. In Case II the magnitude of settlement is approximately 75% of the observed value, and in Case III the result of analysis coincides well with the observation. At the toe of slope (point B), the time-rates of settlements calculated in Case I and Case II are slower than those of the observations. The calculation in

Case III agrees relatively well with the measurements, but the magnitude of settlement at the end of banking is smaller than the value of measurement. One of the reasons may be that some plastic settlements occur in addition to elastic and consolidation settlements in the vicinity of the toe of slope.

Fig. 16 illustrates the variation of settlement in each layer under the center of embankment. In every case of layer I, both the magnitudes and time-rates of settlements coincide well with the observations. The settlements after the end of banking are small. The observed settlement in layer II is approximately 50cm at the end of banking and the increment of settlement at 80 days later is approximately 40cm. The value calculated in Case III coincides completely well with the measurement, whereas the value in Case I and Case II, respectively, is approximately 50% and 70% of the measurements. The time-rates of settlement in layer II differ much according to the permeability coefficient. This fact suggests that it is necessary to pay attention to the measurement of the permeability coefficient in spite of the fact that the soil is organic. In layer III, the calculation in every case agrees well with the measurement qualitatively, but the magnitudes of settlements would be analyzed at smaller values than those of the measurements since the larger value of elastic modulus might be estimated.

As can be seen from Fig. 11, the moduli of elasticity estimated from m_v are lower than those from E_{50} . The mean value of elastic moduli predicted from both E_{50} and m_v was used in this analysis. However, it appears to be better to use the moduli of elasticity estimated from only m_v judging from the results of investigations.

Fig. 17 and Fig. 18 show the vertical and horizontal displacements, respectively, at points 3m, 6m and 12m away from the toe of slope. The vertical displacements calculated in Case I agree well with the observation, contrary to the settlements as shown in Figs. 12 and 13. The heaves calculated in Case III are approximately twice the measured values. The results of the analyses show the peculiar nature of a continuum of which the larger settlements under embankment occur, the more moves up the material away from the toe of slope. Then, in analyzing the consolidation problems on the assumption that the foundation soil is a continuum, the vertical displacements at the parts away from the toe of slope are likely to be estimated greater than the actual values. Another reason may be that the effects of plastic settlements in the vicinity of the toe of slope that occur actually cannot be expressed in this analysis which is based on the assumption that the soil is elastic. The horizontal displacements calculated in every case agree well with the observation.

CONCLUSIONS

The results which have been ascertained in this paper and the problems which have been left for a future study may be summarized as follows:

- (1) The intensity of both the pre-compression load and the applied load of embankment exerts an influence on the determination of moduli of elasticity and permeability coefficients.
- (2) It may be preferable to employ the Poisson's ratio of $\nu=1/3$.
- (3) As previously pointed out, the field time-rate is faster than indicated by calculation when the permeability coefficient obtained from the oedometer test is used.
- (4) It appears to be better to use the moduli of elasticity estimated from m_v than those predicted from E_{50} .
- (5) Judging from the excess pore water pressure distribution as shown in Fig. 13, the conventional method based on the one-dimensional theory should be used within only the territory of 1/5-2/5 of the crown width under the center of embankment.

(6) Since shear failure is apt to occur in the vicinity of the toe of slope, it should be necessary to pay attention to this in the stage of banking.

(7) The calculated settlements and pore water pressures coincide well with the measurements when the permeability coefficient can be predicted accurately in this analysis.

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NOTATION

- C_v = coefficient of consolidation
- E_{ijkl} = components of the elasticity tensor
- E_{50} = tangent modulus
- F_i = components of the body force vector
- K_0 = coefficient of earth pressure at rest
- k_{ij} = components of permeability tensor
- m_v = coefficient of volume compressibility
- n_i = direction cosine of outward normal to the boundary
- P_e = effective overburden pressure
- P_0 = pre-compression load
- S = boundary surface
- t = time
- u_i = rectangular cartesian components of displacement vector
- $\underline{u}$ = set of components of nodal point displacements for entire finite element system
- u_w = excess pore water pressure
- $\underline{u}_w$ = set of nodal point excess pore water pressures for entire finite element system
- V = volume of region
- v_i = components of the stream velocity vector of water relative to soil
- x = spatial co-ordinate vector
- ϵ_{ij} = components of strain tensor
- $\mathcal{Q}(t)$ = functional based on the variational principle
- $\delta\mathcal{Q}(t)$ = first variation
 - ν = Poisson's ratio with respect to effective stress
 - $\bar{\nu}$ = Poisson's ratio with respect to total stress
 - $\bar{\sigma}_{ij}$ = components of symmetric effective stress tensor
 - σ_v' = effective vertical stress
 - δ_{ij} = Kronecker delta
- $\underline{\phi}_u^n, \underline{\phi}_{u_w}^n$ = set of, respectively, displacement and pore water pressure interpolation functions

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