ON THE INFLUENCE OF PROGRESSIVE FAILURE ON THE BEARING CAPACITY OF SHALLOW FOUNDATIONS IN DENSE SAND

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ABSTRACT

In designing the foundations for large-scale structures, it is important to take into account the scale effect of bearing capacity of foundation soils. In this paper the mechanism of the scale effect of bearing capacity of dense sand is investigated for shallow foundations. A series of loading tests using Toyoura sand are carried out in a centrifuge by which stresses in small-scale models can be made approximately identical to those in actual foundations. Deformations of the sand are measured by a set of electronic devices and slip lines in the sand mass are observed immediately after the peak load by an X-ray apparatus. It is found out by these observations that shearing strains in the foundation sand differ very much from place to place, which leads to a conclusion that the progressive failure generally exists in dense sand and that the assumption of constant shearing strain, which has been adopted in existing bearing capacity theories, cannot be accepted except for some limited cases. Kötter's equations are solved by combining a unique relationship between the angle of shear stress and shearing strains for the sand, obtained from plane strain shear tests, with observed shearing strains in models. Resulted bearing capacities compare well with measured values for the sand in which shearing strains differ from place to place along the slip line. It is also shown that the rigidity of specimen containers gives significant influence to the magnitude of observed bearing capacities.

Key words: angle of internal friction, bearing capacity, deformation, model test, plane strain, progressive failure, residual strength, sand, shallow foundation, slip surface

IGC: E3/D6/H1

INTRODUCTION

It seems to be a recent trend that the size of structures is being increased, so that it demands accurate investigations on the bearing capacity and the settlement behaviour of foundation soils. Among many problems in this field, the scale effect of footings on the bearing capacity has apparently been given very little attention.

As long as authors are aware, de Beer (1965 A) is the first to have made comprehensive remarks on the problems of scale effect. He summarized experimental data published by

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various authors and concluded that for sand a bearing capacity factor concerning the self-weight (N_f) decreased with the increase of footing width. As a reason for this de Beer referred to a curvature of the Mohr-Coulomb envelope commonly observed for dense sand. He maintained that, since for wider footings mean normal stress became larger along slip lines in foundation soils, the maximum angle of shearing stresses ($\phi'_{\max}$) became smaller for wider footings.

Graham and Stuart (1971) solved Kötter's equation by taking into account the variation of $\phi'_{\max}$ with the mean normal stresses in sand. They reported a reasonable agreement between their computations and experimental results. Their conclusion, however, should be discounted to some extent because they utilized the angle of shearing stress obtained by triaxial tests for their computations, while the experiments were mostly in the plane strain condition.

De Beer (1965 B) gave another interpretation for explaining the scale effect of footings in dense sand by applying the idea of progressive failure proposed by Muhs (1963). He inferred that the mean distortion at rupture was larger under a wider footing than a narrower one and that the average shearing strength along a final slip line was less for a wider footing.

Although this remark by de Beer is considered to have shed light upon a very important aspect of the scale effect in the bearing capacity problem, it has not so far been given full experimental evidence.

The authors have been tackling the problem of progressive failure of dense sand these several years by making use of a centrifuge (Yamaguchi et al. 1972). Synchronizing a set of electronic devices which consist of a TV-camera, a video tape recorder and a TV-tracker to this centrifuge enables us to determine the magnitude of deformations occurring in sand due to penetration of a footing. The authors have already shown quantitatively how the failure progresses in dense sand, by combining the observed deformations with the stress-strain characteristics obtained for the same material by a plane strain shear test apparatus (Yamaguchi et al. 1975 A).

In this paper, bearing capacity factors and settlements corresponding to the maximum bearing capacity for footings in different conditions are obtained for shallow foundations in dense sand with the centrifuge experiments, and the experimental results are compared with theoretical bearing capacities calculated by taking into account the mobilized angle of shearing stress. All the bearing capacities given in this paper were calculated by using generalized Kötter's equation which is considered to be able to yield the most rigorous bearing capacities for soil in the plane strain condition (Yamaguchi 1969). Some comments are also given to the residual strength of foundation soils.

EXPERIMENTAL INVESTIGATIONS

Apparatus

1. *Plane strain shear test*

Since the bearing capacity experiments in this paper are in the plane strain condition, stress-strain relationships utilized to calculate theoretical bearing capacities should be determined by plane strain shear tests.

The authors pointed out in the last report that there were considerable differences between stress-strain characteristics derived with a conventional triaxial test apparatus and that by triaxial tests with lubricated ends (Yamaguchi et al. 1975 B). This technique of lubrication was applied to the plane strain shear tests. As a lubricant a mixture of silicone grease with castor oil proved to be most effective.

To prepare a very uniform specimen in plane strain tests, a sample former similar to El-Sohby's sliding former was developed (El-Sohby 1964). With this former, a very satis-

Table 1. Physical properties of Toyoura sand

Specific gravity of grain G_s	2.66
Effective size D_{10}	0.13 mm
Uniformity coefficient D_{60}/D_{10}	1.38
Maximum porosity n_{max}	49.9%
Minimum porosity n_{min}	37.9%

factory fitting of a rubber membrane to a specimen could be achieved. Toyoura sand obtained from the same lot was used both for plane strain tests and for centrifuge experiments. The relative density for plane strain tests was so adjusted that it became almost identical to that for the bearing capacity experiments. Physical properties of the sand are given in Table 1.

The outline of the plane strain test apparatus is illustrated in Fig. 1. Tests were carried out in a triaxial cell with a large diameter. To remove the influence of friction of the piston rod from measured stress values, a waterproof load cell was installed inside the cell. A height gauge was attached to the base platen of the apparatus. Using this with a dial gauge and a steel dummy sample of very accurate dimension enabled us to measure the axial strains of a sample with a high accuracy. It was concluded that the errors involved in the measurement did not exceed $\pm 0.01\%$.

2. Bearing capacity experiment

a) Model footing and model foundation.....Model footings with three different width of 2cm, 3cm and 4cm were prepared. They were made of duralumin in order to reduce the self-weight. Particles of Toyoura sand were pasted to the bottom of each footing with a epoxy-resin adhesive to simulate rough contact with foundation sand.

In a previous series of tests it had been found out that sand particles were likely to sneak into narrow gaps between the footing and the side walls of a steel specimen container, and that this led to very unreasonable values of bearing capacity presumably because of the increase of the side friction. In a new series of tests, therefore, flexible sponge tapes were attached to two lateral faces of each footing along the periphery, which successfully prevented sand particles from coming into the gaps. Lubrication was also performed by creating thin film of silicone grease at lateral faces of the footing.

The model ground consisted of vibrated layers of Toyoura sand. The size of steel container in which the model ground was prepared is given in Fig. 2. Each layer was compacted by a concrete vibrator for one and a half minutes into the thickness of 15mm. The side walls of the container were cleaned with acetone and sprayed with a dry lubricant consisting of a fluorine mixture prior to each loading test. The average characteristics of the completed model ground is listed in Table 2.

b) Centrifuge.....Details of the centrifuge apparatus are not given here, because they have already been introduced in one of previous reports (Yamaguchi et al. 1973). The

Fig. 1. Plane strain shear test apparatus

Fig. 2. Steel specimen container for centrifuge

Table 2. Average characteristics of model ground

Unit weight γ	1.60 g/cm ³
Water content w	0.12%
Porosity n	39.7%
Relative density D_r	87.0%

Table 3. Specification of centrifuge

Radius of rotation	118.0 cm
Speed of rotation	100~600 r. p. m.
Exerted radial acceleration	10~300 G
Maximum load	5000 kg
Loading jack Maximum stroke	50 mm
Loading speed	0.5 mm/min

apparatus was constructed in 1970 by referring to an apparatus developed by Mikasa (1965). General information on this centrifuge are given in Table 3.

The load applied to the model footing was measured by a load cell mounted on a loading jack. Two types of strain gauges were used for the load cell. One was foil gauges for measuring large load and the other was semiconductor gauges for small load. The settlement of the footing was recorded by a deformation gauge with a linear differential transformer.

Problems Concerning Apparatus

1. Centrifuge

As has been pointed out by Pokrovski (1940) and Roscoe (1968, 1970), the centrifuge has such an advantage that even in a small-scale model test the magnitude of stress in a specimen becomes almost identical to that in an actual foundation. In other words, the stress similarity can be established between a model with a geometrical reduced scale of $1/N$ and a prototype by giving the model the acceleration of N times the gravitational acceleration (G) with the centrifuge (Yamaguchi 1969). Roscoe made a comprehensive remark on the similarity law between a model and a prototype, and concluded that as long as a tested material remained in the elastic range, the similarity law held for deformations as well as for stresses. But once plastic deformations appear in the model, it becomes doubtful that the similarity for deformation still exists. Since it is inferred that the plastic deformations prevail in authors' experiments with the penetration of the footing into the model ground, the direct transformation of the observed settlement of a model into that of a prototype may not be justifiable. However, the general deformation pattern encountered in the bearing capacity problem, is considered to be a type of contained flow. Therefore, it is probable that rough similarity holds even for the deformation up to such a stage that overall deformations of the model ground do not become catastrophic.

The second problem comes from the structure of the centrifuge. Since the acceleration is given by the centrifugal force, it acts in the radial direction and its magnitude increases in proportion to the distance from the center of rotation. Due to these effects, in the case of the authors' centrifuge the unit weight of sand near the bottom of the model ground is 20% larger than that near the surface, and the circumferential force equivalent to 15% of the total overburden pressure acts at the outer edge of the specimen.

To evaluate the influence of these effects, Kötter's bearing capacity was calculated both for a centrifuge model in which the acceleration works in the radial direction increasing in proportion to the distance from the center of rotation and a prototype in which the uniform acceleration works perpendicularly to the surface. The results are presented in Fig. 3. This shows that the difference in bearing capacity does not exceed even 5%, although the shape of a slip line is slightly different from each other.

On the basis of the above discussions, the authors take a view that the results obtained from the tests on the centrifuge model with a geometrical reduced scale of $1/N$ under the acceleration of NG can be reasonable predictions of those by large-scale model or prototype experiments.

Fig. 3. Calculated bearing capacities and slip lines for a centrifuge model and for a prototype

2. Specimen container

One lateral face of a specimen container used in the previous series of tests was a composite glass plate. This was because the movements of optical targets placed between the glass plate and sand were measured by a set of electronic devices. In tests with surcharge thickness equal to the breadth of a model footing, marked differences were found out between the load-deformation curves by 3 cm wide footing and those by 4 cm wide footings, although the load-deformation relationship was quite similar for 2 cm footings and 3 cm footings. For 2 cm and 3 cm footings the peak of the load-deformation curve could be always located, while for 4 cm footings the peak was not clearly recognized. It was confirmed that this always happened when applied total load exceeded 1,000 kg, and that this was the case with the 4 cm footings with the ratio of depth of embedment to the footing breadth equal to unity. Having envisaged that the lateral expansion of the specimen container reached a considerable amount for load greater than 1,000 kg, the authors constructed

Fig. 4. Load-deformation curves obtained with an old and a new container

a specimen container which was completely made of steel. This container was provided with a vertical and horizontal rib to give a very rigid structure. The load-deformation curves obtained with the new container are compared in Fig. 4 with those by old one for 3 cm footings with the depth of embedment equal to the breadth. It shows that there exists a significant difference between them, and it can be concluded that the general shape of load-deformation curves is very sensitive to the rigidity of containers. This was pointed out also by Ko and Davidson (1973). For 2 cm footings, the difference was quite insignificant. And for tests with smaller depth of embedment, all the load-deformation curves were fairly similar.

Experimental results given in this paper other than those shown in Fig. 10 were from the tests using the new steel container.

Results

1. Plane strain tests

Tests were performed for seven different ambient pressure of 0.5, 1.0, 2.0, 3.0, 4.0, 4.5 and 5.0 kg/cm². Up to ambient pressure of 3 kg/cm² a unique relationship was obtained between the angle of shearing stress $\{\phi' = \sin^{-1}(\sigma_1' - \sigma_3' / \sigma_1' + \sigma_3')\}$ and the shearing strain ($\gamma' = \varepsilon_1 - \varepsilon_3$) with little dependency upon the magnitude of ambient pressure. This did not hold for pressures exceeding 3 kg/cm², i.e., the higher the ambient pressure, the smaller the maximum angle of shearing stress ($\phi_{p\max}'$) and the larger became the strains corresponding to $\phi_{p\max}'$. However, the maximum mean normal stress which occurred in the specimen in the tests under the ambient pressure of 4 kg/cm² was supposed to be not less than 12 kg/cm², because the ratio of σ_1' / σ_3' and that of σ_2' / σ_3' were approximately equal to 6 and 2 respectively. According to preliminary computations (Yamaguchi et al. 1975 A), the authors concluded that this value of mean normal stress appeared in the scale

Fig. 5. Obtained relationship between the angle of shear stress ($\phi' = \sin^{-1} \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$) and the shear strain ($\gamma' = \varepsilon_1 - \varepsilon_3$)

of authors' experiments only in a small area beneath a footing. Therefore the unique relationship obtained from the tests with ambient pressure not exceeding 4 kg/cm² (Fig. 5) was used for the authors' bearing capacity computation. In Fig. 5 the stress-strain characteristics derived from triaxial tests are also given. This shows that the maximum angle of shearing stress for plane strain tests ($\phi_{p\max}'$) is larger by 5° than that for triaxial tests ($\phi_{t\max}'$), which leads to the similar conclusion to that by Cornforth (1964) that $\phi_{p\max}'$ is about 10% larger than $\phi_{t\max}'$. All these results are summarized in Table 4.

Table 4. Observed angle of shearing stress

	Plane strain	Triaxial (lubricated)	Triaxial (conventional)
$\phi'_{\max}$	46°	41°	41°
ϕ_r' (residual)	40°	36°	33°

2. Bearing capacity

Loading tests with the centrifuge (denoted as FC-test) were conducted by imposing three different centrifugal acceleration of 10, 20 and 40 G on the model ground. A series of loading tests were also performed in the gravitational field of 1 G (denoted as FG-test or small-scale model test). The ratio of the depth of embedment (D_f) to the breadth of the footing (B) was selected as 0, 0.5 and 1.0. Four or five tests were carried out for each combination of two parameters, the acceleration and the ratio of D_f/B . Typical load-deformation curves are shown in Fig. 6. A bearing capacity factor N_{rq} is plotted against a parameter $\gamma B/E_q$ in Fig. 7. The factor N_{rq} is a non-dimensionalized parameter defined by the equation

$$N_{rq} = \frac{q}{\frac{\gamma}{2} B}$$

where q denotes the bearing capacity of the foundation soils and γ the unit weight of the

Fig. 6. Typical load-deformation curves ($D_f/B=1$)

sand. The parameter $\gamma B/E_q$ is the one introduced by de Beer (1965 A), where E_q denotes Young's modulus of a mother rock of the sand used in tests. As the same type of sand was utilized throughout the experiments, the value of E_q was taken as unity for convenience. Relationship between the ratio of the settlement at the peak load (S_f) to the footing breadth (B) and the parameter $\gamma B/E_q$ is given in Fig. 8, and the values of initial tangent of the load-settlement curves (K_r) are also plotted against $\gamma B/E_q$ in Fig. 9 (Fig. 4).

DISCUSSION OF THE SCALE EFFECT

It is shown in Fig. 7 that N_{rq} assumes large values for narrow footings and decreases with an increase of the parameter $\gamma B/E_q$. This phenomenon is considered to be related to the scale effect. The result implies that small-scale tests do not reflect the bearing capacity of actual foundation soils. It is interesting to note that experimental results by de Beer and Vesic added in Fig. 7 show fair agreement with the authors' observations, although it is doubtful whether the values of E_q are equal for all these experiments. The reason why the former is slightly less than the latter might be attributed to the fact that the experiments by de Beer as well as Vesic were not in the exact plane strain condition.

The relative maximum settlement (S_f/B) in Fig. 8 is considered to be a parameter representing average strains which take place in sand mass. The value of S_f/B increases as the value of $\gamma B/E_q$ increases. Although the extent of the scale effect increases equally for both cases of N_{rq} and S_f/B with the decrease of D_f/B , the general trend of the effect is completely opposite to each other. In other words, large values of N_{rq} corresponds to small values of S_f/B and small values of the former to large values of the latter. And at a very similar value (≈ 0.15) of $\gamma B/E_q$ both N_{rq} and S_f/B converge to constant. Experimental results by de Beer and by Vesic again compare well with those of the authors, while experiments by Muhs give considerably different results.

It has been said that bearing capacity formulae generally underestimate the bearing capacity for small-scale models and overestimate that for prototypes. This conclusion seems to have been based on the use of the maximum angle of shearing stress in triaxial condition ($\phi'_{t\max}$). However, Fig. 6 shows that the Kötter's theoretical bearing capacity using the

Fig. 7. Relationship between N_{rq} and $\gamma B/E_q$

Fig. 8. Relationship between S_f/B and $\gamma B/E_q$

Fig. 9. Relationship between K_r and $\gamma B/E_q$

FG-test and FC-test. And the location of final slip lines observed with a X-ray apparatus is also shown. It is clearly demonstrated that at the peak load ($S_f=1.3\text{mm}$ for FG-test, $S_f=4.8\text{mm}$ for FC-test) the magnitude of shear strains along the observed slip line differs very much for each series of the tests (Fig. 10(I)(b) and (II)(b)).

Fig. 10. Observed distribution of shear strains in sand ($D_f/B=1$)

For FG-test the maximum shear strain is found to be 6 or 7% at the peak, and the shear strains in almost all the regions along the slip line remain at around 5% which corresponds to the value of shear strain at the maximum strength in the plane strain shear test (γ_f'). This means that the average angle of shearing stress along the slip line is not so much different from the maximum angle of shearing stress (ϕ_p'). It is natural therefore, that the use of a constant angle of shearing stress in theories leads to reasonable estimation of bearing capacity of small-scale models.

While in FC-test the shear strain shows considerable local variation along the slip line and its magnitude is very much greater than that in FG-test as can be seen from the comparison of Fig. 10(I)(b) with Fig. 10(II)(b). And the strain near the footing exceeds γ_f' and reaches the so-called residual state as is confirmed from Fig. 5. These observations suggest that the assumption of constant shear strain is not acceptable for actual foundation soils and that the use of ϕ_p' in theories is not justifiable for estimating their bearing capacities. The average angle of shearing stress along the slip line which can be calculated by combining Fig. 5 with Fig. 10 is considerably smaller than ϕ_p' . This may explain why theories extremely overestimate the bearing capacity of actual foundation soils if ϕ_p'

maximum angle in the plane strain condition (ϕ_p') gives a reasonable estimation for bearing capacities of small-scale models. Ko and Davidson (1973) obtained a similar result with respect to small-scale models.

On the contrary for FC-tests, the use of ϕ_p' in theories results in pronounced overestimation. This discrepancy between theories and experiments can be explained by considering the mechanism of the progressive failure.

Fig. 10 represents the distribution of observed shear strains taking place in the foundation soil at various loading steps for

is put into use.

In Fig. 11 the residual bearing capacity factor $\{N_{\gamma q}\}_r$ is plotted against the parameter $\gamma B/E_q$, and it shows that the scale effect is not so conspicuous as in $N_{\gamma q}$ and S_f/B . Especially for a surface foundation there exists practically no scale effect. Kötter's bearing capacity calculated using the angle of shearing stress at residual state (ϕ'_{pr}) agrees reasonably well with measured bearing capacities (Fig. 6). This is considered to be due to the fact that in almost all the regions along the slip line shearing strains at the residual state of load-deformation curves exceed 10% beyond which the residual state is observed in $\phi' \sim \gamma'$ relationship. In other words the assumption of constant shear strains may be applicable to the foundation soils at this stage.

From the above considerations it becomes evident that theories can only evaluate the bearing capacity of foundation soils in which the condition of constant shear strains is satisfied, if the constant angle of shearing stress is utilized.

It is known that the local variation of the angle of shearing stress can be incorporated into the integration of Kötter's equations (Yamaguchi 1969). As the shape of the final slip line was successfully observed with an X-ray apparatus in the authors' experiments (Photo. 1), the integration could be easily executed along the slip line allowing the value of ϕ' to vary. The numerical integration was started at a point where the slip line met the soil surface and proceeded along it to the apex of a triangular wedge which was observed beneath the footing, which yielded the values of principal stresses at the apex. To

Fig. 11. Relationship between residual bearing capacity factor $(N_{\gamma q})_r$ and $\gamma B/E_q$

Photo. 1. Slip lines observed by X-ray

Photo. 2. Configuration of a wedge formed beneath a footing

obtain the shape of the wedge, X-ray photographs were taken before starting the loading test and immediately after the maximum bearing capacity was recorded. These photographs were superimposed into one photograph (Photo. 2) by setting the base of the footing as a datum line. White spots in the photograph are images of lead shot buried in sand. The photograph gives the general configuration of the wedge found beneath the footing and the angle between the base of the footing and a side of the wedge approximately as 60° for the case of $D_f/B=1$ irrespective of the footing breadth. Stress values at the edge of the footing were obtained by considering the side of the triangular wedge as a slip line. And

assuming the linear variation of stresses along the side of the wedge gave components of forces acting on the sides. The total bearing capacity was determined by equilibrating these forces. The bearing capacity thus calculated is compared with measured values in Fig. 6 which shows a marked improvement in comparison with theoretical bearing capacity based on $\phi_p'_{\max}$, although the general agreement is yet to be reached. The discrepancy between the improved theoretical value and observed bearing capacity is considered to have arisen because the exact plane strain condition was not satisfied in the authors' plane strain shear test apparatus. It was found out by a precise measurement that the lateral expansion of confining plates of the apparatus reached as much as 1/10 mm for the ambient pressure of 4 kg/cm². It is imagined that this defect resulted in a larger value of the strain at the peak strength. By adopting a more rigid structure without increasing the friction between the confining plates and sample sand, it is probable that the strain at the peak strength decreases to some extent. And this will clearly lead to narrowing the gap between the improved calculation and observations.

It is important to note that the bearing capacity calculated with $\phi_t'_{\max}$ shows reasonable agreement with observed values for FC-tests (Fig. 6). The success so far reported in evaluating the bearing capacity of foundation soils in the plane strain condition with the use of $\phi_t'_{\max}$ in theories is supposed to owe to the fact that $\phi_t'_{\max}$ is approximately equal to ϕ'_{pr} . However this coincidence should be considered fortuitous as has been indicated by Rowe (1969).

As can be seen from Fig. 10, the average strain along the slip line for FG-tests is considerably smaller than that for FC-tests. This difference in the shearing strain distribution will explain the scale effect with respect to the relative maximum settlement S_f/B (Fig. 8), because the parameter S_f/B , as was stated before, is considered to be an index representing the average strain in sand mass.

The parameter K_r in Fig. 9 is equivalent to the subgrade reaction coefficient (Fig. 4), and with respect to this also exists the scale effect. De Beer (1965 B) stated that the settlement of a wider footing must be larger than that of a narrower one at the initial stage of loading because the elastic compression was dominant for very small loads. His prediction is strongly supported by the results shown in Fig. 9.

CONCLUSIONS

From the results of this work, following conclusions can be drawn.

(1) With the increase of the value of $\gamma B/E_q$, the bearing capacity factor (N_{rq}) and the modulus of subgrade reaction (K_r) decrease and relative maximum settlement (S_f/B) increases, and each of them converges to a constant value approximately at 0.15 of $\gamma B/E_q$ ($\div 100$ cm of the breadth of footing, considering $\gamma=0.0016$ kg/cm² and $E_q=1$ kg/cm²) for Toyoura sand utilized in the experiments. The extent of the scale effect becomes more marked as the ratio D_f/B becomes greater.

(2) The scale effect exists also for the bearing capacity factor at the residual state $\{N_{rq}\}_r$ but for the case of no surcharge it is practically negligible.

(3) In almost all the regions along the final slip lines, shear strains observed in FG-tests are not very much different from the value at the peak strength (γ_f') but those in FC-tests exceed γ_f' . This explains why the application of the maximum angle of shearing stress in the plane strain condition ($\phi_p'_{\max}$) gives a reasonable estimation for small-scale models, leading to a serious overestimation for large-scale models.

(4) Kötter's bearing capacities calculated with the residual angle of shearing stress (ϕ'_{pr}) agree well with the observed bearing capacities at the residual state. This is considered because shearing strains in almost all the regions along the final slip lines reach

the value at the residual state of $\phi' \sim \gamma'$ relationship in the plane strain shear tests.

(5) Bearing capacities calculated by incorporating the variation of angle of shearing stress with shear strains into Kötter's equation gives a reasonable estimation of bearing capacities of foundation soils in which shearing strains differ very much from place to place along the slip line.

(6) The shape of load-deformation curves is very sensitive to the rigidity of specimen containers.

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NOTATION

- ϕ' = angle of shearing stress
- $\phi'_{\max}$ = maximum angle of shearing stress
- $\phi'_{t\max}$ = maximum angle of shearing stress in triaxial condition
- $\phi'_{p\max}$ = maximum angle of shearing stress in plane strain condition
- ϕ'_r = angle of shearing stress at residual state
- ϕ'_{pr} = angle of shearing stress at residual state in plane strain condition
- $\sigma'_1, \sigma'_2, \sigma'_3$ = major, intermediate, minor effective principal stress
- ϵ_1, ϵ_2 = major, minor principal strain
- γ' = maximum shear strain ($=\epsilon_1 - \epsilon_3$)
- γ'_f = maximum shear strain at peak strength
- B = breadth of footing
- D_f = depth of embedment of footing
- γ = unit weight of sand
- E_q = Young's modulus of mother rock of sand
- q = intensity of bearing capacity
- N_r = Terzaghi's bearing capacity factor concerning self-weight
- N_{rq} = bearing capacity factor $\{=q/(B/2)\gamma\}$
- $\{N_{rq}\}_r$ = bearing capacity factor at residual state
- S_f = settlement of footing at peak load
- K_r = coefficient of subgrade reaction
- D_r = relative density
- n = porosity
- $n_{\max}$ = maximum porosity
- $n_{\min}$ = minimum porosity
- D_{10} = effective size
- D_{60} = diameter at 60%
- w = moisture content
- G_s = specific gravity of grain

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