COMPUTER SOLUTIONS FOR THE STABILITY OF RESERVOIR SLOPES

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ABSTRACT

Prediction of the stability of slopes under various water conditions and for a wide range of loading and soil conditions can be made with the help of a digital computer. Reservoir slopes may reach a very critical state immediately after very rapid drawdown. At present, this condition is analyzed assuming an "instantaneous" drawdown event. This paper presents a method for obtaining the slope stability under a drawdown condition with a defined time-elevation history.

The analyses are based on the simplified form of Bishop's model with circular failure surface and vertical slices. Two options are available for determining the pore pressure. In the time-dependent drawdown analysis, a differential equation equates the volume of water that flows out of the slope in a given time with the product of the velocity of flow, area of flow, and time interval. The solution of this equation yields the position of the phreatic surface with time. This information helps determine the pressures along a trial circular sliding surface, and the variation of the factor of safety with time.

The factor of safety decreases rapidly with the lowering of the reservoir level, followed by a slow and nonlinear increase to the steady state value.

Key words: computer application, earthfill, reservoir, safety factor, slope stability, soil water

IGC: E6/E7

INTRODUCTION

During the planning and development of reservoirs, a condition generally not assessed is the stability of the slopes bounding the reservoir pool. Accordingly, the consequences of reservoir filling and fluctuations, on developments on or adjacent to these slopes, are unknown. Although failures of reservoir slopes are not very common, the results can be catastrophic. The overall object of the research reported in this paper was to develop a systematic technique for identifying reservoir slopes of potential instability.

The technique encompasses two levels of investigation. First it is desired to locate the potentially troublesome areas by a general-level system. This can be accomplished by examining a number of regional factors, e.g. hydrology, geology, and topography etc. for the proposed area. Secondly, a detailed analysis, a specific-level system is needed for any zone highlighted by general-level system, to ascertain whether instability of an individual slope would indeed exist. The specific-level system should be sufficiently versatile to accommodate irregular slopes, complexities of the subsurface and failure surface geometry, con-

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ditions of variable internal water pressure, and ground surface boundary loadings.

This paper concerns itself with the second phase, the development of the specific-level, computer-assisted analysis for the prediction of stability of a reservoir slope. Two models are common in the quantitative geotechnical approach to two-dimensional slope stability problems, viz., the circular arc sliding surface (often solved as a method of slices) and the sliding block surface. This paper concentrates on the former.

GENERAL NATURE OF SLOPE STABILITY

A general problem solution would include a systematic search for the critical sliding surface, examining every position of every shape of surface. Such a solution is not practical, and it is common to assume a shape and search for the critical position of it. The circular shape is a convenient and much-studied one: Fellenius (1927), Taylor (1948), Bishop and Morgenstern (1960), and Spencer (1967). The convenience of the digital computer has made it possible to solve the circular problem more quickly and completely and has encouraged investigators to examine other simple shapes and even irregular shapes (Morgenstern and Price (1965), Wilson (1970), Spencer (1969), Carter, Lovell and Harr (1971)).

Difficulties and uncertainties are not limited to the mechanics of the analysis; accurate soil parameter input is a major concern. Johnston (1969) and Lee and Singh (1968) have shown that shear strength parameters (c and ϕ) of the same soil may vary up to 30% or more when tested in different laboratories.

The dependent variable in conventional slope stability analysis is the strength reduction factor, F , usually called the factor of safety; and defined as the ratio of the available shearing resistance along the most critical sliding surface of assumed shape to the shearing resistance required to satisfy equilibrium. A value of the factor of safety of unity indicates incipient failure.

The presence and movement of ground water is frequently an important factor in slope stability. Such water can both increase the forces tending to produce failure, and decrease the forces acting to prevent sliding. A prominent example of the effect of water forces occurs in the upstream slope of an earth dam or in a reservoir slope when the impounded water level is rapidly dropped, i.e., the rapid drawdown problem. An analytical procedure to determine the factor of safety of a slope under drawdown is presented in this paper. While the drawdown may be rapid, it can not actually be instantaneous, and in the proposed solution method, the phreatic surface has a real or estimated time-elevation history. The factor of safety can be determined throughout this drawdown history.

It is also of interest to analyze for the stability of reservoir slopes under partial drawdown, reservoir filling, and indeed, any fluctuation of the reservoir water level with time. The methods and computer programs developed in this study contribute to an improved capacity in making such assessments of stability.

In order to accommodate potentially complex variations in: boundary geometry and loading, soil weights and shearing parameters, and water forces, it is customary to divide the body segregated by the assumed sliding surface, into vertical slices. (See Fig. 1.) With a computerized solution, the slices can be as thin and numerous as desired. During transient flow condition, unbalanced side force may act on a slice, the magnitude of which will depend on the seepage velocity. In practice the seepage velocity and the side force may be small. But the change in pore water pressure may drastically reduce available shearing force affecting stability of a slope. Based on this observation instability due to lateral seepage force is neglected in this analysis and the study is based on no-flow condition.

Method of Analysis

Fig. 2 shows a typical slice in equilibrium under a no-flow condition.

Fig. 1. Free body divided into vertical slices

Fig. 2. A typical slice with effective and water forces

U = water force

W_w = equivalent water weight of slice

W' = buoyant weight of slice

$W_w + W' = W$ = total weight of slice

S_r = shearing force at base of slice required for equilibrium

S_a = available shearing force at base of slice

$F = S_a/S_r$ = strength reduction factor or factor of safety.

Adding all forces in the vertical direction on a slice, and setting the sum equal to zero for equilibrium,

$$\sum F_v = 0$$

$$(N' + U_a) \cos \alpha + S_r \sin \alpha + Y_L = W + U_\beta \cos \beta + Y_R$$

Substitute $S_a/F = S_r$ and let $S_a = C_a' + N' \tan \phi_a'$, where $C_a' = (c_a')(\text{area})$ and $\tan \phi_a'$ are representative of the shearing parameters in the Mohr-Coulomb effective stress (force) form of the shearing resistance equation. Solve for N' and,

$$N' = \frac{W + U_\beta \cos \beta - U_a \cos \alpha + Y_R - Y_L - \frac{C_a' \sin \alpha}{F}}{\cos \alpha + \frac{\sin \alpha \tan \phi_a'}{F}} \quad (1)$$

Considering the water forces on the sides of the slice, one can write,

Force	Moment Arm
$U_{L1} = \frac{h^2}{2} \gamma_w$	$\bar{a} + \frac{b}{2} \sin \beta + \frac{2}{3} h$
$U_{L2} = h \gamma_w \left(H - h + \frac{b}{2} \sin \beta \right)$	$\bar{a} + \frac{b}{2} \sin \beta + \frac{h}{2}$
$U_{R1} = \frac{h^2}{2} \gamma_w$	$\bar{a} - \frac{b}{2} \sin \beta + \frac{2}{3} h$

$$U_{R2} = h r_w \left(H - h - \frac{b}{2} \sin \beta \right) \quad \bar{a} - \frac{b}{2} \sin \beta + \frac{h}{2}$$

X_L' and X_R' = effective side forces

Taking moments of all the forces about any arbitrary moment center, summing for all slices and setting equal to zero, for equilibrium,

$$\begin{aligned} & \sum (N' + U_a) (\bar{y} \sin \alpha - \bar{x} \cos \alpha) + \sum W \bar{x} + \sum U_\beta (\bar{x} \cos \beta - \bar{a} \sin \beta) \\ & - \sum S_r (\bar{x} \sin \alpha + \bar{y} \cos \alpha) - \sum Y_L \left(\bar{x} - \frac{b}{2} \right) + \sum Y_R \left(\bar{x} + \frac{b}{2} \right) \\ & - \sum \frac{h^2}{2} r_w \left(\bar{a} + \frac{b}{2} \sin \beta + \frac{2}{3} h \right) - \sum h r_w \left(H - h + \frac{b}{2} \sin \beta \right) \cdot \left(\bar{a} + \frac{b}{2} \sin \beta + \frac{h}{2} \right) \\ & + \sum \frac{h^2}{2} r_w \left(\bar{a} - \frac{b}{2} \sin \beta + \frac{2}{3} h \right) + \sum h r_w \left(\bar{a} - \frac{b}{2} \sin \beta \right) \cdot \left(H - h - \frac{b}{2} \sin \beta + \frac{h}{2} \right) \\ & - \sum X_L' \left(\bar{a} + \frac{b}{2} \sin \beta + \frac{2}{3} h \right) + \sum X_R' \left(\bar{a} - \frac{b}{2} \sin \beta + \frac{2}{3} h \right) = 0 \end{aligned} \quad (2)$$

For very thin slice, X_L' and X_R' are very nearly same and can be neglected.

Summation of moments due to the forces Y_L and Y_R over all the slices are cancelled among each other. Similarly, summation of moments due to forces U_{L1} , U_{L2} , U_{R1} and U_{R2} over all the slices are also cancelled. The resulting expression is the same as the development by Carter et al. (1971).

Following Carter et al. (1971) further, the final iterative expression for the factor of safety is given by :

$$F_1 = F_0 + \frac{\sum \bar{y} \frac{A_1 - F_0 A_2}{F_0 + A_3}}{\sum \bar{y} \frac{A_1 + A_2 A_3}{(F_0 + A_3)^3}} \quad (3)$$

where : F_0 = initial factor of safety

F_1 = iterated factor of safety

$$A_1 = \frac{C_a'}{\cos \alpha} + \tan \phi_a' \left[W - \frac{U_a}{\cos \alpha} U_\beta (\cos \beta + \bar{k} \sin \beta \tan \alpha) \right]$$

$$A_2 = W \tan \alpha + U_\beta (\cos \beta \tan \alpha - \bar{k} \sin \beta)$$

$$A_3 = \tan \alpha \tan \phi_a'$$

$$\bar{k} = \frac{\bar{a}}{\bar{y}} \text{ (Fig. 2)}$$

Determination of equilibrium water forces and pressures are straight forward, once the free water surface has been located. Estimation of non-equilibrium pressures and forces are more difficult, but for ordinary conditions, Skempton (1954) proposed that the change in pore pressure be conveniently defined as

$$\Delta u = B [\Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3)]$$

where A and B are pore pressure parameters, and $\Delta \sigma_1$ and $\Delta \sigma_3$ are the changes in major and minor principal stresses in the triaxial test. Where $\Delta \sigma_1$ and $\Delta \sigma_3$ are in a constant defined ratio, the change may be defined in terms of $\Delta \sigma_1$, only, as $\Delta u = \bar{B} \Delta \sigma_1$. The development which follows assumes that $\bar{B}$ can be adequately defined, and uses the approach of Little and Price (1958) to determine the factor of safety for cases of transient flow. The iterative expression for factor of safety is :

$$F_1 = F_0 \left\{ 1 - \frac{\sum W \sin \alpha - \sum \frac{c'b + W(1 - \bar{B}) \tan \phi_a'}{F_0 \cos \alpha + \tan \phi_a' \sin \alpha}}{\sum W \sin \alpha - \sum \frac{[c'b + W(1 - \bar{B}) \tan \phi_a'] \tan \phi_a' \sin \alpha}{[F_0 \cos \alpha + \tan \phi_a' \sin \alpha]^2}} \right\} \quad (4)$$

Location of Exit Point

Fig. 3 shows the model which can be adapted to the analysis of drawdown on a reservoir slope. In the figure, let BC represent a vertical no-flow boundary, e.g., an impervious core. The reservoir level, initially at H_c , falls instantaneously to h_0 ; section $ABCA$ is initially fully saturated. The location of the phreatic surface, at some time t after drawdown, intersects the slope at an elevation " a " and the vertical no-flow boundary at an elevation h_c . In an interval dt , the phreatic surface falls a distance da , such that the new position is displaced parallel to the previous position.

Fig. 3. Location of phreatic surface after drawdown: After Dvinoff and Harr (5)

The material in $ABCA$ is assumed to be homogeneous, with a porosity n , and directional permeabilities represented by the coefficients k_x and k_y . Assuming that the increment of volume in Fig. 3 is horizontal, the volume of water removed in dt per unit thickness is,

$$dV = n(H_c - a) \cot \alpha da + \frac{n}{2} \cot \alpha (da)^2$$

The second term on the right hand side is small and is neglected.

Assuming the flow through $ADEA$ to be essentially horizontal, the discharge rate (found by Pavlovsky) is (Harr (1962)),

$$Q = -k_x(a - h_0) \tan \alpha \left(1 + \ln \frac{a}{a - h_0} \right)$$

Since the quantity of water removed in any time increment dt is equal to the change in the volume of water in section $ADEA$,

$$Qdt = dV$$

Substituting for Q and dV in the above expression and integrating, one can write

$$-\int_0^t \frac{k_x \tan^2 \alpha}{n} dt = \int_{H_c}^a \frac{(H_c - a)/(a - h_0)}{1 + \ln \frac{a}{a - h_0}} da \quad (5)$$

Let $h_0 = 0$ and integrate,

$$\frac{k_x t}{H_c n} \tan^2 \alpha = \frac{a}{H_c} - 1 - \ln \frac{a}{H_c}$$

Location of exit point at any time can be determined from this formula.

For $h_0 \neq 0$, Dvinoff and Harr (1971) have developed a computational scheme for solving the general equation.

Fig. 4. Flownet for drawdown condition ; After Terzaghi and Peck (18)

The above solution derived directly for an earth dam with a vertical impervious core (Dvinoff and Harr (1971)), is applied in this work to generate phreatic surfaces in reservoir slope sections. Fig. 4 shows part of a typical flow net for a homogeneous subsurface under drawdown conditions. It is seen that a streamline AB effectively divides the flow into two regions : (a) flow towards the slope, and (b) flow away from the slope. The streamline AB is almost vertical, and almost directly under the highest point of the slope. Based on the example, it is essentially correct to replace the streamline AB by a fictitious impervious core, and the method of Dvinoff and Harr (1971) may be employed to determine flow characteristics. The position of the fictitious core is a function of the slope, the rate of drawdown, and the slope of the initial phreatic surface.

Fluctuating Headwater

Conditions for rising water are schematically shown in Fig. 5. A typical slice for this case is shown in Fig. 6.

Fig. 5. Flow domain for rising water level

Fig. 6. A typical slice in the slope during rising water condition

The following assumptions are made :

1. Flow is two dimensional.
2. Inertia forces are not significant compared to viscous forces, and can be neglected.
3. Fluid is continuous.
4. Air escapes freely from pores.
5. Soil does not deform ; porosity and coefficients of permeability are constant.
6. Darcy's Law is valid.
7. Dupuit's assumptions are valid.

In Fig. 6 during rising water, where,

$$q_x = \text{flow across } AB$$

$$q_x + \frac{\partial q_x}{\partial x} dx = \text{flow across } CD$$

q_y = flow across AC

$q_y + \frac{\partial q_y}{\partial y} dy$ = flow across BD

dx = width of slice

h = average height of phreatic surface in slice

dh = increase in the average phreatic surface in time dt

Flow into the element :

$$q_{in} = q_x + q_y$$

Flow out :

$$q_{out} = q_x + \frac{\partial q_x}{\partial x} dx + q_y + \frac{\partial q_y}{\partial y} h$$

From the conditions of continuity, the change in storage volume must be equal to the change in height times the area. Using Dupuit's assumption, the equation can be written as,

$$n \frac{\partial h}{\partial t} dx = - \left[\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} h \right]$$

which simplifies to,

$$\frac{\partial h}{\partial t} = \frac{1}{n} \left[k_x h \frac{\partial^2 h}{\partial x^2} + k_x \left(\frac{\partial h}{\partial x} \right)^2 + k_y h \frac{\partial^2 h}{\partial y^2} \right] \quad (6)$$

The above equation can be further simplified to the form,

$$\frac{\partial h}{\partial t} = \frac{\bar{h}}{n} \left[k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} \right] \quad (7)$$

where $\bar{h}$ is the average height of the phreatic surface above datum at any time. This equation has been solved by finite difference technique, and the phreatic surface can be located at any time, as well as the total head at all points in the flow domain.

Once the elevation of phreatic surface near the impervious core and the location of exit point are known at a given time, the set of equations can be simultaneously used to analyze for a fluctuating water table. The programs are capable of choosing the correct sub-routines for rising or drawdown conditions, depending upon the given reservoir history.

Since the rising reservoir level takes a relatively long time to achieve much of a rise in phreatic surface in the reservoir slope, this case is not very important for stability. Also, in the rising water cases, flow is in a direction to improve rather than decrease slope stability (see Fig. 5).

SAMPLE PROBLEMS AND DISCUSSION OF RESULTS

To check the reliability of the programs, sample problems have been solved both manually and by computer. The difference in the values of F were within 3% for all cases checked.

Two of the illustrative hypothetical problem solutions follow.

Steady State

Fig. 7 shows the geometry and soil properties of the sample problems. For simplicity, only a two layer system is considered, viz., a softer layer underlain by a harder layer. Fig. 8 shows the boundary conditions for three different water conditions.

Fig. 8(a) shows the assumed steady state condition, where the reservoir surface and the phreatic surface inside the slope are at elevation R . A complete and very rapid drawdown condition is shown in Fig. 8(b), where the reservoir elevation changes from R to zero, while the elevation of the phreatic surface in the slope remains unchanged.

Fig. 7. Sample problem profile

Fig. 8. Boundary conditions for different water cases

Fig. 9. Variation of minimum F for complete and very rapid drawdown

Fig. 9 is a plot of the minimum factor of safety, F , (by grid search option) as a function of the ratio (R/H) , where R is defined as above, and H is the height of slope. The increase of F with increase in R is shown on right hand side for the steady state, where the weight of water acting on the sloping surface tends to increase the resisting moment. A series of curves of F for the complete and very rapid drawdown condition is shown on the left hand side. The different curves are generated for several values of the pore pressure parameter $\bar{B}$, and also for the pore pressure defined by the product of height of phreatic surface in the slice and the unit weight of water. As $\bar{B}$ increases, pore pressure increases, resulting in a lower value of F .

Fig. 9 also shows the reduction of F from the steady state condition (reservoir elevation $= R$) to the complete and very rapid drawdown condition (reservoir elevation drops from R

to zero). The difference is represented by the distance between the right hand and left hand curves and is a maximum for $(R/H=1)$. The example is thus for a slope which is fairly stable in the steady state, but which would be predicted to fail were the reservoir level rapidly drawdown from a large value of R .

Transient State

To illustrate the usefulness of the transient state analysis, the same problem has been chosen, except that the complete drawdown now takes place at a uniform rate in two days. The boundary conditions are shown in Fig. 8(c), where the initial reservoir elevation and the phreatic surface is calculated by a computerised iterative technique following procedure described by Dvinoff and Harr (1971). The subsequent variation of the minimum factor of safety (F) with time is shown in Fig. 10 for several values of $\bar{B}$. The corresponding phreatic surfaces are shown in Fig. 11.

Fig. 10 shows values of F which sharply dropped almost immediately after the drawdown started. The values of F then gradually increase non-linearly with time due to the drainage of water from the slope. The nature of this variation will depend on the details of the drawdown, the height and inclination of the slope, and the horizontal and vertical permeabilities of the soil in the slope. Fig. 10 also shows the effect of $\bar{B}$. For $\bar{B}$ values close to unity, the effect on factor of safety is prominent; on the other hand, for values less than 0.8, the effects are smaller.

The curves in Fig. 10 are extrapolated back to zero time after drawdown to approximate the values of F for very rapid and complete drawdown. Compare these values of F with those in Fig. 9, and for a given $\bar{B}$ value. Checks are quite good for $\bar{B}$ less than 0.8. But for values of $\bar{B}$ greater than 0.8, the transient state analysis yields the lowest, i.e., most critical value.

The transient solutions are thus particularly useful in checking the effect of drawdown rate on the factor of safety against reservoir slope failure. Curves like those in Fig. 10 can be generated and used to predict the rates of "recovery" of slope stability from controlled drawdowns.

Fig. 10. Variation of minimum factor of safety with time for controlled rate drawdown

Fig. 11. Variation of phreatic surface with time for controlled drawdown

SUMMARY

The stability of reservoir slope is influenced in an important way by the magnitude and

rate of change in the adjacent reservoir water elevation. Computer solutions for such problems have been developed based upon the simplified Bishop's method of slices model.

Of particular interest is the analysis for a time-dependent reservoir lowering; the induced lowering of the phreatic surface in the adjacent slope; and the consequent effects on the slope stability. The relationship between the volume of water which flows out of the slope in a small period of time and the change in elevation of the phreatic surface is a partial differential equation. Solution of this equation defines the position of the phreatic surface with time, which, in turn helps to define the water pressures needed in the stability analysis.

Representative problems have been solved, and some of the findings are given below. The factor of safety in general increases with the sliding circle radius, keeping the center fixed. (It is not affected much by a low water table.) Very rapid and complete drawdown yields lower factors of safety, while the lowest factor of safety is developed for very rapid and complete drawdown and accompanying pore pressure increases. The reduction of F increases with increasing values of $\bar{B}$. For the time-dependent drawdown, the factor of safety increases in a nonlinear fashion with increasing time after drawdown. The lowest value is close to the value for the very rapid drawdown case, but may be lower. Such analyses may be applied to control drawdown rates in order to maintain an appropriate factor of safety against slope instability.

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NOTATION

- a =elevation of the free surface
- b =width of slice
- $\bar{B}$ =pore pressure parameter
- c_a =Mohr-Coulomb strength intercept
- F =factor of safety
- h =height of phreatic surface from base of slice
- $\bar{h}$ =average height of phreatic surface from base of slice
- h_0 =drawdown elevation of reservoir
- H =height of slope
- H_c =initial height of water in slope
- k_x, k_y =coefficient of horizontal and vertical permeability of soil
- n =porosity
- q =quantity of flow
- Q =rate of discharge
- R =elevation of the reservoir before drawdown
- S_a =available shearing force
- S_r =shearing force required for equilibrium
- t =time
- u =pore pressure
- U =water force
- V =volume

- W = total weight of slice
 W' = bouyant weight of slice
 W_w = weight of slice as if it were water
 X' = effective side force due to soil
 Y' = shear force on side of slice
 α = angle with horizontal at base of slice
 β = slope angle
 $\Delta\sigma_1, \Delta\sigma_3$ = change in major and minor principal stresses
 ϕ_a = Mohr-Coulomb strength angle

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