SIMPLE METHOD OF ANALYSIS FOR LIQUEFACTION OF SAND DEPOSITS DURING EARTHQUAKES

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ABSTRACT

Effects of irregular series of earthquake-induced cyclic stresses and the K_0 -value at the time of consolidation on the liquefaction resistance of sand are discussed by examining the results of several series of cyclic triaxial tests that have been performed at the University of Tokyo. Relationships are established in a graphical form between the applied shear stress and the residual pore water pressure, that is, the pore water pressure which remains after through application of irregular time histories of shear stress. On the basis of these relationships, with the effects of irregularity of wave form and the K_0 -value being incorporated, a practical method is proposed to evaluate pore water pressures as well as factor of safety against liquefaction that can develop in a horizontal deposit during an earthquake. The method is applied to analyze the liquefaction of a sand deposit in Niigata where a detailed investigation was recently carried out for cyclic strength of undisturbed samples.

Key words: earthquake, liquefaction, pore pressure, sand

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INTRODUCTION

Practical methods for evaluating whether or not a given soil deposit will develop liquefaction during an earthquake are the subject of major concern for those engineers working in seismically active regions in the world. A simple method meeting this requirement was first proposed by Seed and Idriss (1971) based on the detailed case history studies and the analysis of the cyclic strength data obtained in the laboratory. However, this method does not provide a means of evaluating a partial loss of stability of the ground resulting from a buildup of intermediate pore water pressure before a full development of liquefaction. Even when an earthquake shaking is not strong enough to produce complete liquefaction, a saturated soil deposit will give a rise in pore water pressure and lose partly its strength, and this could be a cause of hazardous failure for many civil engineering structures. Then, it becomes necessary to have a simple method that can be used for estimating to what extent pore water pressures build up and how far the stability is reduced in a sand deposit during an earthquake. It is the objectives of this paper to discuss this aspect of the problem and to present a simple method for assessing the stability of the deposit both in pre-liquefied and liquefied states.

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RESIDUAL PORE PRESSURE

In the complicated time history change of shear stress, it is always possible to locate a biggest spike where the maximum shear stress, $\tau_{\max}$, occurs. The overall amplitude of the irregular shear stress change may be evaluated in terms of the maximum shear stress thus defined. So, the maximum stress ratio defined as $\tau_{\max}/\sigma'_0$ could be the most expedient variable for representing the relative magnitude of the shear stress with reference to the initial confining pressure, σ'_0 .

When dynamic tests are carried out employing an irregular time history of load, it is expedient to repeat tests by changing the maximum stress ratio, $\tau_{\max}/\sigma'_0$. In the case of dynamic torsion shear tests, a test is performed first by setting the torsional load intensity at an appropriate level which is small enough not to induce liquefaction in the specimen throughout the entire duration of irregular loading. In the torsional test result shown in Fig. 1, the maximum stress ratio employed in the first test was 0.306. The time history used was the N-S component of the acceleration record obtained at the time of the Niigata earthquake in 1964. In the next test, the level of the torsional load intensity was raised a little and the load having the same wave form as before but with a greater amplitude was applied to a new specimen prepared under identical conditions. In the example demonstrated in Fig. 1, the maximum stress ratio used in the second test was 0.365. In this way, similar tests were repeated, each time with increased load level, until a state of liquefaction was eventually attained as shown in Fig. 1 (b) in which the induced pore pressure was equal to the initial confining pressure.

The observation of the pore water pressure behavior during each of the above tests indicates that the pore pressure increases gradually as the loading continues, but it stops increasing at the time when the maximum stress is encountered and remains almost constant

Fig. 1. Recorded accelerations, pore pressures, and torsional shear strains, as the stress ratio is increased ($D_r=55\%$)

Fig. 2. A relationship between the maximum stress ratio and residual pore pressure ratio for a wave form of the Niigata earthquake

thereafter although the specimen is still undergoing stress changes with lesser amplitude. It is, therefore, known that the wave shape of irregular loading after the passage of the maximum shear stress has substantially nothing to do with the pore pressure buildup. It is also known that the factors influencing the development of pore pressure are the magnitude of the maximum stress ratio, $\tau_{\max}/\sigma'_0$, and the wave form before the advent of the maximum shear stress. The maximum pore pressure that has been attained under a given irregular loading will be called residual pore pressure and denoted by u_r .

It will then be necessary to find a relationship between the maximum shear stress and the residual pore pressure for a given irregular loading. This relationship may be more conveniently expressed in terms of the maximum stress ratio, $\tau_{\max}/\sigma'_0$, and the residual pore pressure ratio, u_r/σ'_0 . By reading off the residual pore pressure value from the data as shown in Fig. 1, it is possible to obtain a plot of such relationship for the Niigata wave form as demonstrated in Fig. 2. In this type of plot, it is also possible to determine a critical maximum stress ratio at which the developed pore pressure becomes equal to the initial confining stress, that is, the maximum stress ratio at liquefaction, $\tau_{\max, l}/\sigma'_0$.

WAVE FORMS IN IRREGULAR LOADING

The shear stress induced in the level ground during an earthquake by the upward propagation of shear waves varies with time both in magnitude and frequency. It is expected that the wave forms in these irregular changes in shear stress could have some influence on the way in which pore water pressures are developed in saturated sand deposits. The previous study (Ishihara-Yasuda, 1975) has indicated, however, that the response of sand to pore water pressure buildup under random loading conditions did not differ appreciably although the load patterns were in full variety in detail, and the response could be classified roughly into two groups according to whether the load pattern has an appearance like constant-amplitude vibration or that like a single impulse. A rule was then suggested as illustrated in Fig. 3 to classify a given time history of shear stress change into either of vibration type or shock type loading. In this rule, only the wave shape prior to the passage of the maximum shear stress is considered meaningful, because as demonstrated before the shear stress changes after the maximum spike has passed have virtually nothing to do with the pore pressure buildup. Consider only the part of the time history which has the same sign as the maximum shear stress. If there exists, preceding the maximum, only one or two peaks whose amplitude is greater than 60% of the maximum, the wave form is referred to as being of shock type. Conversely, if more than three peaks having the amplitude greater than 60% of the maximum appear in the trace of time history on the side of the maximum stress, then the wave is referred to as being of vibration type.

Fig. 3. Classification of wave forms in irregular loading

MAXIMUM STRESS RATIO AND RESIDUAL PORE PRESSURE

Although the dynamic triaxial test possess a disadvantage that the pore pressure response is not symmetrical between compression and extension stroke (Ishihara-Yasuda, 1972), it still is the most useful tool to perform a comprehensive amount of tests to be used for practical purposes. In view of this, several series of dynamic triaxial tests were carried out using different wave forms to determine the relationship between the maximum stress ratio and the residual pore pressure ratio. Shown in Fig. 4 are the results of these tests in which the relationship is represented separately for each of two classes of wave forms defined above. In Fig. 4, the CM-test refers to a triaxial test in which the maximum stress in the irregular trace of axial loading is made to orient towards the triaxial compression side, whereas the EM-test refers to the test with the irregular trace of loading oppositely oriented. These tests aimed primarily at obtaining data for relatively small ranges of the maximum stress ratios. It should be noted that there is a lower limit in the maximum stress ratio below which no residual pore pressure can be generated whatsoever. For the relative density of 43% used in the present test, this lower limit was about 0.035. This value is found to coincide with the similar data obtained by Shibata (1970) using a triaxial test apparatus. In Fig. 4, average relationships are indicated by solid curves, which show that a greater stress ratio is always required in shock type of loading than in vibration type to raise a given amount of residual pore pressure. The average curves indicated in Fig. 4 will be used hereafter as the characteristic relationships between the stress ratio and residual pore pressure for the range of stress ratio smaller than that inducing liquefaction. For the stress ratio great enough to cause liquefaction, separate tests using a triaxial torsion device were conducted, as described in the following section, to define the effects of wave form more accurately.

Fig. 4. Relationship between maximum stress ratio and residual pore pressure ratio

LIQUEFACTION POTENTIAL UNDER UNIFORM AND IRREGULAR LOADING

Several series of dynamic torsion shear tests were conducted using different wave forms of acceleration records taken at the time of the Niigata (1964) and Tokachioki (1968) earthquakes. These wave forms are considered as being typical of those obtained on the sand deposits during fairly big earthquakes. The maximum stress ratios, $\tau_{max,i}/\sigma'_0$ required

to cause liquefaction in the specimens with a relative density of 55% were then determined for each of the two types of wave forms as defined above. These stress ratios are shown in Table 1 (Ishihara-Yasuda, 1975) and will be considered to represent the dynamic strength under irregular loading conditions. On the other hand, similar torsion shear tests were performed on specimens with identical relative density using stress cycles with constant amplitude. The stress ratio, τ_{dl}/σ'_0 , required to induce liquefaction under 20 cycles will be chosen as representing the dynamic strength under uniform loading conditions.

One of the ways of appraising the effect of randomness of the wave forms on the liquefaction potential would be to compare the stress ratio, $\tau_{\max, l}/\sigma'_0$ under irregular loading with the stress ratio, τ_{dl}/σ'_0 under 20 cycles of uniform loading. Then, the ratio $\tau_{dl}/\tau_{\max, l}$ was calculated for each of the wave forms used and demonstrated in Table 1. This ratio will hereafter be called reduction factor. Table 1 indicates that the reduction factor associated with the shock type of loading ranges from 0.50 to 0.63 with an average of 0.55, whereas it takes an average of 0.70 in the case of vibration type of loading. The average values of reduction factor as above will often be used whenever it is necessary to transform the results of uniform loading tests into the liquefaction potential which could have been obtained under random loading conditions. Although the stress ratios themselves as above may be subject to some errors pertinent to the particular details of the torsion shear device used, the reduction factor may offset the errors because it is defined as a ratio of two quantities having the same degree of inherent errors.

Table 1. Reduction factors for two classes of wave form

Type of wave	Site	Component	Max. acceleration (gal)	Reduction factor	
				Measured	Average
Shock type loading	Niigata***	N S	155	0.54	0.55
		E W	159	0.53	
	Muroran**	N S	95	0.63	
	Hachinohe*	N S	235	0.50	
Vibration type loading	Aomori**	N S	56	0.71	0.70
		E W	86	0.71	
	Hachinohe**	E W	30	0.68	

* Main shock, Tokachioki earthquake (1968)

** After shock, "

*** Main shock, Niigata earthquake (1964)

EFFECT OF THE COEFFICIENT OF EARTH PRESSURE AT REST

It has been recognized that the initial stress conditions as represented by the K_0 -value have a large influence on the resistance to liquefaction of saturated sands. Finn, Pickering and Bransdy (1971) showed that the effect of the K_0 -value at the time of anisotropic consolidation on the cyclic stress ratio causing initial liquefaction can be properly evaluated if the mean normal stress is used as an index for specifying overall confinement of the sand. In addition, data in support of this observation was also provided by Seed and Peacock (1971) using a simple shear device for overconsolidated sand specimens having different values of K_0 . Ishibashi and Sherif (1974) also confirmed this observation by means of a torsional shear test device. The mechanism by which initial K_0 -values influence the liquefaction of sand was studied by Ishihara and Li (1972) and Ishihara, Iwamoto, Yasuda and

Takatsu (1977) using a torsional shear device, who showed that when a saturated sand consolidated under K_0 -conditions is subjected to horizontal cyclic shear stresses with the lateral deformation perfectly confined, the lateral stress will change to produce an isotropic state of stress upon liquefaction. On the basis of this observation, it was shown that the cyclic stress ratio causing liquefaction in an isotropically consolidated sample, $(\tau_t/\sigma_v')_I$ is related with the cyclic stress ratio at liquefaction under an anisotropic consolidation, $(\tau_t/\sigma_v')_A$, in the following manner,

$$\left(\frac{\tau_t}{\sigma_v'}\right)_A = \frac{1+2K_0}{3} \left(\frac{\tau_t}{\sigma_v'}\right)_I \quad (1)$$

where, σ_v' is the vertical effective stress and τ_t the amplitude of cyclic shear stress.

CYCLIC TRIAXIAL STRENGTH OF LABORATORY PREPARED SAMPLES

The most widely used laboratory procedure to evaluate the liquefaction characteristics of sands is the stress-controlled cyclic triaxial strength test on laboratory prepared samples. In this type of tests, samples are generally prepared by what is called pouring method, where the sand is saturated initially in a container, sedimented through water into a water-filled forming mold, and then densified to a desired density by some means such as vibration. Although there is evidence suggesting that specimen preparation methods influence measured liquefaction behavior (Ladd, 1974; Mulilis, et al., 1977), the pouring method probably is the most reasonable duplication of the way in which natural sands are deposited under water. In view of this, the results of cyclic triaxial tests on sand samples prepared by the pouring method were collected from the literature and demonstrated in Fig. 5, together with the corresponding list of test details shown on Table 2. The most of the sands tested are fine uniform sands, and it appears likely that the difference in liquefaction resistance due to the variation in materials may be no more than the errors induced by testing details. The test results shown in Fig. 5 are in terms of the cyclic stress ratio causing initial liquefaction with a relative density of 50%. In case individual tests were run with relative densities other than 50%, the cyclic stress ratios were modified so as to correspond to those at the 50% relative density on the basis of the assumption that the stress ratio at liquefaction changes in direct proportion to relative density values.

Fig. 5. Summarized relationship between cyclic stress ratio and number of cycles to initial liquefaction

Table 2. Characteristics of uniform fine clean sands tested in triaxial test apparatus (Ref. Fig. 5)

Sands	Grading		Void ratio		References	Symbol
	D_{50}^*	U_c^{**}	$e_{\max}$	$e_{\min}$		
Sacramento river sand	0.20 (mm)	5.00	1.03	0.61	Seed-Lee (1966)	⊙
Monterey sand	0.63	1.10	0.83	0.53	Peacock-Seed (1968)	△
Ottawa sand	0.40	2.00	0.82	0.50	Finn et al (1971)	▽
Toyoura sand	0.17	1.44	0.96	0.64	Shibata-Yukitomo (1970)	□
Toyoura sand	0.20	1.65	0.99	0.61	Ogawa et al (1969)	●
Toyoura sand	0.17	1.44	0.99	0.61	Ishihara-Watanabe (1976)	▼
Niigata sand	0.50	1.90	1.01	0.59	Yoshimi-Ohoka (1972)	-----
Niigata sand	0.23	2.34	0.99	0.56	Ishihara-Watanabe (1976)	■
Ongishima sand	0.28	1.94	0.85	0.58	Saito et al (1974)	▲
Senjyu sand	0.28	1.8	—	—	Kuribayashi et al (1976)	-----
Tokyo Bay sand	0.15	3.0	—	—	Kuribayashi et al (1976)	-----

* Grain size corresponding to 50% passage by weight

** Uniformity coefficient

Although there exist scatters in the data, the average cyclic stress ratio required to induce initial liquefaction in 20 cycles may be read off from Fig. 5 as being 0.21 for the relative density of 50%. This value will be considered as a standard cyclic strength for uniform fine clean sands tested by cyclic triaxial test apparatus. Based on the assumption of proportionality between liquefaction strength and relative density, D_r , the following empirical relationship can be derived :

$$\sigma_{al}/(2\sigma'_0) = 0.0042 D_r \quad (2)$$

where σ_{al} is the amplitude of cyclic axial stress required to cause initial liquefaction under 20 cycles and σ'_0 is the initial confining pressure.

RELATIONSHIP BETWEEN RESIDUAL PORE WATER PRESSURE RATIO AND MAXIMUM STRESS RATIO

In order to evaluate the residual pore water pressure in a deposit during an earthquake, it is necessary to have a relationship between the residual pore water pressure ratio and the maximum stress ratio for a given sand. This can be established in the following fashion if the cyclic stress ratio causing liquefaction under 20 cycles of uniform loading, $\sigma_{al}/(2\sigma'_0)$, is made known. For a given value of cyclic stress ratio, $\sigma_{al}/(2\sigma'_0)$, the maximum stress ratio at liquefaction, $\tau_{\max, l}/\sigma'_0$ can be directly calculated by dividing $\sigma_{al}/(2\sigma'_0)$ either by 0.55 or 0.70 depending upon the type of wave form being considered. In the diagram plotting the relationship between the maximum stress ratio versus the pore water pressure ratio, the value of $\tau_{\max, l}/\sigma'_0$ thus determined is used to locate a point at liquefaction such

Fig. 6. Diagram illustrating a procedure for constructing a curve representing a relationship between the maximum stress ratio and residual pore pressure ratio

cyclic stress ratio given by Eq. (2) may be used. Incorporating the reduction factor listed in Table 1, the cyclic stress ratio can be rewritten in terms of the maximum stress ratio as shown in Table 3. The relationship between the maximum stress ratio and the pore water pressure ratio for the uniform fine sand can readily be established based on the data in Table 3 by following the procedure as described above. The relationships are demonstrated

Table 3. Maximum stress ratio for uniform fine sands

Wave form	Shock type	Vibration type
$\frac{\tau_{max, l}}{\sigma'_0}$	$0.00764 D_r$	$0.0060 D_r$

as the point A in Fig. 6. Then, a curved line having the same shape as that in Fig. 4 is drawn through Point A leftwards. This curve is demonstrated in Fig. 6 for illustration. Once this type of relationship is established, it becomes possible to evaluate the residual pore water pressure which would be produced under any given time history change in shear stress. For instance, given a value of the maximum stress ratio, a point can be located on the curved line in Fig. 6 such as Point B, whereupon it is easy to read off a residual pore water pressure ratio on the abscissa.

For a uniform fine clean sand the average

Fig. 7. Relationship between maximum stress ratio and residual pore pressure ratio (Standard curves for uniform fine clean sands)

in Fig. 7 for different values of relative densities.

The cyclic stress ratio as summarized in Fig. 7 is in terms of the stress ratio at isotropic consolidation. Since a soil element in a horizontal deposit is more or less anisotropically consolidated, it will be preferable to transform the stress ratio, $\tau_{\max}/\sigma'_0$, in Fig. 7, into the stress ratio in terms of the vertical effective stress, that is, $\tau_{\max}/\sigma'_v$. This transformation can easily be worked out by using the relationship established in Eq. (1). Although Eq. (1) was derived for the stress ratio large enough to bring about liquefaction, there are good reasons to believe that it also holds valid for any stress ratio smaller than that causing liquefaction. Then, the maximum stress ratio, $\tau_{\max}/\sigma'_v$, corresponding to each values of $\tau_{\max}/\sigma'_0$ shown in Fig. 7 can readily be calculated by simply multiplying the value of $\tau_{\max}/\sigma'_0$ by $(1+2K_0)/3$.

DETERMINATION OF MAXIMUM STRESS RATIO INDUCED BY EARTHQUAKES

In view of the fact that most of the data bank concerning recorded motions of earthquakes contains acceleration records obtained on the ground surface it will be convenient if time changes in shear stress within the deposit can be assessed from a knowledge of acceleration time history specified on the ground surface. The method to this effect was suggested by Seed and Idriss (1971), who deduced a formula similar to the following for the determination of the maximum shear stress at depth, z ,

$$\tau_{\max} = \frac{a_{\max}}{g} \cdot r_d \cdot (\gamma H + (z-H)\gamma_{sat}) \quad (3)$$

where $a_{\max}$ is the maximum ground acceleration, γ and γ_{sat} the unit weights of the soil above and below the ground water table, respectively, and H is the depth of the ground

Fig. 8. Stress reduction coefficient, r_d , versus depth

Fig. 9. Diagram showing $h(z/H)$ versus depth

water table. r_d in Eq. (3) is termed stress reduction coefficient which indicates the rate of decrease in shear stress with depth as compared to that calculated on the assumption that a soil column to the depth in question moves as a rigid body. The average curve for the stress reduction coefficient suggested by Seed and Idriss (1971) is shown in Fig. 8. Also plotted in this figure are the stress reduction coefficients that may be encountered in the deposit when it is put into resonant motion to depths, 10, 13, 15, and 20m. These curves are obtained in Eq. (A.7), (See Appendix).

The vertical effective stress, σ_v' , at depth z is given by

$$\left. \begin{aligned} \sigma_v' &= \gamma H + \gamma'(z - H) \\ \gamma' &= \gamma_{sat} - \gamma_w \end{aligned} \right\} \quad (4)$$

in which γ_w is the unit weight of the ground water. Then, by dividing τ_{max} in Eq. (3) by σ_v' in Eq. (4), one obtains

$$\frac{\tau_{max}}{\sigma_v'} = \frac{a_{max}}{g} \cdot r_d \cdot h\left(\frac{z}{H}\right) \quad (5)$$

where

$$h\left(\frac{z}{H}\right) = 1 + \frac{\frac{\gamma_w}{\gamma} \cdot \frac{z-H}{H}}{1 + \frac{\gamma'}{\gamma} \cdot \frac{z-H}{H}} \quad (6)$$

The function $h(z/H)$ is presented graphically in Fig. 9 versus the depth normalized to H for typical values of γ and γ_{sat} . Thus, by reading off the values of r_d and $h(z/H)$ from Figs. 8 and 9, respectively, it becomes possible to assess for each depth of the deposit concerned the value of τ_{max}/σ_v' that may be induced during an earthquake.

PREDICTION OF RESIDUAL PORE WATER PRESSURE AND LIQUEFACTION

Now that the procedures for representing the cyclic strength of soils are known, together with the method of assessing the cyclic shear stress for a given time history of an earthquake motion, it becomes possible to develop an analysis in which the likelihood of liquefaction or residual pore water pressures that have built up at the end of the main shock of an earthquake can be predicted. The procedures for making the analysis are outlined in a flow chart shown in Fig. 10. Basically, three classes of information as described below must be made known or assumed, if necessary, for this purpose.

In-situ Soil Properties

It is most preferable to conduct cyclic triaxial shear tests on undisturbed samples of soils secured in-situ either by block sampling or some elaborate sampling techniques developed most recently. If it is impossible to conduct tests on undisturbed samples, the cyclic strength may be evaluated based on the results of some indirect field tests such as Standard penetration test and Dutch cone penetration test. Whichever methods are adopted, the cyclic strength should be evaluated in terms of the cyclic stress ratio required to cause initial liquefaction under the application of 20 cycles of constant-amplitude cyclic loading in triaxial shear tests.

Field Conditions

The resistance of in-situ soil deposits to pore water pressure buildup depends on the depth of the ground water table and the in-situ coefficient of earth pressure at rest. These values become necessary when effective confining stresses are calculated at each depth of the deposit in question. At present, it appears generally difficult to precisely measure the

K_0 -value in the field, and therefore, this value may well be assumed to be 0.5 for normally consolidated uncompacted deposit. In case soil deposits are stabilized by means of compaction pile or vibrofloatation, the larger value should be given to the in-situ coefficient of earth pressure at rest.

Time History of Acceleration

The time history of acceleration must be given at the ground surface for computing the shear stresses induced in the ground during a given earthquake. For the simple method described in this paper, it is necessary to specify only the maximum acceleration and the type of waves (Shock or Vibration type) as classified by the rule set up previously.

With all the information as above having been known or specified, it is now possible to go ahead along the flow chart described in Fig. 10.

STEP 1

First of all, the cyclic stress ratio $\sigma_{dl}/(2\sigma'_0)$ causing liquefaction under 20 cycles of uniform loading must be converted to the stress ratio in terms of the maximum shear stress, $\tau_{max, l}/\sigma'_0$. The conversion can be done simply by dividing the cyclic stress ratio by the reduction factor, R_f , which is either 0.55 or 0.70 depending upon whether the given wave is of shock type or vibration type, respectively. Then, by shifting the standard characteristic curve demonstrated in Fig. 7 vertically so that its point on $u_r = \sigma'_0$ is made to coincide with the maximum stress ratio, $\tau_{max, l}/\sigma'_0$, already determined, it is possible to establish a relationship between the maximum stress ratio, τ_{max}/σ'_0 and the residual pore pressure ratio for the soil in question. By simply dividing the ordinate of the curve describing the above relationship by $3/(1+2K_0)$ a modified relationship incorporating the K_0 -effect through Eq. (1) can be obtained between τ_{max}/σ'_0 and u_r/σ'_0 . This completes all the data reduction procedures with respect to the cyclic strength of the soils being considered.

STEP 2

Next step consists of estimating the magnitude of maximum stress ratio, τ_{max}/σ'_0 , that may be applied to the soil element in the deposit when it is subjected to a shaking due to the earthquake as specified above. Based on the information concerning the depth of the ground water table and the unit weight of the soils, the maximum stress ratio τ_{max}/σ'_0 can be computed by Eq. (5) for each depth of the deposit, whereby the stress reduction coefficient shown in Fig. 8 must be referred to.

STEP 3

By locating the computed maximum stress ratio on the curve representing the relationship between τ_{max}/σ'_0 and u_r/σ'_0 , it is now possible to determine the residual pore water pressure ratios for each depth of the deposit. At those depths where u_r/σ'_0 liquefaction develops and otherwise the magnitude of developed pore water pressures is known.

If comparison is made between the calculated maximum stress ratio and the maximum

Fig. 10. Flow chart for evaluating residual pore pressures and factor of safety against liquefaction

stress ratio inducing x percentage of pore water pressure buildup, it is possible to define a factor of safety F_x for a given degree of pore water pressure buildup, as follows,

$$F_x = \frac{\text{maximum stress ratio inducing } x \text{ percent pore water pressure}}{\text{maximum stress ratio due to earthquake shaking}} \quad (7)$$

Of particular interest would be the factor of safety against liquefaction, F_l , which may be defined as

$$F_l = \frac{\tau_{\max, l} / \sigma_v'}{\tau_{\max} / \sigma_v'} \quad (8)$$

EXAMPLE PROBLEMS

Homogeneous Deposit

Suppose a homogeneous sand deposit having a constant relative density and K_0 -value of 0.5 throughout the depth. The ground water table is assumed to be at a depth of 1.0m. It is further assumed that the sand has the cyclic strength of 0.21 in terms of the cyclic stress ratio at a relative density of 50% under 20 cycles of uniform loading. So, the typical pore water pressure characteristics as shown in Fig. 7 can be used for computing residual pore pressures. Suppose this deposit is shaken by an earthquake having a maximum ground surface acceleration of $a_{\max}/g=0.15$ or 0.20 with the wave form of shock type. The residual pore water pressures that are computed on the basis of the above assumptions by using the procedures described in the foregoing pages are demonstrated in Fig. 11. Also shown in Fig. 12 are the distribution of factor of safety against liquefaction as computed by Eq. (8). Fig. 11(a) and 12(a) show that when the deposit consisting of a sand with 50% relative density is subjected to a shock type shaking with a maximum ground surface

Fig. 11. Residual pore pressure distribution in uniform deposits having constant relative densities

Fig. 12. Distribution of factor of safety against liquefaction in uniform deposits having constant relative densities

acceleration of 0.15 liquefaction develops at depths between 6 and 10m but with an increased relative density of 60% nowhere liquefaction occurs. If the same deposit is shaken with a shock type motion with an increased maximum ground acceleration of 0.20, the deposit must be as dense as 70% relative density in order to be safe against liquefaction. It should be noted that there always are some depths in the deposit where pore water pressures are most liable to develop. This is approximately 5 to 10m from the ground surface. The shallow portion of the deposit is less liable to liquefaction because of the increasing effect of the confining stress due to the soil weight above the ground water table. As indicated by the stress reduction coefficient in Fig. 8, the shear stress induced by earthquake shaking is reduced with depth of the deposit. This accounts for the decreasing susceptibility to liquefaction below the depth of about 15m as exemplified in Figs. 11 and 12.

Liquefied Deposit in Niigata

Using a large diameter sampler developed recently (Ishihara-Silver, 1977), undisturbed lumps of loose sands (20cm in diameter and 80cm long) were secured from bored holes drilled near the Shinano river in the city of Niigata where liquefaction of the ground developed extensively at the time of the 1964 Niigata earthquake. The Standard Penetration Resistance measured at this site is shown in Fig. 13. After the big lump samples have been completely drained, small brass tubes 5cm in diameter and 10cm long were inserted into the big sand block to provide small specimens for cyclic triaxial testing. The specimens were frozen in the field and carried to the laboratory. Cyclic triaxial shear tests were performed under the confining stresses corresponding to the in-situ overburden pressure. The cyclic stress ratio, $\sigma_{dl}/(2\sigma'_0)$, required to cause initial liquefaction, 5% and 10% double amplitude axial strains, were determined for each lump sample taken from different depths.

Fig. 13. Soil profile at a site near the Shinano river in the city of Niigata

Fig. 14. Cyclic stress ratio, $\sigma_{al}/(2\sigma'_0)$, causing liquefaction under 20 cycles (A site near the Shinano river in the city of Niigata)

Fig. 15. Computed distribution of residual pore water pressure ratio based on the acc. records in the 1964 earthquake (A site near the Sinano river in the city of Niigata)

Fig. 16. Computed factor of safety against liquefaction based on the acc. records in the 1964 earthquake (A site near the Shinano river in the city of Niigata)

Fig. 14 shows the distribution, through the depth of the sand deposit, of the cyclic stress ratios at the critical conditions as above under 20 cycles of load application.

Although the site investigated is about 2km away from the place where the acceleration records were obtained in the 1964 earthquake, the recorded motion will be used without modification for the liquefaction analysis of the site being considered. The cyclic stress ratios demonstrated in Fig. 14 and the recorded ground surface accelerations were used as input data for the scheme of liquefaction analysis as outlined in Fig. 10. A close observation of the acceleration record shown in Fig. 1(a) indicates that an abrupt increase occurred in the period of the motion just before the maximum acceleration is encountered. This fact appears to indicate that the stiffness of the sand deposit decreased as a result of gradual increase in pore water pressure, and the natural frequency of the deposit became equal to the predominant frequency of the base motions of the earthquake shaking. In other words, it may well be said that the liquefaction was accompanied by the occurrence of resonance in the deposit. Judging from the penetration resistance shown in Fig. 13, the depth of the surface layer having experienced the resonance appears to be about $h=13\text{m}$. Therefore, the stress reduction coefficient at resonance corresponding to this depth, shown in Fig. 8, will be used to compute the stress ratio to which the soil at this site must have been subjected during the 1964 earthquake. The result of liquefaction analysis made in accordance with the procedures outlined in Fig. 10 is shown in Figs. 15 and 16, where it is noted that liquefaction developed at all depths investigated except around the depth 4.5m. Visual observation of the large diameter sample showed that the sand changed its color from brown to dark gray around 4.5m depth, indicating that the historical origin of the deposit was apparently different between above and below this depth. Investigation of old literature record has disclosed that the site investigated had been the flood plain of the Shinano river and was filled around 1955. It may, therefore, be speculated that the hard deposit at depth 4.5m must be the ground surface soil which had existed before it was overlaid with the blown sand in 1955. In fact, it should be mentioned that, in the large diameter sample, several strips of the dark gray sand was visible through the vertical cross section of the brown compacted sand, indicating that the liquefied dark gray sand had intruded into the overlying brown sand by the upward seepage flow after the liquefaction. This observation, together with the historical records of the land formation, seems to substantiate the validity of the analysis result as demonstrated in Fig. 15 and 16.

CONCLUSIONS

By incorporating many laboratory test results as to the cyclic triaxial tests on saturated samples of sands, a relationship that may be claimed as an average was established between the cyclic stress ratio and the residual pore water pressure. This relationship permits pore water pressures to be assessed that will develop in a horizontal sand deposit during a given shaking of earthquake. Also, it was made possible to evaluate factor of safety against liquefaction through the depth of a deposit. An example problem in which an uniform deposit with 50% relative density is subjected to a shaking with a maximum horizontal acceleration of 0.15g showed that liquefaction occurs at depths between 6 and 10 meters. Also, an analysis of a liquefied site in the city of Niigata showed that the river side deposit is vulnerable to liquefaction if an earthquake similar to that in 1964 hits the site again.

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APPENDIX

Suppose a horizontal deposit is subjected to a horizontal motion,

$$u = u_0 \cos\left(\frac{\omega z}{V_s}\right) \cdot e^{i\omega t} \quad (\text{A.1})$$

where u denotes the horizontal displacement, u_0 the displacement at the ground surface, V_s the velocity of shear wave propagation, and ω is the angular frequency of motions. The horizontal shear stress, τ , at depth, z , is calculated from Eq. (A.1) as follows.

$$\tau = \mu \frac{\partial u}{\partial z} = -\frac{u_0 \mu \omega}{V_s} \sin \frac{\omega z}{V_s} \cdot e^{i\omega t} \quad (\text{A.2})$$

where μ is the shear modulus of soil. The acceleration on the ground surface is obtained by differentiating Eq. (A.1) twice with respect to time, and by putting, $z=0$,

$$\frac{\partial^2 u}{\partial t^2} = -\omega^2 u_0 \cdot e^{i\omega t} \quad (\text{A.3})$$

If the soil column above the depth, z , is assumed to move as a rigid mass, the shear stress τ_d acting on the bottom of this column must be equal to the acceleration on the ground surface times the mass of the column.

$$\tau_d = -\omega^2 u_0 \rho z \cdot e^{i\omega t} \quad (\text{A.4})$$

Then, the stress reduction coefficient is obtained by dividing Eq. (A.1) by Eq. (A.4).

$$\frac{\tau}{\tau_d} = \frac{\tau}{u_0 \omega^2 \rho z} = \frac{\sin\left(\frac{\omega z}{V_s}\right)}{\left(\frac{\omega z}{V_s}\right)} \quad (\text{A.5})$$

If the surface layer with a thickness, h , is put into resonance, then the angular frequency, ω_0 , corresponding to the first mode of vibration is determined from Eq. (A.1),

$$\omega_0 = \frac{\pi}{2} \cdot \frac{V_s}{h} \quad (\text{A.6})$$

Therefore, the stress reduction coefficient at the time when resonance develops in the deposit to a depth, h , is given by introducing ω_0 into Eq. (A.5).

$$r_d = \frac{\tau}{u_0 \omega_0^2 \rho z} = \frac{\sin\left(\frac{\pi z}{2h}\right)}{\frac{\pi z}{2h}} \quad (\text{A.7})$$

This relationship is plotted in Fig. 8.

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