A METHOD FOR DETERMINATION OF ANISOTROPIC COEFFICIENTS OF PERMEABILITY AND SPECIFIC STORAGE FROM PUMPING TEST DATA

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ABSTRACT

The approximate solution of unsteady groundwater flow toward a partially penetrating well in an anisotropic confined aquifer is described and a method for analyzing field data with a partially penetrating well in an anisotropic confined aquifer is given.

A distinction of this method is that anisotropic coefficients of permeability can be easily determined by using graphical techniques. An application of this method is illustrated by analyzing data from groundwater basins in Okayama City in Japan.

Key words: anisotropy, ground water, in-situ test, permeability, pumping test

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INTRODUCTION

To obtain the formation constants from pumping test data, Theis' Method or Jacob's Method are often used. Recently The applications of the F.E.M. and F.D.M. to analyze seepage problems have been developed and these methods easily take into account the anisotropy of permeability coefficients. But there are very few works on analyzing pumping test data with considering the anisotropy.

It is helpful to use the partially penetrating well for taking into account the anisotropy in the analysis since, as illustrated in Fig. 1, water moving toward the pumping well has to converge in some manner into the open well from all of the parts of the aquifer. This diversion of flow-lines from the horizontal direction leads to a more complicated pressure distribution pattern around the pumping well comparing with the case of a completely penetrating well. However these flow-lines include the vertical component of velocity, the anisotropy of permeability can be taken into account by considering the effects of partial penetration.

Hantush (1962, a, b) derived formulas for the drawdowns in observation wells and surface-piercing piezometer near a partially penetrating well in a confined aquifer, and compared his results with field tests. A thermal conduction model by Javandell and Witherspoon (1967) verified Hantush's analytical results. However a method how to determine anisotropic coefficients of permeability has not been given yet in their works.

In this paper, a method for determining anisotropic coefficients of permeability and

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Fig. 1. Partially penetrating wells in a confined aquifer

specific storage is given by analyzing pumping test data with a partially penetrating well in a confined aquifer.

DRAWDOWN EQUATION AND SOLUTION

To analyzing the problem of flow toward a partially penetrating well, the following conditions are prescribed.

- (1) The aquifer is homogeneous, anisotropic and extends infinitely with impermeable clay layers above and below.
- (2) The aquifer shall also be considered to be horizontal and water saturated at all times.
- (3) The permeability coefficients of main aquifer in the horizontal and vertical directions have different, but constant values, k_r , k_z respectively.
- (4) The well is of a vanishingly small radius and has the discharge at constant rate.

The differential equation that describes the fluid movements in porous media is given by

$$k_r \frac{\partial^2 h}{\partial r^2} + k_r \frac{1}{r} \frac{\partial h}{\partial r} + k_z \frac{\partial^2 h}{\partial z^2} = S_s \frac{\partial h}{\partial t} \quad (1)$$

where r and z are coordinates as shown in Fig. 1, t is time, and S_s is the specific storage. Then the hydraulic head (h) is a function of r , z , and t .

The initial and boundary conditions to be imposed on the solution are as follows :

$$h(r, z, 0) = H \quad (\text{head initially constant}) \quad (2)$$

$$h(\infty, z, t) = H \quad (\text{head at infinity remains constant})$$

$$\left. \begin{aligned} \frac{\partial h}{\partial z}(r, 0, t) &= 0 \quad (\text{no flow across upper boundary}) \\ \frac{\partial h}{\partial z}(r, b, t) &= 0 \quad (\text{no flow across lower boundary}) \end{aligned} \right\} \quad (3)$$

$$\lim_{r \rightarrow 0} 2\pi k_r r \int_0^l \frac{\partial h}{\partial r} dz = -Q$$

$$(\text{flow rate into a well with zero radius remains constant}) \quad (4)$$

Hantush (1962, a) studied this problem for a more complicated situation where there was also clay leakage into the aquifer that was being pumped. By imposing the condition of no clay leakage, i.e., Eq. (3), the Hantush's solution can be simplified to

$$\zeta^* = W(u_r) + f(u_r, r^*/b, l/b, z/b) \quad (5)$$

where

$$r^* = \sqrt{k_z/k_r} r, \quad \zeta^* = 4\pi k_r b \zeta / Q, \quad \zeta = H - h, \quad u_r = (r^*)^2 / 4(k_r/S_s)t \quad (6)$$

and ζ is the drawdown of piezometric surface at any time and at any point in aquifer, b is the thickness of aquifer. Noting that

$$W(u_r) = \int_{u_r}^{\infty} \exp(-\omega) / \omega d\omega \quad (\text{the well function}) \quad (7)$$

and

$$f(u_r, r^*b, l/b, z/b) = (2b/\pi l) \sum_{n=1}^{\infty} (1/n) \cos(n\pi z/b) \sin(n\pi l/b) \\ \times \int_{u_r}^{\infty} \exp[-\omega - (n\pi r^*)^2 / (4\omega b^2)] d\omega / \omega \quad (8)$$

For relatively large values of time, the function $f(u_r, r^*/b, l/b, z/b)$ can be replaced by $2K_0(n\pi r^*/b)$ for all practical purpose in which case Eq. (8) becomes independent of time as follows:

$$f(u_r, r^*/b, l/b, z/b) \doteq f_s(r^*/b, l/b, z/b) = (4b/\pi l) \sum_{n=1}^{\infty} (1/n) \\ \times K_0(n\pi r^*/b) \cos(n\pi z/b) \sin(n\pi l/b) \quad (9)$$

where K_0 is the zero-order modified Bessel function of the second kind.

Similarly for relatively large values of time, the function Eq. (7) becomes

$$W(u_r) \doteq \ln(t/r^{*2}) - \ln(S_s/2.25k_r) \quad (10)$$

From Eqs. (9), (10), the approximate solution of drawdown (ζ) becomes for relatively large values of time

$$\zeta = (Q/4\pi k_r b) [\ln(t/r^{*2}) - \ln(S_s/2.25k_r) + f_s(r^*/b, l/b, z/b)] \quad (11)$$

if an aquifer is isotropic ($k_r = k_z = k$, i.e. $r = r^*$), Eq. (11) becomes Hantush' solution (1962, a), moreover if the pumping well completely penetrated ($l = b$), Eq. (11) becomes equal to the Jacob's approximate solution. Eq. (11) is rewritten with common logarithms

$$\zeta = (2.30Q/4\pi k_r b) [\log(t/r^2) - \log(S_s k_z \exp(-f_s)/2.25k_r^2)] \quad (12)$$

It is evident that the third term in Eq. (11) is the effect of partial penetration of the pumping well.

METHOD FOR DETERMINATION OF ANISOTROPIC COEFFICIENTS OF PERMEABILITY AND SPECIFIC STORAGE

To determine the formation constants, it is necessary to use the following values.

- (a) Average pumping rate (Q)
- (b) Thickness to a confined aquifer (b)
- (c) Radial distances of each observation well (r)
- (d) Penetrations of pumping well (l) and observation wells (z).
- (e) Head that are measured in observation wells at some distance from the pumping well (h).

For relatively large values of time, the drawdown is given by Eq. (12) in which the ultimate semilogarithmic straight line forms. Because the second term of this equation is constant with time. It is clear that Jacob's method can be applied if the numerical values of f_s can be obtained.

The procedure to determine the formation constants is as follows.

Horizontal Permeability Coefficient (k_r)

Plotting pumping data obtained in each observation well on the common semilogarithmis

Fig. 2. Semi-log plot of drawdown data

paper, the ultimate straight lines are gotten for relatively large values of time. If the confined aquifer is homogeneous these lines become parallel as shown in Fig. 2. To obtain the slope ($m = \Delta\zeta/\text{cycle}$) of these lines the horizontal permeability can be calculated from Eq. (12)

$$k_r = 2.30Q/4\pi bm \quad (13)$$

Vertical Permeability Coefficient (k_z)

Extending these lines to the zero-drawdown axis, one can obtain their time intercepts ($\delta_i = t/r_i^2$) for each observation well on the zero-drawdown axis as shown in Fig. 2.

From Eq. (12) these values of δ_i are described

$$\begin{aligned} \delta_1 &= \alpha \exp(-f_{s1}) \\ \delta_2 &= \alpha \exp(-f_{s2}) \\ \delta_3 &= \alpha \exp(-f_{s3}) \end{aligned} \quad (14)$$

where $\alpha = S_s k_z / 2.25 k_r^2$.

The ratio of above values can be written

$$\begin{aligned} \beta_{12} &= \delta_2 / \delta_1 = \exp(f_{s1} - f_{s2}) \\ \beta_{13} &= \delta_3 / \delta_1 = \exp(f_{s1} - f_{s3}) \end{aligned} \quad (15)$$

Interpolating known factors ($l/b, z/b, r/b$) in Eq. (9), one can prepare a graph of f_s versus k_z/k_r for each observation well as shown in Fig. 3. Since in most anisotropic aquifers, the horizontal coefficient of permeability is larger than the vertical one, then can be treated as $k_z/k_r < 1$.

Calculating the values of $\exp(f_{s1} - f_{sj})$ versus k_z/k_r the graph of β_{ij} versus k_z/k_r can be gotten as shown in Fig. 4.

From the gotten values of time intercepts (δ_i) and in Eq. (15) the point of β_{ij} can be easily decided on the ordinate of the graph, as a result, the ratio (a) of k_z/k_r can be determined. Having obtained the horizontal conductivity and the ratio k_z to k_r , the vertical conductivity k_z may be calculated from

$$k_z = a k_r \quad (16)$$

Specific Storage (S_s)

By using the ratios (a) of k_z to k_r in Fig. 3, one can get a value of f_{si} for each observation well and determined the specific storage (S_{si}) from following equation

$$S_{si} = 2.25(k^2/k_z)\delta_i \exp(f_{si}) \quad (17)$$

Fig. 3. Relation of f_s versus k_z/k_r Fig. 4. Relation of β versus k_z/k_r

EXAMPLE ANALYSIS OF PUMPING TEST DATA

The following discussion is on example calculations for this method given above. The pumping test data are taken from a real aquifer project that is located in Okayama city in Japan. The geologic conditions obtained from well logs are shown in Fig. 5. The water level of the sand-gravel layer under G.L. -13m is different from that of the upper gravel layer; therefore the sand-gravel layer is revealed a confined aquifer. The thickness of the sand-gravel layer is $b=50$ m.

The depth of penetration in each observation well is $z=20$ m and that of penetration in the pumping well is $l=25$ m. The pumping test was performed on this project using an average rate of $Q=7 \times 10^3 \text{ cm}^3/\text{sec}$ for a period of 30 hours. Water levels were measured in observation wells.

The parameters l/b , z/b are obtained for the pumping well and the observation wells

$$l/b = 25/50 = 0.5,$$

$$z/b = 20/50 = 0.4$$

Fig. 5. Geologic conditions of the field

and (r/b) can be calculated with radial distances from the pumping center to each well

$$\text{No. 1} \quad r_1/b = 5.0/50 = 0.1$$

$$\text{No. 2} \quad r_2/b = 8.7/50 = 0.17$$

$$\text{No. 3} \quad r_3/b = 15/50 = 0.3$$

Fig. 6 shows the result of pumping data plotted on the semilogarithmic paper. Horizontal

Fig. 6. Semi-log plot of drawdown data

Fig. 7. Relation of f_s versus k_z/k_r

Fig. 8. Relation of β versus k_z/k_r

permeability can be determined from Eq. (13)

$$k_r = 2.30Q/4\pi bm = (2.30 \times 7.0 \times 10^3)/(4 \times 3.14 \times 5.0 \times 10^3 \times 40) \\ = 6.41 \times 10^{-3} \text{ (cm/sec)}$$

The values of time intercepts (δ_i) for each observation well are obtained as follows :

$$\delta_1 = 2.3 \times 10^{-7} \text{ sec/cm}^2$$

$$\delta_2 = 6.3 \times 10^{-7} \text{ sec/cm}^2$$

$$\delta_3 = 1.5 \times 10^{-6} \text{ sec/cm}^2$$

From these values the ratios (β_{ij}) are

$$\beta_{12} = \delta_2/\delta_1 = 2.74$$

$$\beta_{13} = \delta_3/\delta_1 = 6.52$$

Interpolating known factors (l/b , z/b , r_i/b) in Eq. (9), Fig. 7 can be prepared for each observation well, and the graph of β_{ij} versus k_z/k_r can be easily gotten as shown in Fig. 8. By using values of β_{12} , β_{13} , the ratio (a) of k_z to k_r can be determined on Fig. 8, i.e. $a = k_z/k_r = 0.105$, then the vertical permeability coefficient is calculated with Eq. (16) as following,

$$k_z = 0.105 \times 6.41 \times 10^{-3} \text{ cm/sec} = 6.73 \times 10^{-4} \text{ cm/sec}$$

Deciding the ratio (a) on the abscissa in Fig. 7, the value of f_{st} for each observation well and the specific storage (S_{st}) are determined from Eq. (17). The average value of S_s is

$$S_s = 9.73 \times 10^{-7} \text{ cm}^{-1}$$

CONCLUSIONS

A method of analyzing field data with a partially penetrating well in an anisotropic aquifer is given. Since this method is an application of Jacob's method which has been commonly used and it is possible to calculate numerically the values of the second term in Eq. (5), that is, Eq. (9), anisotropic coefficients of permeability and specific storage can

be easily determined.

The conclusions obtained in this paper are as follows :

(1) The unsteady analytical solution for groundwater flow to a partially penetrating well in an anisotropic confined aquifer is shown.

(2) By using this solution a method is provided to evaluate and easily to determine the anisotropic coefficients of permeability and the specific storage.

NOTATION

a =ratio of k_z to k_r

b =thickness of aquifer, dimension L

$f = f(u_r, r^*/b, l/b, z/b) = (2b/\pi l) \sum_{n=1}^{\infty} (1/n) \cos(n\pi z/b) \sin(n\pi l/b) W(u_r, n\pi r^*/b)$

$f_s = f_s(r^*/b, l/b, z/b) = [f, \text{ with } 2K_0(n\pi r/b) \text{ replacing } W(u_r, n\pi r^*/b)]$

h =hydraulic head, dimension L

H =initial hydraulic head, dimension L

k_r =horizontal coefficient of permeability, LT^{-1}

k_z =vertical coefficient of permeability, LT^{-1}

l =depth of penetration of the pumped well, L

m =slope of the ultimate straight line which is constructed by plotted pumping test
($=\Delta\zeta/\text{cycle}$)

Q =constant well discharge, L^3T^{-1}

r =radial distance measured from center of well, L

$r^* = r\sqrt{k_z/k_r}$

S_s =specific storage (volume of water released from storage by a unit volume of the aquifer under a unit head decline), L^{-1}

t =time since pumping started, T

$u_r = r^{*2}/4(k_r/S_s)t$

$W(u_r) = \int_{u_r}^{\infty} \exp(-\omega)/\omega d\omega$ (the well function)

$\alpha = S_s k_z / 2.25 k_r^2$

$\beta_{ij} = \delta_j / \delta_i = \exp(f_{si} - f_{sj})$

δ_i =time intercept extend the ultimate straight line to the zero-drawdown axis, TL^{-2}

ζ =drawdown in a observation well, L

ζ^* =dimensionless drawdown ($=4\pi k_r b \zeta / Q$)

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